CONTENTS

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U-QA

1 Averages

I. INTRODUCTION

If a player has scored 3, 112, 42 and 56 runs in his first four matches, how many runs can he be expected to score when he goes out to bat in the next match? The answer is around 53 and is called his average score.

By definition, an average is the most likely middle value of a data set. It is the value that each element of the set would take if all the elements of the set were to be the same. The average of the elements of a set can be calculated as the sum of all the values in the set divided by the total number of values.

$$\text{Average} = \frac{\text{Sum of all values in the data set}}{\text{Number of values in the data set}}$$

Example 1: A group of 5 friends scored 10, 12, 16, 20 and 18 in a class test. Find the average score of the students.

Solution: $\text{Average} = \frac{10 + 12 + 16 + 20 + 18}{5} = 15.2$

Hence, the average score is 15.2.

This implies that if any student from this class now takes the exam, he/she can be expected to obtain approximately 15 marks in that test.

☞ REMEMBER:

- If the value of each item in a group is increased/decreased by the same value x, then the average of the group also increases/decreases by x. For instance, if the income of each person in a group increases by Rs. 10, the average income of the group also increases by Rs. 10. This is valid only when the value of each item increases/decreases by the same amount.
- If the average age of a group of people is x years, then their average age after n years will be $(x + n)$ and their average age n years ago would have been $(x - n)$ years. This is because with each passing year, each person's age increases by 1 and vice versa.
- If the value of each item in a group is multiplied/divided by the same value x (where $x \neq 0$ in the case of division), then the average of the group also gets multiplied/divided by x.
- The average of a group always lies between the smallest value and the largest value in that group.
- If the average of a set is x, and an element having a value n is added, such that $n > x$, then the average of the new set is greater than x. On the other hand, if $n < x$, then the average of the new set is less than x.
- Conversely, if the average of a set is x, and an element having a value n is removed from the set, such that $n > x$, then the average of the new set is less than x. On the other hand, if $n < x$, then the average of the new set is greater than x.

Example 2: In a class, the average weight of students is 60 kg. If a student weighing 68 kg joins the class, the average weight increases by 1 kg. How many students were there in the class initially?

Solution: Let n be the original number of students in the class.

Hence, the total weight of all the students in the class was $60n$.

Now, when one student weighing 68 kg joins the class, the new average becomes 60 + 1 = 61 kg and the total weight of the students becomes $60n + 68$.

$$\therefore \frac{60n + 68}{n + 1} = 61$$

$\therefore 60n + 68 = 61n + 61$

$\therefore n = 7$

Hence, there were 7 students in the class initially.

Example 3: The average age of 5 students of a class is 16 years. Two new students, aged 20 and 18 years, are added to the group. What will be the new average age of the group?

Solution: Total age of 5 students = 5 × 16 = 80 years
After two students are added, age of the group becomes = 80 + 20 + 18 = 118 years.
So, new average age = 118/7 = 16.85 years

Example 4: The average weight of 10 oarsmen in a boat increases by 1 kg when one of the men who weighs 70 kg is replaced by a new man. What is the weight of the new man?

Solution: Let the original average weight of the 10 oarsmen be x kgs.
So, the total weight of the 10 oarsmen is $10x$ kgs.
After the replacement of an oarsman weighing 70 kg by another oarsman, the new average weight becomes $(x + 1)$ kg.
So, the new total weight of the group = $10(x + 1)$.
Let the weight of the new man be y kg.
So,

$$\therefore x + 1 = \frac{10x - 70 + y}{10}$$

$\therefore 10x - 70 + y = 10x + 10$

$\therefore y = 80$ kg

Thus, the weight of the new man is 80 kg.

Alternatively,

Since one particular oarsman is replaced by another, the total number of oarsmen remains the same i.e. 10.
Since the average weight of 10 men increases by 1 kg, the total weight of the group increases by 10 × 1 = 10 kg
Since the weight of the remaining 9 men remains constant, the weight of the new man = 70 + 10 = 80 kg

Example 5: The average weight of a school football team (consisting of 22 members including a goal keeper) decreases by half a kilogram if the goal keeper is not included. What is the goal keeper's weight, if the average weight of the team initially was 60 kg?

Solution: The average weight of the team (with the goalkeeper) is 60 kg.
So, the total weight of the team = 60 × 22 = 1320 kg
The average weight of the team (without the goalkeeper) is 59.5 kg.
The total weight of this team = 59.5 × 21 = 1249.5 kg
∴ Weight of goal keeper = 1320 – 1249.5 = 70.5 kg

Example 6: (CSAT 2011) A student on her first 3 tests received an average score of N points. If the student exceeds her previous average score by 20 points on her fourth test, then what is the average score for the first 4 tests?

(a) $N + 20$ (b) $N + 10$ (c) $N + 4$ (d) $N + 5$

Solution: Average for the first three tests = N
∴ The sum of scores in the first three tests = $3N$
Score in the fourth test = $N + 20$

$$\therefore \text{Final average} = \frac{\text{Sum of scores in four tests}}{4}$$

$$= \frac{3N + N + 20}{4} = N + 5$$

Hence, **option d.**

II. OTHER CONCEPTS

A. ARITHMETIC MEAN (AM)

Arithmetic Mean is the representative value of a given set of values. It is the standard average (seen above), often also called the "mean". It is calculated by dividing the sum of all the values by *the total number of values.*

$$\text{AM} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

Example 7: The profits registered by a leading telecom operator for the past five years are Rs. 30 lakhs, Rs. 42 lakhs, Rs. 45 lakhs, Rs. 48 lakhs and Rs. 52 lakhs. Their closest rival had registered profits of Rs. 28 lakhs, Rs. 46 lakhs, Rs. 50 lakhs and Rs. 57 lakhs for the last four years. Which company registered better profits (in terms of average)?

Solution: Since the profits are to be compared in terms of average, the company with the higher average
profit has better profits.

For the first company, the average profit $= \frac{30 + 42 + 45 + 48 + 52}{5} = \text{Rs. } 43.4 \text{ lakhs}$

For the rival company, the average profit $= \frac{28 + 46 + 50 + 57}{4} = 45.25 \text{ lakhs}$

Since the average profit of the rival company is higher, the rival company has registered better profits (in terms of average).

B. WEIGHTED AVERAGES

If a person invests 30% of his money in Gold, 25% of his money in the stock market, 40% of his money in fixed deposits and the remaining 5% in a savings account; and wants to know the return obtained on his entire investment, he needs to make his calculations keeping in mind the different proportions allocated to various categories. This can be done using the concept of "Weighted Averages". The term 'weight' stands for the relative importance that is attached to the values.
The average in such a case is called the weighted average and is given by the following formula.

$$\text{Weighted Average} = \frac{w_1x_1 + w_2x_2 + w_3x_3 + \cdots + w_nx_n}{w_1 + w_2 + w_3 + \cdots + w_n}$$

where $w_1, w_2, w_3, \ldots$ are the weights of the respective values.

☞ REMEMBER:

If all the weights are equal, then the weighted average is the same as simple average or arithmetic mean.

Example 8: A test was given to divisions A, B, and C of grade 8. The average scores of divisions A, B, C were 25, 30 and 35 respectively. The strength of divisions A, B, and C was 20, 24 and 30 respectively. Find the average scores of grade 8.

Solution: Since the strength of each division is different, the average score cannot be found directly. Instead, use the weighted average to find the average scores of grade 8.

$$\text{Weighted Average Score} = \frac{(25 \times 20) + (30 \times 24) + (35 \times 30)}{20 + 24 + 30} = \frac{2270}{74} = 30.7$$

Hence, the average score of grade 8 is 30.7.

Example 9: Mumbai gets wheat from different regions. If it gets 1200 kg from Nasik at the rate of Rs. 30 per kg, 1500 kg from Ratnagiri at the rate of Rs. 25 per kg and 2000 kg from Kolhapur at the rate of Rs.32 per kg, find the average cost of wheat procured by Mumbai.

Solution: Since the quantities procured from various regions are different, the weighted average of the cost is to be found.

$$\text{Weighted Average Cost} = \frac{(1200 \times 30) + (1500 \times 25) + (2000 \times 32)}{1200 + 1500 + 2000} = \frac{137500}{4700} = 29.25$$

Hence, the average cost is Rs 29.25 per kg.

Example 10: The average age of a husband, wife and their daughter ten years ago was 25 years and that of the husband and wife 6 years ago was 37 years. What is the present age of the daughter (in years)?

Solution: The total present age of the husband, wife and their daughter = (25 + 10) × 3 = 105 years
The total present age of the husband and wife = (37 + 6) × 2 = 86 years
∴ Daughter's present age = 105 – 86 = 19 years

Example 11: The difference between the ages of two persons is 10 years. Fifteen years ago, the elder one was twice as old as the younger one. Find the present age of the elder person?

Solution: Let the ages of the younger and older person be x years and $(x + 10)$ years respectively.
$\therefore (x + 10) - 15 = 2(x - 15)$
$\therefore x = 25$
∴Present age of the elder person = $x+10$
= 25 + 10 = 35 years

TEST 1

1. The average age of A, B, C and D is 26. The average age of B and D is 28. What is the average age of A & C?

   (a) 23 (b) 24 (c) 25 (d) 26 (e) None of these

2. The average of 3 numbers A, B and C is 15. If 5 is added to A, the average of A & B becomes 16. What is the value of C?

   (a) 12 (b) 15 (c) 18 (d) 21 (e) None of these

3. The average age of 3 brothers Akash, Bharat and Chandra is 34. If their sister Dimple is included in the calculation, then the average age becomes 29. What is the age of Dimple?

   (a) 14 (b) 15 (c) 16 (d) 17 (e) None of these

4. The average age of Priya and her daughter Ruchi is 32 and the average age of Shiv and his son Prem is 36. If Prem and Ruchi are 22 and 18 years old respectively, what is the average age of Shiv and Priya?

   (a) 45 (b) 46 (c) 47 (d) 48 (e) None of these

5. In a cricket T20 tournament, the 1^{st} 4 matches had the 1^{st} innings score of 141, 147, 162 & 178. If the 1st innings score in the 5^{th} match was 7 less than the average, what was the score in the 5^{th} match?

   (a) 151 (b) 152 (c) 153 (d) 154 (e) None of these

6. 15 numbers are arranged in a random order. The average of the 15 numbers is 54. The average of the first 8 numbers is 64 and the average of the last 8 numbers is 60. Find the 8th number.

(a) 182 (b) 152 (c) 214 (d) 91 (e) 180

7. Two different batches of students of a coaching class with average marks 80 and 90 respectively are combined to form a third batch. The average marks of the classes now changes to 84. Find the ratio of the number of students in the first batch to that in the second batch.

(a) 2 : 3 (b) 4 : 3 (c) 5 : 4 (d) 3 : 2 (e) 5 : 6

8. The average age of 40 students and a teacher is 24 years. The average age of only the students is 0.5 years less than the average age of the 40 students and the teacher. Find the age of the teacher in years.

(a) 40 years (b) 42 years (c) 44 years (d) 45 years (e) 46 years

9. The average weight of 6 students increases by 2 kg if a student who weighs 48 kg is replaced by another student. What is the weight of the new student?

(a) 36 kg (b) 46 kg (c) 50 kg (d) 12 kg (e) 60 kg

10. The average age of a group of friends is 25 years. If four new friends with an average age of 21 years join the group, the average of the entire group becomes 23 years. How many friends were there in the group initially?

(a) 4 (b) 5 (c) 6 (d) 2 (e) 3

TEST 2

11. The average of temperatures on Sunday, Monday, Tuesday and Wednesday is 48° and the average of temperatures on Monday, Tuesday, Wednesday and Thursday is 49°. If the ratio of temperatures on Sunday and Thursday is 12 : 13, then find the temperature recorded on Sunday.

(a) 45° (b) 48° (c) 44° (d) 49° (e) 50°

12. The combined average monthly salary of employees in department A and department B of factory XYZ is Rs.1,200 and the average monthly salary of 200 employees in department A is Rs.900. If the average monthly salary of employees in department B is Rs.1,400, find the number of employees in department B.

(a) 200 (b) 400 (c) 150 (d) 300 (e) 350

13. A group of 15 boys and 12 girls went for a picnic. The average age of the boys was 25 while that of the girls was 20. Find the average age of the group.

(a) 22.87 (b) 22.78 (c) 23 (d) 22 (e) 23.78

14. The average of a student in the first 50 tests is p and his average in the next 10 tests is q. If his overall average is $p + 2$, what is q in terms of p?

(a) $p + 6$ (b) $p + 8$ (c) $p + 10$ (d) $p + 12$

15. Average age of 20 students is 9 years. If the age of the teacher is included the average increases by 2. Find the age of the teacher?

(a) 51 years (b) 50 years (c) 52 years (d) 53 years (e) None of these

16. Average age of three brothers is 10 years. If the age of the father and mother is also considered, the average increases by 13 years. If the father is 5 years elder to the mother, what is the age (in years) of the father?

(a) 42 (b) 44 (c) 45 (d) 40 (e) None of these

17. The average age of a couple married five years back was 24 years at the time of their marriage. The average age of the family now is 20 years. It is known that the couple has one child. What is the age of the child in years?

(a) 2 (b) 3 (c) 4 (d) 1 (e) 5

18. A batsman's score in an innings was 2.5 times the average of his score in the first two innings. If his average score in the three innings put together was 90, what was his score in the third innings?

(a) 50 (b) 40 (c) 120 (d) 60 (e) 150

19. The average runs scored by 10 players of a cricket team are 26. If the captain's runs are included, the average increases by 4. What is the captain's score?

(a) 60 (b) 50 (c) 80 (d) 40 (e) 70

20. A class of 25 students took a science test. 10 students had an average score of 80. The other students had an average score of 60. What is the average score of the whole class?

(a) 66 (b) 68 (c) 70 (d) 64 (e) None

TEST 3

21. The average age of 5 members of a family is 25 years. If the average age of two members in that family is 13 years, what is the average age (in years) of the other three members of the family?

(a) 33 (b) 34 (c) 30 (d) 31 (e) None of these

22. If Jason purchased two suits for Rs. 179 each and three suits for Rs. 189 each, what is the average price (in Rs.) that Jason paid for each suit?

(a) 185 (b) 186 (c) 183 (d) 184 (e) 187

23. The average of the test scores of a class of p students is 70, and the average of the test scores of a class of n students is 92. When the scores of both classes are combined, the average is 86. What is the value of (p/n)?

(a) 4/9 (b) 2/5 (c) 3/8 (d) 3/10 (e) 4/11

24. In a cricket match, the top five batsmen together scored 30 more runs than the bottom six batsmen taken together. What is the difference in the average runs scored by the top five batsmen and the average runs scored by the bottom six batsmen if the total score of the eleven batsmen is 210?

(a) 6 (b) 8 (c) 10 (d) 9 (e) None of these

25. In a group of 11 players, the average age of four players is 32. The average age of the next three is 24 and that of the remaining players is 20. What is the average age of the 11 players?

(a) 24 (b) 22 (c) 23 (d) 25

26. The average price of four items is Rs. 2,200. What is the price of the costliest item if the prices of these items are in the ratio 1 : 5 : 3 : 2?

(a) Rs. 3,750 (b) Rs. 4,800 (c) Rs. 3,980 (d) Rs. 4,000

27. What is the average of all the prime numbers between 50 and 80?

(a) 70.15 (b) 66.14 (c) 69.2 (d) 68.52

28. The average of 5 consecutive even numbers is 32. Find the smallest of these numbers.

(a) 28 (b) 26 (c) 30 (d) 32

29. 7 numbers - $x, x + 1, x + 2, x + 3, x + 4, x + 5, x + 6$ - are placed from left to right and their mean is found to be 14. What is the mean of the last 4 numbers?

(a) 13 (b) 17.2 (c) 14 (d) 15.5

30. The average of ten numbers is p and m is the average of five out of these ten numbers. If n is the average of remaining five numbers, which of these expressions is equal to p?

(a) $\frac{10}{(m+n)}$ (b) $m + n$ (c) $\frac{(m+n)}{2}$ (d) $\frac{(m+n)}{10mn}$

31. In three given numbers, average of the second and third number is less than the average of the first and third number by 17. What is the difference between the first and second number?

(a) 36 (b) 38 (c) 30 (d) 34

32. The arithmetic mean of 52 numbers is 30. If each number is decreased by 2, what will be the new arithmetic mean?

(a) 25 (b) 29 (c) 28 (d) 26

33. In a test, Jay's marks were wrongly entered as 75 instead of 60. This increased the average marks of the class by 0.5. What is the number of students in this class?

(a) 50 (b) 35 (c) 45 (d) 30

34. Arithmetic mean of a group of students is 65. If 30% of them secured an average score of 68 and 25% secured an average score of 60, what is the average score of the remaining 45% students?

(a) 64.2 (b) 67.25 (c) 64.81 (d) 65.77

35. The total marks of all the students in a particular class are $(x^2 + 4x + 3)$. There are $(x + 1)$ students in the class. If the average marks of a student are 49, how many students are there in the class?

(a) 46 (b) 47 (c) 48 (d) 49

2 Percentages

I. INTRODUCTION

A Percentage is used to represent a part of or a fraction of the whole. It is a way to describe a number as a fraction with the denominator 100. "Percent" implies "for every hundred" and is denoted by the symbol '%'.

To write a fraction or decimal as a percentage, convert it to an equivalent fraction with a denominator of 100.

The conversion of some common percentages into fractions and vice versa is as shown in the table below.

$\frac{1}{2}$ = 50%	$\frac{1}{3}$ = 33.33%	$\frac{1}{5}$ = 20%	$\frac{1}{7}$ = 14.28%
$\frac{1}{4}$ = 25%	$\frac{1}{6}$ = 16.67%	$\frac{1}{10}$ = 10%	$\frac{1}{14}$ = 7.14%
$\frac{1}{8}$ = 12.5%	$\frac{1}{9}$ = 11.11%	$\frac{1}{15}$ = 6.66%	$\frac{1}{11}$ = 9.09%
$\frac{1}{16}$ = 6.25%	$\frac{1}{12}$ = 8.33%	$\frac{1}{20}$ = 5%	$\frac{1}{13}$ = 7.69%
$\frac{1}{32}$ = 3.125%	$\frac{1}{33}$ = 3.33%	$\frac{1}{40}$ = 2.5%	$\frac{1}{17}$ = 5.88%

Example 1: A's income is 40% of B's income. If B's income is Rs. 25,140, then what is A's income?

Solution: A's income = 40% of 25140.

This can be calculated as $\frac{40}{100} \times 25140 =$ Rs. 10,056

It can also be calculated as 0.4 × 25140 = Rs. 10,056

A third way to calculate this is to reduce the given percentage to the lowest possible ratio.

40% ≡ 40/100 ≡ 2/5

∴ A's income = (2/5) × 25140 = 50280/5 = 10056

Thus, A's income is Rs. 10,056

If a value x corresponds to 100%, then $0.1x$ corresponds to 10% and $0.01x$ corresponds to 1%. On the other hand, $2x$ corresponds to 200%, $100x$ corresponds to 10000% and so on.

Example 2: Student A scores 30 out of 75 marks. Student B scores 25 out of 60. Who performed better in percentage terms?

Solution : Student A's percentage $= \frac{30}{75} \times 100 = 40\%$

Student B's percentage $= \frac{25}{60} \times 100 = 41.67\%$

Hence, student B performed better in percentage terms.

Example 3: (CSAT 2011) In a group of persons, 70% of the persons are male and 30% of the persons are married. If two-sevenths of the males are married, what fraction of the females is single?

(a) 2/7 (b) 1/3 (c) 3/7 (d) 2/3

Solution: Percentage of males = 70%
∴ Percentage of females = 30%
Percentage of married people = 30%
∴ Percentage of unmarried people = 70%

Now, $\frac{5}{7}^{th}$ of males are unmarried.

Let the fraction of females who are single be x.

$\therefore 70 = 70 \times \frac{5}{7} + 30 \times x$

$x = \frac{2}{3}$

Hence, **option d.**

II. PERCENTAGE INCREASE AND DECREASE

Percentages are often used to indicate changes in a quantity. A percentage is a good measure to compare the change in two different quantities depending on the initial (or base) value of the quantity. For instance, if two people A and B have a salary of Rs. 100 and Rs. 10 respectively, and if both get an increment of Rs. 10, the increment of B is greater in percentage terms (even though the actual increment is the same). Thus, percentage change helps compare two similar quantities.

$$\text{Percentage change} = \frac{\text{Final quantity} - \text{Initial quantity}}{\text{Initial quantity}}$$

$$= \frac{\text{Final quantity} - \text{Initial quantity}}{\text{Initial quantity}}$$

Note that the formula mentions final and initial value (and not greater and lesser value). This implies that the final value can be less than, equal to or greater than the initial value.

Example 4: If the sales of a company grew from Rs. 200 Crores to Rs. 450 Crores, then what is the growth registered by the company in the given time period?

Solution : Growth rate $= \frac{(450 - 200)}{200} \times 100 = 125\%$

If a quantity increases by a%, then its value gets multiplied by $(100 + a)/100$
Similarly, if a quantity decreases by a% then its value gets multiplied by $(100 - a)/100$

Example 5: A's salary is 20% more than B's salary. By what percentage is B's salary less than A's salary?

Solution: Let B's salary be 100.
Then, A's salary is 100 × 1.2 = 120.

B's salary is $\frac{(120 - 100)}{120} \times 100$

$= \frac{20 \times 100}{120}$% less than A's salary.

Hence, B's salary is 16.67% less than A's salary.

Alternatively,
If you are comfortable with expressing percentages as ratios, this question can be answered directly without any calculation above.
20% corresponds to 1/5
Thus, A's salary is 1/5th greater than B's salary

Observe from the solution given above, that B's salary is 16.67% less than A's salary.
16.67% corresponds to 1/6
Thus, B's salary is 1/6th less than A's salary.
Thus, to generalize in percentage terms, if a quantity A is $1/n^{th}$ greater than another quantity B, then quantity B is $1/(n + 1)$th less than quantity A.

A. ABSOLUTE VALUE CHANGE VERSUS PERCENTAGE CHANGE

The absolute value change denotes the actual change that occurs in the measure of a quantity, whereas percentage change is the absolute change with respect to the measure of the original quantity (unless otherwise stated).

Example 6: If the cost of a product increases from Rs. 500 in 2000 to Rs. 750 in 2001, then calculate the absolute value change and the percentage change of the product between the two years.

Solution: The absolute value change
= |Final Value - Original Value| = 750 - 500 = Rs. 250

$$\text{The percentage change} = \frac{\text{Final Value} - \text{Original Value}}{\text{Original Value}} = \frac{250}{500} \times 100 = 50\%$$

Note that two quantities may have different percentage change for the same value of absolute value change, depending on the original value.
When the final value > original value, the percentage change is positive. Such a percentage change is also called percentage increase.
When the final value = original value, the percentage change is zero.
When the final value < original value, the percentage change is negative. Such a percentage change is also called percentage decrease.

B. PERCENTAGE POINT CHANGE VERSUS PERCENTAGE CHANGE

Consider the following example: The interest rate of a bank increased from 11% in 2003 to 12.5% in 2004. Here a percentage point is defined as the difference between the two percentage values.
In such a case,
The percentage point change from '03 to '04 = 12.5 - 11 = 1.5

$$\text{The percentage change from '03 to '04} = \frac{1.5}{11} \times 100 = 13.36\%$$

REMEMBER:

In financial and business jargon, the word 'basis points' or 'bps' is used very often.
1% corresponds to 100 basis points
Hence, 100 basis points ≡ 1 percentage point
Thus, in the above example, the interest rate has changed by 1.5 percentage points (or 150 basis points).

C. HOW CHANGES IN THE NUMERATOR AND DENOMINATOR VARY THE OVERALL VALUE OF THE RATIO

1. CHANGES IN THE NUMERATOR

The numerator is directly proportional to the value of the ratio. In fact, the percentage change in the value of the numerator is equal to the percentage change in the value of the ratio.

Example 7: If selling price of certain commodity remains constant, then a 10% decrease in cost price results in 40% increase in profit. Find actual percentage profit.

Solution: Let the original cost price of the commodity be x.
Let the profit earned on this commodity be y.
If selling price remains constant then decrease in cost price will be increase in profit.
Hence, if cost price is decreased by $0.1x$, then profit will also increase by $0.1x$.
Now, this is 40% of the total profit.
Hence, we have,
$0.1x = 0.4y$
Hence, $x = 4y$
Hence, $y = x/4$
Hence, y is 25% of x.
Hence, original profit was 25%.

Example 8: A student took a certain entrance test and scored 180 marks on his first attempt and 250 marks on his second attempt. What was the percentage change of his marks between the two attempts? Assume that the total marks in the examination remains the same for both the attempts.

Solution:
Let the total amount of marks for the test be X. Hence, the student scored $180/X$ in his first attempt and $250/X$ in his second attempt.
Since the denominator does not change, the percentage change will be equal to the change in the numerator.

Hence, Percentage Change $= \frac{250-180}{180} \times 100 = \frac{70}{180} \times 100 = 38.89\%$

2. CHANGES IN THE DENOMINATOR

The denominator is inversely proportional to the value of the ratio; i.e. if the value of the denominator increases, then that of the ratio decreases and vice versa.
If the price of a commodity increases by a%, then the percentage reduction in the

consumption, so that the expenditure remains the same is : $\frac{a}{a+100} \times 100$

Similarly, if the price of commodity decreases by b%, then the percentage increase

in consumption, so that the expenditure remains the same is: $\frac{b}{100-b} \times 100$

Example 9: The price of petrol increases by 30%. By what percentage should a motorist reduce his consumption of petrol to keep his expenditure constant?

Solution: Reduction in consumption $= \frac{30}{100+30} \times 100 = 23.07\%$

Example 10: The price of a commodity decreases by 20%. By what percentage should the quantity increase so as to keep the revenue constant?

Solution: Increase in quantity $= \frac{20}{100-20} \times 100 = 25\%$

Example 11: Sumaiya generally spent Rs. 800 for buying a month's provision of potatoes. However, lower yield this year caused the cost of one kilogram of potatoes to be increased by 60%. Owing to this, Sumaiya had to buy 30 kg less potatoes than usual. What was the cost of potatoes this year?

Solution: The cost of potatoes increased by 60%.

$\therefore$ The consumption will decrease by $\frac{60}{100+60} \times 100 = 37.5\%$

Sumaiya bought 30 kg potatoes less than usual.
Hence, the percentage decrease in consumption is equivalent to a decrease of 30 kg.

∴ 100% consumption will be equivalent to $\frac{100 \times 30}{37.5} = 80$ kg

∴ Sumaiya's usual ration of potatoes was 80 kg. So, after the increase in the cost of potatoes (i.e. this year), she will buy 80 - 30 = 50 kg
Since her expenditure is Rs. 800, the cost of potatoes this year = 800/50 = Rs. 16

3. CHANGES IN BOTH THE NUMERATOR AND DENOMINATOR

If the numerator increases and the denominator decreases, it is clear that the ratio will increase. Similarly, if the numerator decreases and the denominator increases, then it is apparent that the ratio will decrease. On the other hand, if both the numerator and denominator simultaneously increase/decrease, then it is not quite so apparent how the ratio will change.

Example 12: Ramesh is a wholesaler of Bananas. One day he observed that if he decreases selling price of Bananas by 10% then total sales increases by 15%. Find the percentage increase in Ramesh's total revenue.

Solution: Let x be the per unit selling price of Banana.
Let y be the volume of Bananas sold.
Hence, Ramesh's initial revenue = xy
If the selling price of Bananas decreases by 10%, then total sales increases by 15%.
Hence, Ramesh's new revenue = $0.9x \times 1.15y = 1.035xy$
Hence, Ramesh's revenue increases by;$(1.035xy - xy)/xy \times 100 = 3.5\%$

D. CALCULATING THE PERCENTAGE CHANGE IN A QUANTITY ($a \times b$), WHEN BOTH a AND b CHANGE

Let the original value of an item be $A = a \times b$. This changes to $B = x \times y$ in the next year. So, to find the percentage change between A and B, find the percentage changes between a and x (say p%) and b and y (say q%). So,

$$p = \frac{x-a}{a} \times 100 \text{ and } q = \frac{y-b}{b} \times 100$$

(Here, p and q will be negative if there is a percentage decrease.)
The percentage change can also be calculated as $a + b + (ab)/100$ where a and b is the percentage change in the respective variables. Here the sign of the percentage change is to be considered.

Example 13: Due to the erosion of soil from some parts of his field, a farmer considered increasing the length of his rectangular field by 25% and reducing its breadth by 12%. What will be the percentage change in the area of his plot?

Solution: Since area of a rectangle is the product of its length and breadth, use the above method to solve this problem.
Here, $p = 25\%$ and $q = -12\%$. Thus,

$$100 \xrightarrow{+25\% \text{ of } 100 = +25} 125 \xrightarrow{-12\% \text{ of } 125 = -15} 110$$

Hence, the percentage change in the area of the plot = 110 − 100 = 10%

Alternatively,
$a = +25\%$ and $b = -12\%$.
Hence, percentage change
= 25 + (−12) + [25 × (−12)]/100
= 25 − 12 − 3 = 10%

III. SUCCESSIVE PERCENTAGE CHANGES

Two successive increases on a particular value of $a\%$ and $b\%$ would be equal to a net increase of

$$\left(a+b+\left(\frac{ab}{100}\right)\right)\%$$

In case of decline in growth or a discount, the value of a, b or both is negative.
In general, if there are successive increases of $p\%$, $q\%$ and $r\%$ in 3 stages, then:

$$\text{Total percentage increase} = \left(\frac{100+p}{100}\times\frac{100+q}{100}\times\frac{100+r}{100}-1\right)\times 100$$

Example 14: A shopkeeper increases the price of his new product by 20%. He makes a loss and decreases the price by 35%. Find the total percentage change.

Solution: Percentage change

$$= 20+(-35)+\frac{20\times(-35)}{100} = -22\%$$

Hence, the total percentage decrease is 22%.

Example 15: Ravi's income has increased by 10% over the last year and will be 20% higher next year. If last year his salary was Rs. 15,000, what will it be next year?

Solution: Successive increases of 20% and 10% $= 20+10+\dfrac{20\times 10}{100} = 32\%$

∴ Ravi's salary next year = 15000 × 1.32 = Rs. 19,800

Example 16: After giving successive discounts of 30% and x%, the selling price of a shirt becomes Rs. 98. If the marked price of a shirt is Rs. 200, then find x.

Solution: Total discount offer by the shopkeeper = (200 – 98)/200 × 100 = 51
This is nothing but, $(-30) + (-x) + (-30)(-x)/100$ i.e. $(30x)/100 - (30 + x)$
∴ $30x/100 + 51 = 30 + x$
∴ $x - 30x/100 = 21$
∴ $x = 30$

TEST 1

1. The Indian cricket team won 50% of the matches played in the first two weeks of the NatWest series in Australia. However, at the end of the series its success rate was 75%. If the Indian team had played 6 matches in the first two weeks and they won all the matches they played after the first two weeks, find the total number of matches that they played after the first two weeks.

   (a) 6 (b) 4 (c) 5 (d) 10

2. The price of sugar has increased by 20%. Sneha has decided to only spend 8% more than she initially did on sugar. Find the percentage by which she needs to reduce the sugar consumption.

   (a) 12% (b) 10% (c) 15% (d) 18%

3. In an election between two candidates, one got 55% of the total valid votes. 20% of the total votes cast were invalid. If the total number of votes cast was 7500, the number of valid votes that the other candidate got was:

   (a) 2700 (b) 2900 (c) 3000 (d) 3100 (e) 3300

4. The price of oil decreases by 30%. What should be the increase in consumption so as to keep the expenditure constant?

(a) 44% (b) 42.86% (c) 48.26% (d) 30% (e) 45%

5. The price of a trouser is more than that of a t-shirt by 60%. By what percent is the t-shirt's price less than that of the trouser?

(a) 37% (b) 55% (c) 40% (d) 37.5% (e) 35%

6. Since the price of sugar reduced by 20%, Rohan was able to purchase 10 kg more for Rs. 80. What was the original price per kg of sugar?

(a) Rs. 4 (b) Rs. 10 (c) Rs. 3 (d) Rs. 2 (e) Rs. 1.6

7. Seema brought some chocolates. She distributed 25% of the chocolates among the students of the first standard and 20% of the remaining chocolates among the students of the second standard. If she still had 240 chocolates, how many did she have initially?

(a) 436 (b) 350 (c) 400 (d) 410 (e) None of these

8. In a certain test, Ajay was able to correctly solve 3 out of the 5 questions asked. In the second test, there were 7 questions out of which he got 6 correct. In both the tests, each question carried 10 marks with no negative marking. What is the percentage change in the marks obtained by Ajay in the two tests?

(a) 43% (b) 48.26% (c) 42.86% (d) 46.86% (e) 42%

9. A shopkeeper reduces the price of a product by 12.5%. Thinking that this is not enough, he then reduces it by 10%. When selling the product to a customer, he further gives a discount of 7%. What single discount would have been equivalent to the successive discounts?

(a) 12.5% (b) 21.25% (c) 29.5% (d) 26.76%

10. In a mini zoo, 60% of the animals are monkeys and 70% of the animals have a tail. If 50% of the monkeys have a tail, what percentage of the other animals have a tail?

(a) 40% (b) 100% (c) 50% (d) 90%

TEST 2

11. The hourly wages of a labourer have increased by 15%. Since the increase, the number of hours he works daily has reduced by 12.5%. If, before the increase, he was earning Rs.80 everyday, find the amount he is earning after the increase.

(a) 78 (b) 90 (c) 80.5 (d) 92.5 (e) 92

12. A batsman scored 110 runs which included 3 fours and 8 sixes. A four and six correspond to 4 and 6 runs respectively, and all other runs have to be scored by running between the wickets. What percent of his total score did he make by running between the wickets?

(a) 45% (b) 45.45% (c) 55% (d) 54.63% (e) None of these

13. If $A = x\%$ of y and $B = y\%$ of x, then which of the following is true?

(a) A is smaller than B.
(b) A is greater than B
(c) Relationship between A and B cannot be determined.
(d) If x is smaller than y, then A is greater than B.
(e) None of these

14. What percent of numbers from 1 to 70 have 1 or 9 in the unit's digit?

(a) 1 (b) 14 (c) 20 (d) 21 (e) None of these

15. A student multiplied a number by 3/5 instead of 5/3. What is the percentage error in the calculation?

(a) 34% (b) 44% (c) 54% (d) 64% (e) None of these

16. Ganesh spends 15% of his salary on fuel, 20% on house rent, 40% on other house expenditures and the remaining amount on his children's education. What is the amount spent by him on fuel if his children's education costs him Rs. 5,000?

(a) Rs. 3,000 (b) Rs. 2,000 (c) Rs. 4,000 (d) Rs. 4,500 (e) None of these

17. A's income is 20% more than B's income while B's income is 20% less than C's income. Whose income is the highest among all?

(a) A (b) B (c) C (d) A and C (e) Cannot be determined

18. If the length of a rectangle is increased by 50% and its breadth is increased by 20% what is the net increase in percentage of the area of the rectangle?

(a) 75% (b) 100% (c) 80% (d) 70% (e) None of these

19. Ravi's initial salary was increased by 50% and then reduced by 50%. What is the net increase or decrease in his initial salary?

(a) Decreases by 25% (b) Increases by 20% (c) Decreases by 20%
(d) Decreases by 30% (e) No change

20. When prices are reduced by 40%, the sales increase by 60%. What is the net effect on revenue earned?

(a) No change (b) Decreases by 4% (c) Increases by 4%
(d) Decreases by 6% (e) Increases by 6%

TEST 3

21. In a Chinese city, the population increases in the first year by 10%, increases in the second year by 10% and decreases in the third year by 10%. If the population now is 100000, what is the population after three years?

(a) 98100 (b) 108000 (c) 121000 (d) 108900 (e) None of these

22. In an examination, Raju got 12 out of the first 16 questions correctly. Out of the remaining questions he got only 25% of the questions correctly. All the questions have equal marks and he got 50% marks in all. What is the total number of questions, if there is no negative marking?

(a) 32 (b) 28 (c) 50 (d) 36 (e) 40

23. A student must get 40% marks to pass in an exam. A student gets 30 marks and fails by 6 marks. What are the maximum marks of the exam?

(a) 80 (b) 100 (c) 70 (d) 75 (e) 90

24. If we decrease the selling price of milk by 20%, by what percent should the sales be increased so that the total revenue remains the same?

(a) 20% (b) 15% (c) 30% (d) 22% (e) 25%

25. The value of a machine, purchased exactly two years ago, depreciates at the rate of 8% every year. Its present value is Rs. 1045.304. At what price was it purchased?

(a) Rs. 1,235 (b) Rs. 1210.12 (c) Rs. 1248.501 (d) Rs. 1,100

26. Rishi gets a commission of 6% on sales up to Rs. 5,000 and 5% on all sales exceeding Rs. 5,000. If Rishi has collected Rs. 15,150 for his company after deducting his commission, what is his commission amount?

(a) Rs. 950 (b) Rs. 780 (c) Rs. 980 (d) Rs. 850

27. If the numerator of a fraction is increased by 15% and its denominator is decreased by 12%, the value of a fraction becomes 5/4. Find the original fraction.

(a) $\frac{12}{15}$ (b) $\frac{13}{15}$ (c) $\frac{22}{23}$ (d) $\frac{21}{25}$

28. Three–fourth of one-third of two-fifth of a number is 5. What is 30% of that number?

(a) 5 (b) 10 (c) 15 (d) 20

29. The sum of two numbers is $\frac{5}{4}$ of the first number.

The second number is what percent of the first number?

(a) 25% (b) 15.5% (c) 10.25% (d) 20%

30. If one number is 60% of the second number and 5 times the sum of their squares is 170, what are the numbers? Assume that both numbers are positive integers.

(a) 4, 7 (b) 3, 7 (c) 9, 4 (d) 5, 3

31. 25% of machine A's daily production is equal to 45% of machine B's daily production. If machine B makes 2000 products everyday, what is machine A's daily production?

(a) 3100 (b) 3250 (c) 3600 (d) 3750

32. Three years ago, the population of a town was 320000. If it has increased by 2%, 2.5% and 5% respectively in the last three years, what is the present population of this town?

(a) 361256 (b) 351288 (c) 371204 (d) 368524

33. Amit has to get 40% of the total marks to qualify in an examination. In the first paper he scores 70 out of 140 and in the second paper, he scores 25 out of 60. How much should he score out of 100 in the third paper to just qualify in the exam?

(a) 33 (b) 29 (c) 25 (d) 34

34. A new house worth Rs. 19,68,300 is constructed on land worth Rs. 51,200. If the value of the house and land respectively depreciates and appreciates at 20% p. a, in how many years will the value of both be equal?

(a) 8 (b) 9 (c) 10 (d) 11

35. The price of apples decreases by 15%. As a result, Reena increases her consumption by 20%. The percentage change in expenditure is

(a) –5% (b) 5% (c) –2% (d) 2% (e) 10%

3 Interest and Growth Rates

I. INTRODUCTION

Money borrowed today is repaid with a higher amount tomorrow. This gives rise to the concept of interest.

The amount of money which the creditor lends initially is known as the **Principal (*P*)** or **Capital** and the time frame for which he lends the money is known as **Time** or **Period (*T* or *n*)**.

The difference between the Principal and the amount of money which the borrower needs to repay at the end of the time period is called the **Interest (*I*)** over the Principal amount. Also, the total money which he repays at the end is termed as the **Amount (*A*)**; in other words,
Amount = Principal + Interest.

The Interest is calculated based on the **Rate of Interest (*R*)**, which is specified in terms of percent per annum (p. c. p. a) unless specified otherwise.

There are two ways in which interest is calculated.

1. Simple Interest (SI)
2. Compound Interest (CI)

II. SIMPLE INTEREST

The interest calculated only on the original principal, for the given time duration, is called Simple interest.

$$\text{Simple Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100} = \frac{P \times R \times T}{100}$$

$$\text{Amount} = \text{Principal} + \text{Interest} = P + I = P + \frac{P \times R \times T}{100}$$

Example 1: Find the simple interest on a principal of Rs. 3,500 at the rate of 4% per annum for a period of 8 years.

Solution: Simple Interest $= \frac{P \times R \times T}{100} = 3500 \times 0.04 \times 8 = \text{Rs. } 1{,}120$

Example 2: If Rs. 12,000 lent at simple interest becomes Rs. 12,800 in two years' time, then how much will Rs. 18,000 become at the end of 5 years at the same rate of simple interest?

Solution: Rs. 12,000 lent at simple interest becomes Rs. 12,800 in two years' time. This amount of Rs. 12,800 includes the original principal of Rs. 12,000 as well. So, the additional amount is the simple interest.
Hence, Simple Interest for 2 years on Rs. 12,000 = Rs. 800.

$$\text{As } SI = \frac{P \times R \times T}{100}, R = \frac{100 \times SI}{P \times T}$$

$$\therefore R = \frac{100 \times 800}{12000 \times 2} = 3.33\%$$

Hence, Simple Interest on Rs. 18,000 for 5 years = 18000 × 0.0333 × 5 = Rs. 3,000
∴ Amount to be paid back after 5 years = 18000 + 3000 = Rs. 21,000

Example 3 Calculate the simple interest on Rs. 75,000 for $2\frac{1}{2}$ years at $15\frac{2}{3}\%$ per annum.

Find the amount to be paid after $2\frac{1}{2}$ years.

Solution: Principal = Rs. 75,000, Rate = $15\frac{2}{3}\% = \frac{47}{3}\%$

$$SI = \frac{P \times R \times T}{100} = \frac{75000 \times 47 \times 2.5}{3 \times 100} = \text{Rs. } 29{,}375$$

Amount = P + SI = 75000 + 29375 = Rs. 1,04,375

III. COMPOUND INTEREST

Simple interest is interest accrued on principal alone, whereas, Compound interest is interest accrued on principal as well as the previous year's interest.

When money is lent at compound interest, at the end of a fixed period, the interest for that fixed period is added to the principal, and this amount is considered to be the principal for the next year or period. This is repeated until the amount for the last period has been calculated. The difference between the final amount and the original principal is the Compound Interest (*CI*). This amount can be calculated using the following formula:

$$\text{Amount} = P \times \left(1 + \frac{r}{100}\right)^n$$

Example 4: If Rs. 12,000 has been lent out at 10% rate of interest, the interest being compounded annually, then what is the interest for the third year?

Solution: The amount at the end of the second year will be the principal for the third year.

$$\text{Amount at the end of the second year} = P \times \left(1 + \frac{r}{100}\right)^n = 12000 \times \left(1 + \frac{10}{100}\right)^2$$

= 12000 × $(1.1)^2$ = Rs. 14,520

The simple interest on this sum will be the interest for the third year.

Interest for the third year = 14520 × 0.1 = Rs. 1,452

Example 5: If the compound interest on a certain sum for 3 years at 4% is Rs. 1,500, then what would be the simple interest on the same sum at the same rate and the same time period?

Solution: Let the sum be Rs. *P*.

$$\therefore CI = P \times \left(\frac{104}{100}\right)^3 - P = 1{,}500$$

$$\therefore (0.124864)P = 1500$$

$$\therefore P = \frac{1500}{0.124864}$$

Simple interest on Rs. *P* for 3 years at 4% is 0.12*P*

$$\therefore SI = \frac{0.12 \times 1{,}500}{0.124864} \approx \frac{1500}{1.04} \approx \text{Rs. } 1{,}442$$

Example 6: Simple interest on a certain sum of money for 2 years at 8% per annum is half the compound interest on Rs. 4000 for 2 years at 10% per annum. The sum placed on simple interest is:

Solution: $\text{C.I.} = \left\{4000 \times \left[1 + \left(\frac{10}{100}\right)^2\right] - 4000\right\}$

∴ C.I. = (4000 × 1.1 × 1.1) – 4000 = Rs. 840

The simple interest remains constant for each year. So, the simple interest obtained per year = 840/2 i.e. Rs. 420

Let the sum placed on simple interest be Rs. *P*

$\therefore 420 = \frac{P \times 8 \times 3}{100}$

$\therefore P = 1750$

Thus, the sum placed on simple interest is Rs. 1,750.

IV. COMPOUNDING MORE THAN ONCE A YEAR

As mentioned earlier, the frequency of compounding can vary. It can be done half yearly (semi-annually), quarterly, monthly etc. When compounding is done more than once a year, the rate of interest for that time period will be less than the effective rate of interest for the entire year.

For half yearly rate, $A = P \times \left(1 + \frac{r/2}{100}\right)^{2n}$

For quarterly rate, $A = P \times \left(1 + \frac{r/4}{100}\right)^{4n}$

For monthly rate, $A = P \times \left(1 + \frac{r/12}{100}\right)^{12n}$

REMEMBER:

In the above formulae $2n$, $4n$ and $12n$ are obtained by taking into consideration the number of time periods in the year for which compounding will occur i.e. for half yearly compounding there will be two time periods in a year when compounding occurs. Similarly, for quarterly there will be four such time periods in a year and for monthly there will be twelve such time periods.

Example 7: What is the difference between the compound interests on Rs. 5,000 for 1.5 years at 4% per annum compounded yearly and half-yearly?

Solution: C. I. when interest compounded yearly $= [5000 \times \left(1 + \frac{4}{100}\right) \times (1 + \frac{0.5 \times 4}{100})$

$= \left(5000 \times \frac{26}{25} \times \frac{51}{50}\right) = \text{Rs.} 5304.$

C. I. when interest compounded half – yearly $= 5000 \times \left(1 + \frac{2}{100}\right)^3$

$= \left(5000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50}\right) = \text{Rs.} 5306.04$

Difference = Rs. (5306.04 – 5304) = Rs. 2.04

V. POPULATION FORMULA

If the original population of a town is P and the annual increase is r%, then the population in n years (P') is $P' = P \times \left(1 + \frac{r}{100}\right)^n$

If the annual decrease is r%, then the population in n years is given by a change of sign in the formula: $P' = P \times \left(1 - \frac{r}{100}\right)^n$

Example 8: The population of a city currently is 30 million. The number has been increasing at a steady rate for the past 10 years.

If it is observed that the rate of increase is 15% every year, then what will be the population of the city 3 years from now?

Solution: P = 30 million, r = 15%, n = 3 years

Hence, using the formula, the population after 3 years is: $30 \times \left(1 + \frac{15}{100}\right)^3$

$\approx$ **45.6 million**

So the population becomes 45.6 million after 3 years.

VI. DEPRECIATION OF VALUE

The value of any asset decreases with time due to any of a number of factors including wear and tear, outdated technology, usage etc. This decrease is called its **depreciation.** If P is the original value and r is the rate of depreciation per year, then the final value (F) after n number of years is given by the formula, $F = P \times \left(1 - \frac{r}{100}\right)^n$

Example 9: Find the approximate original value of the car purchased two years ago. It's current value is Rs. 1,00,000 and is constantly depreciating at the rate of 1% per year

Solution: By using the depreciation formula:

$\therefore F = P \times \left(1 - \frac{r}{100}\right)^n$

$\therefore 100000 = P \times \left(1 - \frac{1}{100}\right)^2$

$\therefore P \approx 102030$

Hence, the original value of the car is approximately Rs. 1,02,030.

Example 10: A building worth Rs. 17,28,000 is constructed on land worth Rs. 7,29,000. After how many years will the value of both be the same, if the worth of the land appreciates at 20% per annum and that of the building depreciates at the rate of 10% per annum?

Solution: Let the number of years be n.

According to the question, $729000 \left(1 + \frac{20}{100}\right)^n = 1728000 \left(1 - \frac{10}{100}\right)^n$

$\therefore 729000 \times (1.2)^n = 1728000 \times 0.9^n$

$\therefore \left(\frac{1.2}{0.9}\right)^n = \frac{1728000}{729000} = \left(\frac{12}{9}\right)^3$

$\therefore n = 3$

TEST 1

1. Reena took a loan of Rs. 1,200 at simple interest for as many years as the rate of interest. If she paid Rs. 432 as interest at the end of the loan period, what was the rate of interest (in %)?

   (a) 3.6 (b) 6 (c) 18 (d) None (e) Cannot be determined

2. A certain amount earns simple interest of Rs. 1,750 after 7 years. Had the interest been 2% more, how much more interest (in Rs.) would it have earned?

   (a) 35 (b) 245 (c) 350 (d) None (e) Cannot be determined

3. Sakshi borrowed some amount at simple interest at 2% p.a. for the first two years, at 4% p.a. for the next three years and at 5% p.a. for the next four years. What amount did she borrow if the total interest paid by her at the end of nine years was Rs.12,600?

   (a) Rs.36,600 (b) Rs.35,000 (c) Rs.37,640 (d) Rs.39,850

4. Albert invested an amount of Rs. 8,000 in a fixed deposit scheme for 2 years at compound interest rate of 5%. What amount will Albert get on maturity of the fixed deposit?

(a) Rs. 8,640 (b) Rs. 8,620 (c) Rs. 8,820 (d) Rs. 8,920 (e) None of these

5. The compound interest on Rs. 30,000 at 7% per annum is Rs. 4,347. The period (in years) is:

(a) 2 (b) 2.5 (c) 3 (d) 3.5 (e) 4

6. Chintamani took a loan of Rs. 50,000 at a rate of 10% per annum at simple interest for 3 years and invested the money at the rate of 10% per annum for the same period, compounded annually. How much money did he gain or lose in the entire transaction?

(a) Gained Rs. 16,550 (b) Gained Rs. 5,000 (c) Gained Rs. 1,550 (d) Neither gained nor lost

7. A bank lent Rs. 4,000 to Manoj at a certain rate of simple interest and Rs. 5,000 to Aditi at simple interest at a rate which is 0.5 percentage points more than that of Manoj. After two years, the bank received Rs. 860 as interest from both of them combined. Find the rate of interest per annum at which the amount was lent to Aditi?

(a) 4.5% (b) 4% (c) 5.5% (d) 5% (e) 8%

8. It takes *n* years, for Rs. 62,500 to amount to Rs. 1,08,000, at 12% per annum compounded annually. Find the value of *n*.

(a) 7 (b) 5 (c) 4 (d) 3 (e) 2

9. A sum of Rs. 5,000 deposited by Mr. A at compound interest doubles after 6 years. What will be its value after 18 years?

(a) Rs. 20,000 (b) Rs. 60,000 (c) Rs. 25,000 (d) Rs. 30,000 (e) Rs. 40,000

10. The difference between the compound interest and the simple interest for 2 years on a certain sum at 10% rate of interest is Rs. 850. Find the principal.

(a) Rs. 8,50,000 (b) Rs. 8,500 (c) Rs. 85,000 (d) Rs. 70,250 (e) None of these

TEST 2

11. The population of country A on 1st Jan, 2008 was 1 billion and it grows at the rate of 10% per year. The population of country B on the same date was 1.5 billion and it decreases at the rate of 10% per year. On 1st January of which year will country A's population become more than that of country B?

(a) 2008 (b) 2009 (c) 2010 (d) 2011 (e) 2012

12. Out of a world population of approximately 6.6 billion, 1.2 billion people live in the richer countries of Europe, America, Japan and this figure is growing at the rate of 25% per year, while the other 5.4 billion people live in the less developed countries and this figure is growing at the rate of 15%. What will be the world population in 5 years if we assume that these growth rates will stay constant for the next 5 years? (round answer to 3 significant digits)

(a) 11.53 billion (b) 12.53 billion (c) 12.03 billion (d) 14.03 billion (e) 14.52 billion

13. There is 60% increase in an amount in 6 years at simple interest. What will be the compound interest of Rs. 12,000 after 3 years at the same rate?

(a) Rs. 2,160 (b) Rs. 3,120 (c) Rs. 3,972 (d) Rs. 6,240 (e) None of these

14. If the simple interest on a sum of money for 2 years at 5% per annum is Rs. 50, what is the compound interest on the same sum at the same rate and for the same time?

(a) Rs. 51.25 (b) Rs. 52 (c) Rs. 54.25 (d) Rs. 60 (e) None of these

15. A car is bought new at Rs. 3,00,000 and its cost depreciates at 20% per annum. What is the value of the car (in Rs.) after 4 years?

(a) 2,00,000 (b) 2,04,000 (c) 1,23,380 (d) 2,50,000 (e) 1,22,880

16. A colony of bacteria contains 25000 bacteria and it increases at 10% per hour. What is the count of bacteria 3 hours from now?

(a) 33295 (b) 33695 (c) 33475 (d) 33375 (e) None of these

17. What is the simple interest on a sum of Rs. 42,700 at 8% p.a. for the period from 6th March 2012 to 4th May 2012?

(a) Rs. 549 (b) Rs. 490 (c) Rs. 560 (d) Rs. 667

18. Anil invested Rs.5,000 at 4% p.a. and Rs.8,000 at 6% p.a. in two different health policies. What is the total compound interest earned by him on the two policies at the end of two years?

(a) Rs.1,396.8 (b) Rs.1,543.5 (c) Rs.1,246.2 (d) Rs.1,450.1

19. What is the compound interest on Rs.40,000 at 10% p.a. for 2.5 years compounded annually?

(a) Rs.12,358 (b) Rs.14,174 (c) Rs.13,361 (d) Rs.11,876

20. If a sum of money becomes $\frac{6}{5}$ of itself in 4 years at simple interest, what is the rate of interest per annum?

(a) 5% p.a. (b) 8% p.a. (c) 10% p.a. (d) 12% p.a.

TEST 3

21. At what annual rate will the simple interest on an amount become double of that amount in 15 years?

(a)14.32% (b)13.2% (c) 15% (d) $11\frac{3}{5}\%$ (e) $13\frac{1}{3}\%$

22. In 2 years, a sum of Rs. 3,468 kept at simple interest amounts to Rs. 4,126. What is the rate of interest?

(a) 9.5% (b) 9.8% (c) 8.2% (d) 8.9% (e) None of these

23. An investment at the rate of 5% per annum for 2 years gives simple interest of Rs. 1,264. What is the amount invested?

(a) Rs. 15,240 (b) Rs. 12,640 (c) Rs. 14,264 (d) Rs. 13,641 (e) None of these

24. A sum of Rs. 25,000 is compounded annually at the rate of 4% p.a. What is the amount obtained after two years?

(a) Rs. 27,140 (b) Rs. 27,040 (c) Rs. 26,925 (d) Rs. 26,900 (e) Rs. 27,000

25. A sum of Rs. 50,000 placed at compound interest doubles after 10 years. What will be its value after 20 years?

(a) Rs. 2,05,000 (b) Rs. 2,25,000 (c) Rs. 2,10,000 (d) Rs. 2,00,000 (e) Rs. 2,15,000

26. What is the simple interest on Rs. 5,454 at the rate of 4% p.a. for 5 years?

(a) Rs. 992 (b) Rs. 1,022 (c) Rs. 986 (d) Rs. 972 (e) Rs. 1,091

27. The current value of a bike purchased 2 years ago is Rs. 72,000. Its value has constantly depreciated at the rate of 2% per year. What was its approximate original value?

(a) Rs. 73,469 (b) Rs. 74,969 (c) Rs. 75,124 (d) Rs. 76,260 (e) Rs. 75,480

28. The current population of a village is 5000. The population in this village increases at a steady rate of 20% per year. What will be the population of this village after 2 years?

(a) 7200 (b) 6800 (c) 7100 (d) 6500 (e) 7000

29. The present worth of Rs. 253 due in 2 years at 6% p.a. compound interest is?

(a) Rs. 233.59 (b) Rs. 228.02 (c) Rs. 231.24 (d) Rs. 225.17 (e) Rs. 222.13

30. The difference between the compound interest and simple interest on a certain amount at 20% p.a. for 2 years is 100. What is the amount?

(a) Rs. 1,000 (b) Rs. 1,500 (c) Rs. 2,000 (d) Rs. 2,500 (e) Rs. 3,000

4 Profit, Loss and Discount

I. PROFIT AND LOSS

The price at which a person buys (or produces) a product is the **Cost Price** (*CP*) of the product with respect to that person and the price at which a person sells a product is called the sales price or the **Selling Price** (*SP*) of the product, again with respect to that person.

When a person is able to sell a product at a price higher than its cost price for him, then he can be said to have earned a **Profit** (*P*).
Profit = Selling price – Cost Price
$P = SP - CP$

Similarly, if a person sells an item for a price lower than its cost price for him, then a **Loss** (*L*) has been incurred.
Loss = Cost price – Selling Price
$L = CP - SP$

If a person sells a product at the same price at which he ought it i.e. at the cost price, the transaction is said to have been conducted on a "no-profit-no-loss" basis. Thus, there is neither profit nor loss in such a transaction.

II. PERCENTAGE PROFIT OR LOSS

If two businessmen make a profit of Rs. 10 each, it would appear that both have gained equally. However, if the first businessman had invested Rs. 100 to gain Rs. 10 and the second had invested Rs. 20 to gain Rs. 10, the gain of the second businessman would be greater (in percentage terms). Thus, the actual gains or losses (even if they are numerically equal) are not comparable by themselves as the investment or the capital of the two businesses (or transactions) may differ. Hence, the comparison of gains and losses can be made by converting them into percentages.

Example 1: Company A earned revenue of Rs. 15 crores with an investment of Rs. 12 crores. On the other hand, Company B earned revenue of Rs. 33 crores with an investment of Rs. 30 crores. Which company made a higher profit (in terms of percentage)?

Solution: Profit = Revenue – Investment
Observe that both companies made a profit of Rs. 3 crores. Thus their performance seems to be similar. That may not necessarily be the case.
Company A made a profit of Rs. 3 crores on an investment of Rs. 12 crores and Company B made the same profit on an investment of Rs. 30 crores. Thus, since company A gained the same amount on a lower investment, it has made a higher profit (by observation). However, if we want to know how much that gain was, we need to find the percentage profit.
Company A earned a profit which was 1/4th of its investment, whereas Company B earned a profit which was 1/10th of its investment.
This concept can be put in terms of percentage profit, mathematically as shown below,

$$\text{Percentage profit for Company A} = \frac{\text{Actual Profit}}{\text{Investment}} \times 100 = \frac{3}{12} \times 100 = 25\%$$

$$\text{Percentage profit for Company B} = \frac{3}{30} \times 100 = 10\% = 10\%$$

Hence, Company A made a higher profit than Company B.

Formulae: Percentage profit $= \frac{\text{Actual Profit}}{\text{Investment}} \times 100 = \frac{\text{Selling Price} - \text{Cost Price}}{\text{Cost Price}} \times 100$

Percentage loss $= \frac{\text{Actual Loss}}{\text{Investment}} \times 100 = \frac{\text{Cost Price} - \text{Selling Price}}{\text{Cost Price}} \times 100$

⚠ IMPORTANT:

Always determine Percentage Profit (or Loss) on the *Cost Price* (or *Investment*) of an item, unless it is stated otherwise in the question.

Example 2: A grocer buys eggs at the rate of Rs. 100 per 144 eggs and sells them at a rate of Rs. 75 per 100 eggs. Find the profit or loss percentage.

Solution: Note that the cost price and selling price cannot be considered as Rs. 100 and Rs. 75 respectively, as those are the rates of buying and selling respectively and more importantly, because the number of eggs bought and sold is not the same. Consequently, first bring the selling price and cost price to a common base and then compare the two. This can be done by finding the unit cost price and unit selling price.

CP of 144 eggs = 100
Hence, CP of 1 egg = 100/144
SP of 100 eggs = 75
Hence, SP of 1 egg = 75/100

$$\frac{\text{SP of one egg}}{\text{CP of one egg}} = \frac{75}{100} \times \frac{144}{100} = \frac{108}{100}$$

$$\text{Percentage profit } = \frac{\text{SP of one egg} - \text{CP of one egg}}{\text{CP of one egg}} \times 100$$

$$= \left(\frac{\text{SP of one egg}}{\text{CP of one egg}} - 1\right) \times 100$$

$$\therefore \text{ Percentage profit} = \left(\frac{108}{100} - 1\right) \times 100 = 8\%$$

Note: If the number of eggs bought and sold was the same, say 75, then the rate of buying and selling could have been considered as the cost price and selling price respectively, as shown below.

CP of 75 eggs = 100
Hence, CP of 1 egg = 100/75
SP of 75 eggs = 75
Hence, SP of 1 egg = 75/75

$$\therefore \text{ \% Profit (or Loss)} = \frac{\left(\frac{75}{75} - \frac{100}{75}\right)}{\left(\frac{100}{75}\right)} \times 100 = \frac{(75 - 100)}{100} \times 100 = \mathbf{25\%}$$

Thus, there is a 25% loss in this case.

Example 3: If the selling price of 40 equally priced books is equal to the cost price of 32 of those books, then what is the profit or loss percentage?

Solution: SP of 40 books = CP of 32 books
Let the CP of 1 book be Rs. x and the SP of 1 book be Rs. y.
Then, $40y = 32x$
$\therefore$ (SP/CP) of 1 book (in Rs.) $= y/x = (32/40) = 4/5$
Since the SP of one book is less than the CP of one book, there is a loss in the transaction.

$$\text{Hence, Percentage Loss } = \left(1 - \frac{\text{SP}}{\text{CP}}\right) \times 100 = \left(1 - \frac{4}{5}\right) \times 100 = 20\%$$

Alternatively,
Let CP of 1 book be Re. 1
∴ CP of 32 books is Rs. 32
∴ SP of 40 books is Rs. 32
But, CP of 40 books is Rs. 40
So, there is a Loss and Loss = 40 - 32 = Rs. 8
∴ Percentage Loss = (8/40) × 100 = 20%

⚠ IMPORTANT:

Percentage Profit can only be calculated when the number of goods sold and the number of goods bought is equal.

Example 4: Sandeep bought a computer for Rs. 30,000 and sold it to Amoy at a loss of 10%. Amoy then sold it to Latif at a loss of 15%. Find the amount paid by Latif to purchase the computer.

Solution: Profit and loss are always calculated on the CP (unless otherwise specified).
Since Sandeep incurred a loss of 10%, he sold it at 90% of the CP.
∴ SP of Sandeep = 30000 × 0.9 =Rs. 27,000
This is the C.P for Amoy.
∴ CP of Amoy = Rs. 27,000
Since Amoy sold it at a loss of 15%, he sold it at 85% of the CP.
∴ SP of Amoy = 27000 × 0.85 = Rs. 22,950
∴ CP of Latif = Rs. 22,950
If two items are *sold for the same SP*, one at a gain of a% and the other at a loss of a%, then there is an overall loss and the loss percentage = $a^2/100$ %.
Note that in this case, the selling price of both items should be the same. Also, the percentage and loss profit should also be the same.

Example 5: Mani sold both his consignments to two different vendors at Rs. 30,000 per consignment. He got a profit of 15% on the first consignment and a loss of 15% on the second consignment.
Find his overall profit or loss percentage.

Solution: Since the SP is same and both profit and loss % in individual transactions are the same, he ends up with an overall loss which is given by $a^2/100$%.

$$\therefore \text{Percentage Loss} = \frac{15 \times 15}{100} = 2.25\%$$

Alternatively,
CP of the first consignment = 30000/1.15 ≈ Rs. 26,087
CP of the second consignment = 30000/0.85 ≈ Rs. 35,294
∴ Total CP = Rs. 61,381 and total SP = Rs. 60,000
Loss = 61381 - 60000 = Rs. 1,381

$$\therefore \text{Percentage Loss} = \frac{1381}{61381} \times 100 = 2.25\%$$

Example 6: A grocer bought 6 dozen eggs for Rs. 80. Later he found 24 eggs to be broken and sold the rest at the rate of 2 eggs for Rs. 5. Find the profit or loss percentage.

Solution: Cost of 6 dozen eggs = Rs. 80
Since 24 eggs are broken, he sells only 4 dozen.
SP of 2 eggs = Rs. 5
∴ SP of 12 eggs (i.e. 1 dozen) = Rs. 30
∴ SP of 4 dozen eggs = Rs. 120

∴ Profit = Rs. 40
∴ Profit Percentage = (40 × 100)/80 = 50%

A. CALCULATING PERCENTAGE PROFIT BY EQUATING THE AMOUNT OF MONEY SPENT AND EARNED

When the amount of money spent and earned are equated, then:

$$\text{Percentage Profit} = \frac{\text{Remaining Goods}}{\text{Sold Goods}} \times 100$$

Example 7: If a book store bought 20 books, and it recovered its investment when it sold 15 of these books; then what will be the store's percentage profit?

Solution: $\text{Percentage Profit} = \frac{\text{Remaining Goods}}{\text{Sold Goods}} \times 100 = \frac{5}{15} \times 100 = 33.33\%$

Alternatively,
Let the Cost Price of each book be Rs. x, and the Selling Price of each book be Rs. y.
Since the sale of 15 books was enough to cover the cost price of 20 books,
∴ $20x = 15y$

$$\therefore x = \frac{3y}{4} \text{ or } \frac{y}{x} = \frac{4}{3}$$

$$\therefore \text{Percentage profit} = \left(\frac{y}{x} - 1\right) \times 100 = \left(\frac{4}{3} - 1\right) \times 100 = \frac{1}{3} \times 100 = 33.33\%$$

Example 8: Arjun bought 20 chocolates for Rs. 10. At what price should he sell each chocolate to get a profit percentage of 25%.

Solution: Cost price of each chocolate= Rs. 10/20 = 50p
To get a profit percentage of 25%, selling price becomes = 1.25 × 0.5 = Rs. 0.625
Thus, he should sell each chocolate at Rs. 0625

III. MARKED PRICE AND DISCOUNT

The difference between the Selling Price of a good and its Cost Price is known as **markup**. The price that is printed on an article or written on the label attached to it is the sum of the Cost Price and the markup, and is called the **Marked Price** (MP) or **List Price** of the item.

i.e. Cost Price + Markup = Marked Price

Markup can either be expressed as an amount (as shown above) or as a percentage of the Cost Price. So, the relationship between CP and MP is

$$CP + \frac{\text{Markup (as a percentage)}}{100} \times CP = \text{Marked Price}$$

$$\text{Hence, Markup (as a percentage)} = \frac{(\text{Marked Price} - \text{Cost Price})}{\text{Cost Price}} \times 100$$

Generally, MP = SP. However, sometimes, in order to increase sales or to sell-off the old stock, retailers reduce the marked price of the article by a certain amount called **Discount**. In this case, the Selling Price will be the reduced price (i.e. price after deducting the discount).
i.e. Selling Price = Marked Price – Discount
Similar to markup, discount can also be represented both as an amount (shown above) and as a percentage.
So, the relationship between MP and SP is

$$\text{Selling Price} = MP - \frac{\text{Discount (as a percentage)}}{100} \times MP$$

$$\textit{Hence, Discount (as a percentage)} = \frac{\text{Marked Price} - \text{Selling Price}}{\text{Marked Price}} \times 100$$

More formulae: Discount Percentage $= \frac{\text{Discount}}{\text{Marked Price}} \times 100$

$$\frac{SP}{MP} = 1 - \frac{\text{Discount Percentage}}{100}$$

Example 9: A pair of jeans was initially marked at such a price that it would have earned the shopkeeper a profit of 25% on the Cost Price. Later, a discount of 10% was offered on the jeans and it was then sold for a net profit of Rs. 100. What was the Cost price for the pair of jeans?

Solution: Since the Marked Price of the jeans would have earned a profit of 25% on the Cost Price,
MP = CP + 25% of CP = 1.25 CP
Later, a discount of 10% was offered.
Hence,
SP = MP – 10% of MP = 0.9 MP
Now, Profit = SP – CP = 100
∴ 0.9MP – CP = 100
∴ 0.9(1.25 CP) – CP = 100
∴ 1.125 CP – CP = 100
∴ CP = 100/0.125 = 800
Hence, the Cost Price of the pair of jeans was Rs. 800.

Example 10: An unscrupulous store owner adds a markup of 40% to the Cost Price and then offers a discount of 10% to please his customers. Apart from this, he also uses false weights to reduce the quantity sold by 20%. What is his profit percentage?

Solution: MP = CP + 40% CP = 1.4 CP
SP = MP – 10% MP = 0.9 MP
∴ SP = 0.9 × 1.4 CP = 1.26 CP
Let the CP of 1 kg be Rs. 1000.
Then, SP = Rs. 1260. However, by using false weights, he sells only 80% of 1 kg = 800 grams.
Hence, if the SP of 800 grams is Rs. 1260; then the SP of 1000 grams will be:

$$\frac{1260 \times 1000}{800} = \text{Rs. } 1575$$

Hence, Profit (per kg) = SP – CP = 1575 – 1000 = Rs. 575

$$\therefore \text{Profit Percentage} = \frac{\text{Profit}}{\text{CP}} \times 100 = \frac{575}{1000} \times 100 = 57.5\%$$

A. SUCCESSIVE DISCOUNTS

When a discount of a% is followed by another discount of b%, then the total discount is given by

$$\left(a + b - \frac{ab}{100}\right)\%$$

In general, if there are successive discounts of p%, q% and r% in 3 stages, then:

$$\text{Total discount} = \left[1 - \left(\frac{100-p}{100} \times \frac{100-q}{100} \times \frac{100-r}{100}\right)\right] \times 100$$

Example 11: A retail store offered a discount of 15% on every item purchased. Later, they announced an additional discount of 20% on every item purchased. Find the total discount percentage availed by the customers.

Solution: The first discount offered (a) = 15%
The second discount offered (b) = 20%
Using the formula for successive discounts,
Discount Percentage = 15 + 20 – (15 × 20)/100 = 32%

Example 12: A shopkeeper allows a discount of 30% to his customers and still gains 15%. Find the marked price of an article which cost him Rs. 6,200.

Solution: CP = Rs. 6,200
His gain is 15% on CP
$\therefore$ SP = 1.15 × 6200 =Rs. 7,130
Since the shopkeeper gives 30% discount on marked price, hence
SP = MP × 0.7
$\therefore$MP = 7130/0.7 = 10185.7
Hence, the Marked Price ≈ Rs. 10,186

Example 13: In a shop, the marked price of an article is worked out in such a way that it generates a profit of 25%. What should be the discount percent allowed on the marked price such that the profit made on the sale of an article is 20%?

Solution: Let the CP of the article be Rs. 100.
$\therefore$ MP = 125 and SP = 120 (to give a 20% profit)
Now, Discount = 125 – 120 = 5
$\therefore$ Discount Percentage = 5 × 100/125 = 4%

TEST 1

1. Find the profit or loss percentage when a shopkeeper marks his goods 20% above the cost price and then allows a discount of 20% on the marked price?

   (a) 2% loss (b) 3% profit (c) 4% loss (d) 2% profit

2. A shopkeeper sold an article at a loss of 8%. Had he sold it for Rs. 540 more, he would have made a profit of 10%. Find the cost price (in Rs.) of the article?

   (a) 1800 (b) 2000 (c) 2500 (d) 3000 (e) 3600

3. During a special sale, the shop assistant was supposed to reduce the marked price of each article by 15%. For a particular item, he made a mistake and increased the marked price by 15%. Consequently for that item, the customer had to pay Rs. 540 more than what he would have paid if the price was correctly reduced by 15%. What was the price paid (in Rs.) by the customer?

   (a) 1800 (b) 2070 (c) 2340 (d) 2000 (e) Cannot be determined

4. A trader gives a discount of 10% on the marked price and in the process still makes a profit of 20% on his cost price. By what percentage did he mark the product over his cost price?

   (a) 15% (b) 20% (c) 26.66% (d) 33.33%

5. If a selling price of Rs. 2,500 results in a 20% discount off the marked price, then the selling price that would result in a 40% discount off the marked price (in Rs.) will be equal to:

   (a) 1875 (b) 2000 (c) 2025 (d) 2075

6. A businessman buys two different kinds of rice which cost him Rs. 30 per kg and Rs. 42 per kg. He mixes them in the ratio 3 : 2 and sells the mixture at the rate Rs. 38 per kg. Find the profit or loss percentage (approximately)?

   (a) 7% profit (b) 9% profit (c) 13% profit (d) 7% loss (e) 9% loss

7. A shopkeeper sells 150 bags for Rs. 7500. By doing so, he gains the cost of 250 bags. Find his percentage gain.

   (a) 40% (b) 50% (c) 67% (d) 16.67% (e) 66.67%

8. When a plot is sold for Rs. 18,700, the owner loses 15%. At what price must that plot be sold in order to gain 15%?

   (a) Rs. 25,100 (b) Rs. 24,700 (c) Rs. 25,300 (d) Rs. 25,500 (e) Rs. 25.700

9. At Vijay stores there are some good offers on the purchase of a laptop.
   Offer 1: Purchase a laptop for Rs. 35,000 and get a discount of 30%.
   Offer 2: Purchase a laptop for Rs. 35,000 and get successive discounts of 20% and 10%.
   Which offer is better for the customer?

   (a) Offer 1 is better (b) Offer 2 is better (c) Both the offers are equally good
   (d) Data insufficient (e) None of these

10. Two plots in Bandra were sold for Rs. 1 crore each. The first plot in Pali Hill was sold at a gain of 12% and the second one at Bandra Reclamation was sold at a loss of 12%. Find the total loss or profit percentage?

    (a) There is neither a profit nor a loss (b) gain of 12% (c) loss of 1.44%
    (d) loss of 144% (e) None of these

TEST 2

11. The marked price of a gift article is Rs. 2750. The shopkeeper allows successive discounts of 10%, 5% and 4%. The selling price of the article is:

    (a) Rs. 2732.08 (b) Rs. 2527.2 (c) Rs. 2720.8 (d) Rs. 2257.2 (e) Rs. 2351.25

12. A trader gets a profit of 25% on an article. If he buys the article at 10% less and sells it for Rs. 2 less, he still gets 25% profit. Find the actual CP of the article.

    (a) Rs. 15 (b) Rs. 16 (c) Rs. 18 (d) Rs. 16.5 (e) Rs. 17

13. The cost price of 20 articles is the same as the selling price of x articles. If the profit is 25%, then the value of x is:

    (a) 15 (b) 16 (c) 18 (d) 20 (e) 25

14. A trader mixes 26 kg of rice at Rs. 20 per kg with 30 kg of rice of another variety at Rs. 36 per kg and sells the mixture at Rs. 30 per kg. His profit percent is:

    (a) No profit, no loss (b) 5% (c) 8% (d) 10% (e) None

15. 100 oranges are bought at the rate of Rs. 350 and sold at the rate of Rs. 48 per dozen. The percentage of profit or loss is:

    (a) (100/7)% Gain (b) 15% Gain (c) (100/7)% Loss (d) 15% Loss (e) None

16. The percentage profit earned by selling an article for Rs. 1920 is equal to the percentage loss incurred by selling the same article for Rs. 1280. At what price should the article be sold to make 25% profit?

    (a) Rs. 2,000 (b) Rs. 2,200 (c) Rs. 2,400 (d) Rs. 1,800 (e) Data insufficient

17. A man saved Rs. 290 when two successive discounts of 10% and 5% were given on a microwave oven. What was the marked price of the microwave oven?

    (a) Rs. 1,800 (b) Rs. 2,500 (c) Rs. 2,200 (d) Rs. 2,400 (e) Rs. 2,000

18. Ramesh and Suresh are two shopkeepers. On a plasma TV which has a marked price of Rs. 20,000, Ramesh offers two successive discounts of 20% and 5% respectively and Suresh offers two successive discounts of 15% and 10% respectively. What is the difference between the discounts offered by Ramesh and Suresh?

(a) Nil (b) Rs. 50 (c) Rs. 100 (d) Rs. 500 (e) None of these

19. Ramesh calculates the profit percentage on the selling price instead on the cost price and obtains the profit percentage as 50%. What is the actual profit percentage?

(a) 100% (b) 50% (c) 75% (d) 60% (e) 80%

20. Raju makes a profit of 8% when he offers two successive discounts of 10% on a particular product. If the total discount offered is Rs. 190, find the cost price of the product.

(a) Rs. 2,000 (b) Rs. 1,500 (c) Rs. 750 (d) Rs. 900 (e) Rs. 800

21. A sells an article to B for Rs. 1,100 at a 10% profit and B sells it back to A at a 10% loss. What is the overall gain or loss for A in the entire transaction?

(a) No gain no loss (b) Loss of 1% (c) Gain of 11% (d) Loss of 4% (d) Gain of 4%

22. In a certain deal Virat Enterprises got 20% margin on the selling price of the land. If the cost price of the land is Rs. 1,000 per square foot, what is its selling price per square foot?

(a) Rs. 1,200 (b) Rs. 1,250 (c) Rs. 1,400 (d) Rs. 1,300 (e) None of thes

23. Rajat used to buy wheat at Rs. 10 per kg and sell it at a 20% profit. But, in this month, Rajat had to buy the wheat at Rs. 11 per kg and sell it at the original selling price. Find the new profit percentage.

(a) 10% (b) 15% (c) 8% (d) 9.09% (e) None of these

24. A shopkeeper gives 20% discount on the printed price of a book. Raju is a good bargainer and he gets 10% discount on the already discounted price of the book. If the shopkeeper still makes 8% profit in the transaction, by what percentage is the printed price more than the cost price of the book?

(a) 40% (b) 48% (c) 45% (d) 50% (e) Cannot be determined.

25. A shopkeeper pretends to sell his goods at cost price but uses a weight of 950 grams per kg. What is his gain percent?

(a) $7\frac{3}{19}\%$ (b) $4\frac{8}{19}\%$ (c) $6\frac{4}{19}\%$ (d) $5\frac{5}{19}\%$

TEST 3

26. A shopkeeper sold an article at a discount of 10% on a marked price of Rs. 2,400, thereby making a profit of 10%. What is the cost price of the article?

(a) Rs. 1,964 (b) Rs. 1,832 (c) Rs. 1,868 (d) Rs. 1,924 (e) None of these

27. Mahesh sells his flat for Rs. 75 lakhs and makes a profit of 20%. Had he sold it for Rs. 65 lakhs, what would have been his percentage loss or gain?

(a) 15% (b) 4% (c) 8% (d) 7% (e) 13%

28. Anisha bought a saree worth Rs. 1,450 for Rs. 1,150. What percentage discount did she get on the saree?

(a) 21 (b) 22 (c) 23 (d) 24 (e) 25

29. A shopkeeper purchased 30 kg potatoes for Rs. 528 and sold the whole lot at Rs. 20 per kg. What was his loss or gain percentage?

(a) 12% (b) 14% (c) 16% (d) 17% (e) 15%

30. Raghav buys a laptop and a mobile phone for Rs. 40,000 each. When he sells them, he makes a profit of 10% on the laptop and loss of 6% on the mobile phone. What is the outcome of the transaction (in terms of profit/loss percentage)?

(a) Loss of 5% (b) Profit of 4% (c) Loss of 3% (d) Profit of 2% (e) No profit no loss

5 Ratio and Proportion

I. INTRODUCTION

Numbers can be used to make comparisons in day-to-day situations. When comparing two or more identical quantities, it becomes easier to do so by finding out how many times one quantity is greater than or less than the other.

II. RATIOS

Ratios are useful when making comparisons. One of the values is divided by the other to find the value of one quantity in terms of the other. A ratio can also be expressed as a fraction that one quantity is of the other. The ratio of two terms a and b is denoted by $a : b$ and is equal to a/b where a is called the **antecedent** and b is called the **consequent**.

For instance, if a ratio of expenditure over two items A and B is given as 3 : 4, it means that if the total expenditure is Rs. 7, then the expenditure on item A is Rs. 3 and that on item B is Rs. 4. A major advantage of ratios is that if the total expenditure changes to any value, the value of the expenditure on A and B can be calculated directly (if the ratio becomes the same). Thus, if the total expenditure now becomes Rs. 56, the expenditure on A and B can be easily calculated.

$56 = 7 \times 8$

So, the expenditure on A = $3 \times 8 = 24$

And, the expenditure on B = $4 \times 8 = 32$

This can be applied to any change in the total expenditure.

This can also be done using the concept of common ratio. Thus, if a ratio is given as $a : b$, it implies that the actual value is some common multiple of a and b. To define that common multiple, we define a term called "common ratio", which is generally denoted by k. Thus, if a ratio is $a : b$, the actual values are taken as ak and bk for calculation purposes.

To compare two quantities, ensure that their units are the same.

- The order of the terms in a ratio is important. $a : b$ is not the same as $b : a$. It is the same if $a : b = 1 : 1$
- The two quantities should be of the same unit. For example, 30 marks can be compared with 45 marks but not with Rs. 45.

Example 1: 70 shares have to be distributed among brokers A and B in the ratio 2 : 3. How many shares will each of them get?

Solution: $a : b = 2 : 3$

Let the common ratio be k.

So, the number of shares that A and B get is $2k$ and $3k$ respectively.

Total number of shares = 70

$\therefore 2k + 3k = 70$

$\therefore 5k = 70$

$\therefore k = 14$

Thus, A gets = $2 \times 14 = 28$ shares and B gets $3 \times 14 = 42$ shares.

Alternatively,

The shares have to be distributed in the ratio 2 : 3. Thus, if there are 5 shares in all, A will get 2 shares and B will get 3 shares. This logic will apply irrespective of total number of shares. Hence, A will get $2/5^{th}$ of the shares and B will get $3/5^{th}$ of the shares.

$\therefore$ A gets $2/5 \times 70$ shares = 28 shares

B gets $3/5 \times 70$ shares = 42 shares

Example 2: A sum of money is to be distributed among A, B, C, and D in the proportion of 5 : 2: 4 : 3. If C gets Rs. 1000 more than D, what is B's share?

Solution: Let the shares of A, B, C and D be Rs. $5x$, Rs. $2x$, Rs. $4x$ and Rs. $3x$ respectively.
C gets Rs. 1,000 more than D.
$\therefore 4x - 3x = 1000$
$\therefore x = 1000.$
B's share = Rs. $2x$ = Rs. 2,000.

Example 3: Asha, Altheda and Amata had a total of Rs. 2,750 with them. They decided to divide this money among themselves such that 1/4th of Asha's share was equal to 1/5th of Altheda's share, which in turn was equal to half of Amata's share. How much money did Amata receive?

Solution: Let the ratio of money received by Asha, Altheda and Amata be $x : y : z$.

Hence, $\frac{x}{4} = \frac{y}{5} = \frac{z}{2}$

$\therefore y = 5x/4$ and $z = x/2$
$x + y + z = 2750$

$\therefore x + \frac{5x}{4} + \frac{x}{2} = 2750$

$\therefore \frac{11x}{4} = 2750$

$\therefore x = 1000$
Thus, Asha gets Rs. 1,000.
Atlheda gets (5 × 1000)/4 = Rs. 1,250
And Amata gets 1000/2 = Rs. 500

Ratios can also be expressed in percentages. To express the value of a ratio as a percentage, multiply the ratio by 100.

$1 : 3 = \frac{1}{3}$ is equivalent to $\frac{1}{3} \times 100 = 33.33\%$

A. SCALING RATIOS

Assume that $a : b = 2 : 3$ and $b : c$ is $4 : 3$. In such a case, if we need to find the value of $a : c$, it cannot be directly found. This is because the term common to both ratios i.e. b, does not have the same value in both ratios. Thus, in the first ratio, 2 units of a correspond to 3 units of b while in the second ratio, 4 units of b correspond to 3 units of c. To compare a and c, we equate b in both the ratios.
Since b corresponds to 3 and 4 in the two ratios, find the LCM of 3 and 4 i.e. 12.
Thus, in the first ratio, if b becomes 12 i.e (3 × 4), a will become 2 × 4 i.e. 8
Similarly, in the second ratio, if b becomes 12 i.e. (4 × 3), c becomes 3 × 3 i.e. 9
Thus, $a : c = 8 : 9$
For instance, in the previous example:
$x : y = 4 : 5$ and $y : z = 5 : 2$
Here y is common and has the same value in both ratios, x and z can be directly compared.
$\therefore x : z = 4 : 2$
$\therefore x : y : z = 4 : 5 : 2$
Now, the calculation can be done very easily.

When the ratio between three terms is given as $\frac{1}{a} : \frac{1}{b} : \frac{1}{c} = \frac{1}{x} : \frac{1}{y} : \frac{1}{z}$

Simply take the reciprocal and obtain the ratio i.e. $a : b : c = x : y : z$

The same logic applies when the ratios are given as $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$

On the other hand, if the ratio is given as $a : b : c = \frac{1}{x} : \frac{1}{y} : \frac{1}{z}$

Then, $a : b : c = (y \times z) : (x \times z) : (x \times y)$
The same logic applies when the ratios are given as $ax = by = cz$

Example 4: \$9000 is divided among A, B and C. The ratio of the amount C got to what B got is 1 : 3. The ratio of the amount B got to that A got is 1 : 2. Find the amount that each of them got.

Solution: The amount that B gets is common and is compared to the other two.
Hence, the LCM of 1 and 3 should be taken.
The LCM of 3 and 1 is 3. So, multiply the second ratio by 3.
The two ratios are 1 : 3 and 3 : 6.
∴ The amounts that the three of them get are in the ratio = 1 : 3 : 6
∴ C gets 1/10th, B gets 3/10th and A gets 6/10th of the total amount.
∴ C gets \$900, B gets \$2,700 and A gets \$5,400.

B. COMPARISON OF RATIOS

Consider two ratios $a : b$ and $c : d$.

Now, $a : b$ is greater than $c : d$ if $\frac{a}{b} > \frac{c}{d}$

Multiplying both sides by bd,
$ad > bc$
Hence, $a : b$ is greater than $c : d$ if $ad > bc$ and vice versa.
Thus, to determine which of the two given ratios $a : b$ and $c : d$ is greater, compare $a \times d$ and $b \times c$ where $b > 0$ and $d > 0$.

C. PROPERTIES OF RATIOS

1. When a ratio, say $a : b$, is multiplied with itself, then the new ratio formed, i.e. $a^2 : b^2$, is known as the *duplicate* ratio.
   Also, $a^3 : b^3$ is called the *triplicate* ratio,
   $\sqrt{a} : \sqrt{b}$ is called the *sub – duplicate* ratio and
   $\sqrt[3]{a} : \sqrt[3]{b}$ is called the *sub – triplicate* ratio.
   Moreover, $b : a$ is called the *reciprocal* ratio of $a : b$.

2. Multiplying or dividing the same number (say x) to both the numerator and the denominator of a ratio (say a : b) will not change the value of the ratio: – i.e. $\frac{a}{b} = \frac{a \times x}{b \times x}$ and $\frac{a}{b} = \frac{a/x}{b/x}$

3. Effect of adding or subtracting a number (say x) from the numerator and denominator of a ratio $a : b$:-

i) If $a < b$ or $(a/b) < 1$, then for a positive quantity x,

   Similarly, $\frac{a - x}{b - x} < \frac{a}{b}$

ii) If $a > b$ or $(a/b) > 1$, then for a positive quantity x,

   $a)\ \frac{a + x}{b + x} < \frac{a}{b}$ and

   $b)\ \frac{a - x}{b - x} > \frac{a}{b}$

4. If the numerator and denominator of the ratio $a : b$ are increased by, say, c and d respectively, then the new ratio formed will be equal to the original ratio only if the ratios $a : b$ and $c : d$ are equal
i.e. $a : b = (a + c) : (b + d)$ only if $a : b = c : d$ Thus,
Thus,

i)If $\frac{a}{b} > \frac{c}{d}$, then $\frac{a}{b} > \frac{a+c}{b+d}$ and

ii) If $\frac{a}{b} < \frac{c}{d}$, then $\frac{a}{b} < \frac{a+c}{b+d}$

Example 5: Two numbers are in the ratio 3 : 5 and the difference of their squares is 64. Find the numbers.

Solution: Let the numbers be $3x$ and $5x$.
$\therefore (5x)^2 - (3x)^2 = 64$
$\therefore 25x^2 - 9x^2 = 64$
$\therefore 16x^2 = 64$
$\therefore x = 2$ or -2
Hence, the numbers are 6 and 10 or −6 and −10.

Example 6: In an election for college president, Mehul received 5 votes for every 7 votes Harish got. If Harish got 140 votes, then how many students participated in the election if everyone has voted for either of them?

Solution: Let the number of votes that Mehul got be x
Then, $5/7 = x/140$

$\therefore x = \frac{140 \times 5}{7} = 100$

Hence, the total no of students who voted = 140 + 100 = 240

Example 7: If $\frac{a}{b} = \frac{3}{4}$, then find the ratio of $\frac{5a+3b}{7a-9b}$.

Solution: Substitute the value of $a = 3x$ and $b = 4x$

$$\frac{5a+3b}{7a-9b} = \frac{15x+12x}{21x-36x} = -\frac{27x}{15x} = -\frac{9}{5}$$

III. PARTNERSHIP

Problems on partnerships are an extension of problems on ratios. If two people have invested some amount in a business, the profit (or loss) that they make has to be in the same ratio as their investment and is calculated over the total duration.

Example 8: Ajay and Akshay invest Rs. 2,300 and Rs. 2,500 respectively in a business venture. If the profit at the end of a year is Rs. 3,360, what is Ajay's share of the profit?

Solution: Ajay's share of the profit is in the same ratio as Ajay's investment to the total investment.

$$\therefore \text{Ajay'share} = \frac{2300}{4800} \times 3360 = \text{Rs.} 1610$$

Example 9: A, B and C enter into a partnership in the ratio (7/2) : (4/3) : (6/5). After 4 months, A increases his share by 50%. If the total profit at the end of one year is Rs. 21,600, then B's share in the profit is:

Solution: $A : B : C = \frac{7}{2} : \frac{4}{3} : \frac{6}{5} = \frac{105}{30} : \frac{40}{30} : \frac{36}{30} = 105 : 40 : 36$

For the first four months, the investment of A is $105x$ while for the next 8 months, the investment of A is 1.5 times $105x$.

$\therefore$ A's investment = $(105x \times 4) + (105x \times 1.5 \times 8) = 105x \times 16 = 1680x$

B's investment = $40x \times 12 = 480x$

C's investment = $36x \times 12 = 432x$

$$\therefore \text{B'share} = \frac{480x}{(1680x + 480x + 432x)} \times 21600 = 4000$$

Thus, B gets Rs. 4,000

Example 10: A and B started a company on the first day of the year and put in Rs. 250 and Rs. 400 on a monthly basis. A stopped investing after 4 months and so B ran the company alone for the next 3 months. Now, C invested Rs. 300 in the company to join B and continued his investment for the next five months. At the end of the year, the company made a profit of Rs. 29,200. By how much did B's share of the profit exceed the combined share of A and C's profit?

Solution: A's total investment = 250 × 4 = Rs. 1,000

B's total investment = 400 × 12 = Rs. 4,800

C's total investment = 300 × 5 = Rs. 1, 500

Total investment = 1000 + 4800 + 1500 = Rs. 7300

$$\text{B'share} = \frac{4800}{7300} \times 29200 = \text{Rs. } 19{,}200$$

$\therefore$ A and C's combined share = 29200 – 19200 = Rs. 10,000

$\therefore$ Required difference = 19200 – 10000 = Rs. 9,200.

IV. PROPORTION

The equality of two ratios is called **proportion**. A proportion is an equation that has two equivalent ratios on either side.

In other words, if $a/b = c/d$, then a, b, c and d are said to be in proportion. This equality of ratios is denoted as $\boldsymbol{a : b :: c : d}$.

When a, b, c and d are in proportion, they are called the first, second, third and fourth proportional respectively. a and d are called the **extremes** and b and c are called the **means**. When four numbers are in proportion, the product of the extremes is equal to the product of the means.

$$\text{i.e. if } \frac{a}{b} = \frac{c}{d}, \text{then } a \times d = b \times c$$

where, a and d are the extremes and b and c are the means.

Example 11: A 4 inch long and 6 inch wide photo is scaled proportionally. Find the width of the new scaled photo if it is 6 inch long.

Solution: Let the width of the new photo = x

$$\frac{4}{6} = \frac{6}{x}$$

$$x = \frac{6 \times 6}{4} = 9$$

Hence, the scaled photo will be 9 inches wide.

The concept of proportion is not restricted to only two equal ratios. It can be extended to more than two equal ratios.

If $a/b = c/d = e/f = g/h$, then a, b, c, d, e, f, g and h are said to be in proportion.

Example 12: Find the fraction which bears the same ratio to 1/27 that 3/11 does to 5/9.

Solution: Let x be the fraction.

$$\therefore \frac{x}{\frac{1}{27}} = \frac{\frac{3}{11}}{\frac{5}{9}}$$

$$x = \frac{\frac{3}{11} \times \frac{1}{27}}{\frac{5}{9}} = \frac{1}{55}$$

A. CONTINUED PROPORTION

If $a/b = b/c$, then a, b, and c are said to be in continued proportion. In this case, b is called the **mean proportional** and it is also the geometric mean of a and c, as $b^2 = ac$

Example 13: Three numbers are in continued proportion. Their mean proportional is 10 and the sum of the other two is 29. Find the numbers.

Solution: Let a, b and c be the numbers which are in continued proportion.
Then, $b = 10$
$b^2 = ac = 100$ and $a + c = 29$

$$\therefore a + \frac{100}{a} = 29$$

$\therefore a^2 - 29a + 100 = 0$
$\therefore a^2 - 4a - 25a + 100 = 0$
$\therefore (a - 25)(a - 4) = 0$
$\therefore a = 25$ and $c = 4$ OR $a = 4$ and $c = 25$

B. PROPERTIES OF PROPORTIONS

1. If $a : b :: c : d$ or $a/b = c/d$, then

   i) $\frac{a}{c} = \frac{b}{d}$ ... Alternando Law

   ii) $\frac{d}{c} = \frac{b}{a}$ ... Invertendo Law

   iii) $\frac{a+b}{b} = \frac{c+d}{d}$ ... Componendo Law

   iv) $\frac{a-b}{b} = \frac{c-d}{d}$ ... Dividendo Law

   v) $\frac{a-b}{a+b} = \frac{c-d}{c+d}$ ... Componendo and Dividendo Law

2. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$, then $\frac{a+c+e}{b+d+f} = k$

   Also, if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

   and p, q, r are real numbers, then $\frac{pa^n + qc^n + re^n}{pb^n + qc^n + rf^n} = k^n$

   However, if $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}, \ldots$ are not equal, then $\frac{a+c+e+\cdots}{b+d+f+\cdots}$ lies between the highest

   then $\frac{a+c+e+\cdots}{b+d+f+\cdots}$ lies between the highestand lowest of the given fractions.

Example 14: Solve the following equation: $\frac{(7x+5)^2 + (7x-5)^2}{(7x+5)^2 - (7x-5)^2} = \frac{25}{24}$

Solution: Using the Componendo and Dividendo law, $\frac{(7x+5)^2}{(7x-5)^2} = \frac{49}{1}$

$$\frac{(7x+5)^2}{(7x-5)^2} = \frac{49}{1}$$

Taking square roots, $\frac{7x+5}{7x-5} = \frac{7}{1}$ or $\frac{7x+5}{7x-5} = \frac{-7}{1}$

$7x + 5 = 49x - 35$ or $7x + 5 = -49x + 35$

$42x = 40$ or $56x = 30$

$x = 20/21$ or $x = 15/28$

Example 15: If $(a + 2b + c)$, $(a - c)$ and $(a - 2b + c)$ are in continued proportion, find the mean proportional between a and c.

Solution: Since, $(a + 2b + c)$, $(a - c)$ and $(a - 2b + c)$ are in continued proportion,

$$\frac{a+2b+c}{a-c} = \frac{a-c}{a-2b+c}$$

$\therefore (a - c)^2 = (a + 2b + c)(a - 2b + c) = [(a + c) + 2b][(a + c) - 2b]$

$\therefore a^2 - 2ac + c^2 = (a + c)^2 - (2b)^2 = a^2 + 2ac + c^2 - 4b^2$

$\therefore 4ac = 4b^2$

$\therefore ac = b^2$

Hence, the mean proportional of a and c is b.

TEST 1

1. What number should be added to each term in the ratio 19 : 43, so that it becomes equal to 2 : 3?

   (a) 20 (b) 29 (c) −91 (d) −30

2. A construction company is planning to invest in a road and railway line construction in the ratio 4 : 5. If the amount invested in the railway line construction is 6 million, then how much money did the company invest in the road construction?

   (a) 14 million (b) 10.8 million (c) 4.8 million (d) 2.6 million (e) 7.5 million

3. If the incomes of A and B are in the ratio 3 : 4 and their expenditures are in the ratio 2 : 3, then find the ratio of their savings.

   (a) 1 : 1 (b) 1 : 9 (c) 1 : 2 (d) Cannot be determined

4. The total money collected for New Year celebrations in a certain building was Rs. 20,500. The ratio of the amount contributed by the people of the A wing to that contributed by the people of the B wing was 8 : 5. Also, the ratio of the amount contributed by the people of the B wing to that contributed by the people of the C wing was 2 : 3. Find the amount contributed by the people of B wing.

   (a) Rs. 5,000 (b) Rs. 2,000 (c) Rs. 2,500 (d) Rs. 3,000 (e) Rs. 500

5. 78 is divided into two parts such that the ratio between those two parts is 7 : 6. Find the product of those two parts.

   (a) 1215 (b) 2808 (c) 1512 (d) 3276 (e) 1014

6. Seats for Mathematics, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?

   (a) 2 : 3 : 4 (b) 2 : 4 : 5 (c) 2 : 3 : 5 (d) 3 : 4 : 2 (e) 4 : 5 : 3

7. During the elections for the post of a building society chairman, the ratio of the number of members with Mr. Shah to that with Mr.Raheja was 6 : 5. But 24 members from Mr. Shah's side defected and joined Mr.Raheja. Now the ratio of members with Mr. Shah to that with Mr.Raheja is 2 : 3. Find the number of members siding with Mr. Shah initially.

(a) 90 (b) 15 (c) 75 (d) 240 (e) 30

8. The sum of three numbers is 98. If the ratio of the first to the second is 2 : 3 and that of the second to the third is 5 : 8, then the second number is:

(a) 25 (b) 35 (c) 30 (d) 45 (e) 40

9. Aakash has coins of 50 paise, 25 paise ahd Rs. 1.50 in the ratio 1 : 2 : 3 (Aakash stays in a country where all are valid currency coins. Also, country where 1 Rupee equals 100 paise). How many coins of 25 paise does Aakash have, if he has got Rs.6,600 in all?

(a) 2000 (b) 2200 (c) 2400 (d) 2600 (e) 2800

10. The annual income of Mr. X and Mr. Y is in the ratio 9 : 8 and their expenditures are in the ratio 5 : 4 . If both individually manage to save Rs. 5,000, then B's expenditure is:

(a) Rs. 1,250 (b) Rs. 5,000 (c) Rs. 6,250 (d) Rs. 11,250 (e) Rs. 10,000

TEST 2

11. If $5x - 13y = 3x - 8y$, find the value of $(2x^2 + 3y^2) : (2x^2 - 3y^2)$

(a) 50 : 12 (b) 62 : 39 (c) 25 : 4 (d) 31 : 19

12. On the basis of their performance in a test, Professor Shetty distributed Rs. 798 among Vinod, Vinay and Vinit such that 6 times Vinod's share is equal to 10 times Vinay's share or 5 times Vinit's share. How much does Vinod get?

(a) 228 (b) 238 (c) 240 (d) 275 (e) 285

13. Find the fourth proportional to 3, 5 and 27.

(a) 45 (b) 16.2 (c) 135 (d) 55

14. If $\frac{x-y}{x^2-y^2} = \frac{x^2-y^2}{k}$, then $k = ?$

(a) $(x-y)(x^2-y^2)$ (b) $(xy)(x^2-y^2)$ (c) $(x+y)(x^2-y^2)$ (d) $(x-y)^2(x+y)$

15. A precious stone is accidentally broken into 2 pieces whose weights are in the ratio 4:5. The value of the stone is directly proportional to the square of its weight. What is the ratio of the total value of the original (unbroken) stone to the total value of the broken pieces?

(a) 41 : 81 (b) 81 : 41 (c) 40 : 81 (d) 81 : 40 (e) None of these

16. A and B started a business in partnership investing Rs. 20,000 and Rs. 15,000 respectively. After six months, C joined them with Rs. 20,000. What will be B's share in total profit of Rs. 25,000 earned at the end of 2 years from the starting of the business?

(a) 7500 (b) 8000 (c) 7750 (d) 7250 (e) 8250

17. A began a business with Rs. 85,000. He was joined afterwards by B with Rs. 42,500. For how much period does B join, if the profits at the end of the year are divided in the ratio of 3 : 1?

(a) 6 months (b) 9 months (c) 3 months (d) 8 months (e) 5 months

18. A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets Rs. 1,000 more than D, what is B's share?

(a) Rs. 500 (b) Rs. 1,500 (c) Rs. 2,000 (d) Rs. 2,500 (e) None of these

19. The sum of three numbers is 98. If the ratio of the first number to the second is 2 : 3 and that of the second to the third is 5 : 8, then the second number is:

(a) 20 (b) 30 (c) 40 (d) 48 (e) 58

20. If $x : y = 3 : 5$, then find the value of $(3x + y) : (5x - y)$.

(a) 49 : 25 (b) 7 : 5 (c) 36 : 25 (d) 49 : 36 (e) 36 : 16

TEST 3

21. A and B invest in a business in the ratio 3 : 2. If 5% of the total profit goes to charity and A's share of the total profit is Rs. 855, the total profit is:

(a) Rs. 1,425 (b) Rs. 1,500 (c) Rs. 1,537.50 (d) Rs. 1,576 (e) None of these

22. A, B and C jointly thought of engaging themselves in a business venture. It was agreed that A would invest Rs. 6,500 for 6 months, B would invest Rs. 8,400 for 5 months and C would invest Rs. 10,000 for 3 months. A wants to be the working member for which he was to receive 5% of the profits. The total profit earned was Rs. 7,400. Calculate the share of B in the profit.

(a) Rs. 1,900 (b) Rs. 2,660 (c) Rs. 2,800 (d) Rs. 2,840 (e) None

23. If $a : b = 3 : 4$ and $b : c = 5 : 8$, what is $a : b : c$?

(a) 15 : 20 : 32 (b) 12 : 16 : 24 (c) 9 : 12 : 20 (d) 6 : 10 : 16 (e) None of these

24. $a/b = 3/8$, $b/c = 5/3$, $c/d = 4/5$, find d/a.

(a) 1/2 (b) 1/3 (c) 2 (d) 3 (e) None

25. A, B, C subscribe Rs. 50,000 for a business. A subscribes Rs. 4,000 more than B and B subscribes Rs. 5,000 more than C. Out of a total profit of Rs. 35,000, A receives:

(a) Rs. 8,400 (b) Rs. 11,900 (c) Rs. 13,600 (d) Rs. 14,700 (e) None of these

26. The fourth proportional to 5, 8, 15 is:

(a) 18 (b) 19 (c) 20 (d) 22 (e) 24

27. Two number are in the ratio 3 : 5. If 9 is subtracted from each, the **new numbers** are in the ratio 12 : 23. The smaller number is:

(a) 27 (b) 33 (c) 39 (d) 50 (e) 55

28. Ram and Shyam select two fractions, 11/76 and 9/62, respectively. Who has selected the larger fraction?

(a) Ram (b) Shyam (c) Both fractions are equal (d) Cannot be determined

29. Ganesh brought two identical pizzas. He cut one pizza into 6 equal parts and the other one into 9 equal parts. Ramesh ate 2 pieces from the first pizza and 5 pieces from the other one. Suresh ate 3 pieces from the first one and 3 pieces from the second one. What is the ratio of pizzas eaten by Ramesh and Suresh?

(a) 16/15 (b) 15/16 (c) 14/15 (d) 15/14 (e) None of these

30. Identify the correct option:

(a) $\frac{4}{5} > \frac{5}{6} > \frac{6}{7}$ (b) $\frac{4}{5} < \frac{5}{6} > \frac{6}{7}$ (c) $\frac{4}{5} > \frac{5}{6} = \frac{6}{7}$ (d) $\frac{4}{5} < \frac{5}{6} < \frac{6}{7}$ (e) None of these

31. The ratio of ages of A and B is 11 : 8 and the sum of their ages is 38 years. Find the ratio of ages of A and B after 8 years.

(a) 4 : 3 (b) 6 : 5 (c) 5 : 4 (d) 7 : 5 (e) 3 : 2

32. Eight years ago, Ramesh was twice as old as Suresh. It the ratio of their ages is 3 : 2 now, find Suresh's present age in years.

(a) 8 (b) 15 (c) 16 (d) 24 (e) None of these

33. If $A = \frac{1}{3}, B = \frac{1}{4}$ and $C = \frac{1}{5}$, what is A : B : C?

(a) 15 : 20 : 12 (b) 20 : 15 : 12 (c) 12 : 15 : 20 (d) 15 : 12 : 20

34. Sonal and Jignesh started a business as partners. It was decided that Sonal would invest Rs. 12,000 for 5 months and Jignesh would invest Rs. 10,000 for 3 months. Sonal was also to receive 25% of the total profit as the working partner in the business. If the total profit made was Rs.6,000, how much did Jignesh get?

(a) Rs.2,500 (b) Rs.2,025 (c) Rs.1,550 (d) Rs.1,500

35. Amarjeet has divided his money in such way that the half of it goes to his wife, one-fifth of the remaining goes to his son Karan, and the rest gets equally split among his daughters Manjeet and Manpreet. If Manjeet receives Rs. 2,50,000, what is the amount received by Karan?

(a) Rs.20,600 (b) Rs.16,000 (c) Rs.1,90,500 (d) Rs.1,25,000

36. In a rare coin collection, there is one gold coin for every three non-gold coins. 10 more gold coins are added to the collection and the ratio of gold coins to non-gold coins would be 1 : 2. Based on the information, the total number of coins in the collection now becomes **[UPSC 2013]**

(a) 90 (b) 80 (c) 60 (d) 50

6 Mixtures and Alligations

I. MIXTURES

Two or more items are mixed together to form a mixture. Whenever this happens, the attributes of the mixture are dependent on the attributes of the original items and the proportion in which the items have been mixed. Since the proportion in which the items are mixed may not always be the same, the average value of the attribute is the weighted average of the items being mixed. (The concept of weighted averages has been discussed in detail along with Averages earlier.)
The formula for weighted averages where two items are mixed can be represented as follows:

$$x = \frac{w_1 x_1 + w_2 x_2}{w_1 + w_2}$$

Where x is the weighted average
x_1 and x_2 are the attributes (for example, cost, marks of students etc.) and
w_1 and w_2 are the weights (for example, weight in kilos/litres, number of students etc.)

Example 1: Three varieties of premium basmati rice costing Rs. 60 per kg, Rs. 70 per kg and Rs. 90 per kg are mixed together in the ratio of 2 : 4 : 1 respectively. Find the cost of the resultant mixture.

Solution: Here, the cost of the resultant mixture is the weighted average of the individual costs because the three varieties are mixed in different quantities. The average cost will generally tend to go towards the variety whose proportion is the highest. However, this may not always be true.

So, cost of resultant mixture $= \frac{2 \times 60 + 4 \times 70 + 1 \times 90}{2 + 4 + 1} = \frac{490}{7} =$ Rs. 70 per kg

Observe that because the proportion of the Rs. 70 type rice is more than the other two varieties combined, the average price is closest to Rs. 70 (exactly Rs. 70 in this case).

Example 2: Two varieties of milk, costing Rs. 30 and Rs. 36 per litre, are mixed in a ratio of 3 : 5 to get the final mixture. Find the cost of the final mixture.

Solution: Cost price of the final mixture is; $= \frac{3 \times 30 + 5 \times 36}{3 + 5} = 33.75$

Example 3: A shopkeeper purchased 4 quintals of tea at the rate of Rs. 110 per kg, 2 quintals of tea at the rate of Rs. 140 per kg and another 4 quintals of tea at the rate of Rs. 120 per kg. He mixed the three varieties of tea. At what selling price should he sell the final mixture of tea to get a profit of 20%?

Solution: Cost of the resultant mixture $= \frac{4 \times 110 + 2 \times 140 + 4 \times 120}{4 + 2 + 4} = \frac{1200}{10}$

= Rs.120 per kg
To get a profit of 20%,
SP = 1.2 × CP = 1.2 × 120 = Rs. 144 per kg

Example 4: A goldsmith mixes two types of alloys. He takes 6 kg of the first alloy containing gold and silver in the ratio 3:2 and 18 kg of the second alloy containing gold and silver in the ratio 2:3. What is the ratio of gold and silver in the final alloy mixture?

Solution: Here the attribute is the average proportion of gold in the alloy mixture and weight is the quantity of the alloys.

The proportion of gold in the first alloy is 3/5 and the proportion of gold in the second alloy is 2/5. The weights of the two alloys are 6 kg and 18 kg respectively.

$$\text{Average proportion of gold} = \frac{6 \times \frac{3}{5} + 18 \times \frac{2}{5}}{6 + 18} = \frac{54}{5 \times 24} = \frac{9}{20}$$

Hence, 9 out of 20 parts are gold.
Hence, the other 11 of 20 parts are silver.
∴ Ratio of gold to silver in the final alloy mixture = 9 : 11
Note: The weight of the two alloys could also have been taken in terms of the lowest possible ratio i.e. 1 : 3 instead of 6 : 18

Alternatively,
In case you are not comfortable with the method of weighted averages, you can find the actual amount of gold and silver in each alloy.
Gold in alloy 1 = (3/5) × 6 = 3.6 kg
Silver in alloy 1 = 6 – 3.6 = 2.4 kg
Gold in alloy 2 = (2/5) × 18 = 7.2 kg
Silver in alloy 2 = 18 – 7.2 = 10.8 kg
∴ Total gold in mixture = 3.6 + 7.2 = 10.8 kg
Total silver in mixture = 2.4 + 10.8 = 13.2 kg

$$\therefore \text{Ratio of gold and silver} = \frac{10.8}{13.2} = \frac{9}{11}$$

Example 5: Copper and Zinc are mixed in the ratio of 3 : 2 to form Brass. If the cost price of Copper and Brass are 25 and 27 respectively, find the cost price of Zinc.

Solution: Let price of Zinc be x.

$$\text{Hence, we have, } 27 = \frac{3 \times 25 + 2x}{3 + 2}$$

∴ 27 × 5 = 75 + 2x
∴ x = 30
Hence, cost price of Zinc is 30.

Example 6: (CSAT 2012) Two glasses of equal volume are respectively half and three-fourths filled with milk. They are then filled to the brim by adding water. Their contents are then poured into another vessel. What will be the ratio of milk to water in this vessel?

(a) 1 : 3 (b) 2 : 3 (c) 3 : 2 (d) 5 : 3

Solution: Let each glass be of capacity x.
∴ In first glass, water quantity = $x/2$ and milk quantity = $x/2$.
And in second glass, water quantity = $x/4$ and milk quantity = $3x/4$.

$$\text{So, ratio of milk to water when both the glasses are poured in a vessel} = \frac{\frac{3x}{4} + \frac{x}{2}}{\frac{x}{4} + \frac{x}{2}} = \frac{5}{3}$$

Hence, **option d.**

II. ALLIGATIONS

The basic concept of alligation is the same as that of mixtures, i.e. weighted average. Alligation helps in finding the ratio in which two weights have to be mixed to get a given consistency (or ratio) of the mixture.

$$x = \frac{w_1 x_1 + w_2 x_2}{w_1 + w_2}$$

$\therefore xw_1 + xw_2 = w_1x_1 + w_2x_2$

$\therefore w_1(x - x_1) = w_2(x_2 - x)$

$\therefore \frac{w_1}{w_2} = \frac{(x_2 - x)}{(x - x_1)}$

This is known as **the rule of alligation.**

This is especially useful for question types such as: "How much rice costing Rs. *A* per kg be mixed with rice costing Rs. *B* per kg to get a mixture costing Rs. *C* per kg?" Here, C = x, the greater of A and B corresponds to x_2 and the lesser of A and B corresponds to x_1.

Example 7: How much sugar costing Rs. 6 per kg must be mixed with 30 kg of sugar costing Rs. 9 per kg, so that the resultant mixture costs Rs. 7 per kg?

Solution: If w_1 and w_2 are the weights of the two varieties of sugar, then using the formula for mixtures,

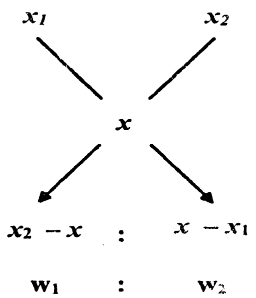
Cost of resultant mixtureis: $7 = \frac{w_1 \times 6 + w_2 \times 9}{w_1 + w_2}$

$\therefore 7w_1 + 7w_2 = 6w_1 + 9w_2$

$\therefore w_1(7 - 6) = w_2(9 - 7)$

$\therefore w_1 = 2w_2 = 2 \times 30 = 60$ kg

The alligation rule can be represented and applied in a pictorial way (alligation cross) as follows:

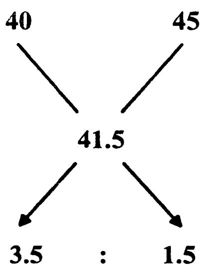


Note that the subtraction need not necessarily be $x - x_1$ and $x_2 - x$. It will be Higher Value - Average Value and Average Value - Lower Value

Example 8: An acid solution with 45% concentration is mixed with another acid solution with 40% concentration. The concentration of the resultant acid solution is 41.50%. In what ratio (acid with 45% concentration: acid with 40% concentration) were the two acid solutions mixed?

Solution: Here, the concentration of the acid in each solution is the attribute and the volume of acid used is the weight.

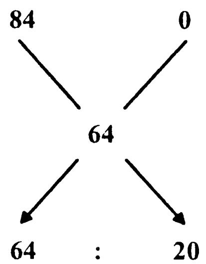
Using the alligation cross,



Hence, the ratio in which the 45% solution was mixed with the 40% solution is 1.5 : 3.5 = 3:7

Example 9: 32 litres of milk and water solution contains 84% milk. How much water should be added to this solution to reduce its concentration to 64%?

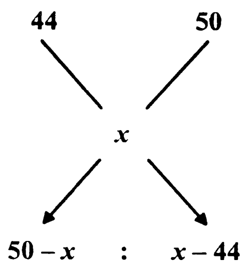
Solution: Here, the solution having 84% milk is mixed with a solution containing only water.
Concentration of milk in the first solution = 84%
Concentration of milk in the second solution = 0% [pure water contains 0% milk]
Using the alligation cross,



Ratio of the two solutions = 64 : 20 = 32 : 10
Since the weight of the first solution is 32 litres, the weight of the second solution (pure water) should be 10 litres.

Example 10: A class of 15 students got an average of 50 marks in an exam, while another class of 30 students got an average of 44 marks in the same exam. If all the students are combined into one class, then what will be the average marks of that class in the exam?

Solution: Let the average marks of the class be x.
Here, the ratio in which the two classes are mixed is known and their individual average is known.



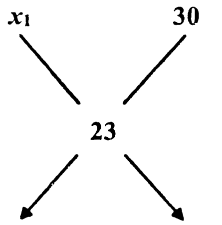
30 : 15

$$\therefore \frac{50-x}{x-44} = \frac{30}{15} = \frac{2}{1}$$

$\therefore 50 - x = 2x - 88$
$\therefore 3x = 138$
$\therefore x = 46$
Hence, the average marks of the new class will be 46.

Example 11: When 7 litres of milk at Rs. 30 per litre is mixed with 3 litres of another brand of milk, the resultant mixture costs Rs. 23 per litre. What is the cost (per litre) of the 3-litre milk brand?

Solution:



7 : $23 - x_1$

3 : 7

$\therefore \dfrac{7}{23 - x_1} = \dfrac{3}{7}$

$\therefore 69 - 3x_1 = 49$

$\therefore x_1 = 20/3 = 6.67$

Hence, the 3-litre brand of milk costs Rs. 6.67 per litre.

TEST 1

1. A shopkeeper mixes two varieties of pulses to get a mixture of pulses. He uses 1 kg and 4 kg of pulses costing Rs. 10 and Rs. 20 per kg respectively. What is the cost of the resultant mixture (in Rs. per kg)?

   (a) 11 (b) 15 (c) 17 (d) 25 (e) 18

2. A solution containing 20% water is mixed with another solution containing 40% water. In what proportion should the two solutions be mixed to get a solution containing 25% water?

   (a) 3 : 1 (b) 1 : 2 (c) 2 : 3 (d) 1 : 5 (e) 3 : 4

3. How many litres of water must be added to 20 litres of 24% solution of wine in water to make it a 10% solution of wine in water?

   (a) 17 (b) 15 (c) 24 (d) 28

4. In what ratio should two alloys with zinc and tin in the ratio 3 : 5 and 5 : 3 respectively, be mixed to get a new alloy containing zinc and tin in the ratio 1 : 1?

   (a) 1 : 1 (b) 5 : 4 (c) 2 : 3 (d) 1 : 4

5. Mr. Lal mixed coffee powder worth Rs. 2 per kg and Rs. 4 per kg and then sold the mixture at Rs. 3.75 per kg, thereby gaining a profit of 50%. In what proportion did he mix the two types of coffee powder?

   (a) 1.5 : 1 (b) 3 : 2 (c) 3 : 1 (d) 1 : 2 (e) 2 : 1

6. Milk contained in a vessel of capacity 72 litres is diluted by replacing it with water twice. After the replacement the ratio of milk to water is 25:11. Find the quantity of water added each time.

   (a) 24 litres (b) 30 litres (c) 12 litres (d) 42 litres (e) 10 litres

7. A teacher teaches two different classes having the same number of students. In one class the ratio of the number of students who passed to the number of students who failed is 3 : 4. In the other class, the same ratio is 4 : 5. The teacher wants to find the total passing percentage of all her students in both classes. The approximate value of this quantity is:

   (a) 41% (b) 44% (c) 47% (d) 50%

8. Mr. Dayal, a shopkeeper, bought two varieties of orange juice at Rs. 50 per litre and Rs. 42 per litre respectively. He mixed them in some proportion to get a drink he called 'Oranj-La', which he sold at Rs. 54 a litre, thereby making a profit of 20%.
How much of the Rs. 50 variant of juice is present in 40 litres of Oranj-La?

(a) 15 litres (b) 20 litres (c) 30 litres (d) 25 litres (e) 32 litres

9. 2 alloys, A and B, have the following composition: A →copper : zinc = 1 : 4
B →copper : gold = 1 : 5
They are mixed in the ratio 2 : 5
What is the ratio of copper to the other metals in the resultant alloy?

(a) 1 : 14 (b) 2 : 14 (c) 3 : 14 (d) 4 : 14

10. Find the ratio in which rice at Rs. 7.20 per kg is mixed with rice at Rs. 5.70 per kg to produce a mixture worth Rs. 6.30 per kg.

(a) 1 : 3 (b) 2 : 3 (c) 3 : 4 (d) 4 : 5 (e) None of these

TEST 2

11. A man mixes some quantity of inferior sugar at Rs. 2.4 per kg with superior sugar at Rs. 4 per kg in the ratio 1 : 3? At what price should he sell the rice to get a 25% profit?

(a) Rs. 3.6 (b) Rs. 4.5 (c) Rs. 5 (d) Rs. 4.2 (e) None of the above

12. There are 20, 50, 40 and 30 employees in departments A, B, C and D respectively. The average salary in departments A, B, C and D is Rs. 30,000, Rs. 20,000, Rs. 25,000 and Rs.15,000 respectively. What is the average salary of all the employees across these four departments?

(a) Rs. 24,462 (b) Rs. 23,820 (c) Rs. 21,785 (d) Rs. 20,684

13. A certain heart stimulant is supposed to contain 2% strychnine. It is prepared from two solutions that contain 10% and 0.1% strychnine respectively. If the amount of heart stimulant to be made is 10 ml, what approximate volume (in ml) of the 0.1% solution is to be used in its preparation?

(a) 1.9 (b) 2.1 (c) 7.9 (d) 8.1 (e) 9

14. 8 litres are drawn from a cask full of wine and replaced by water. This operation is performed three more times. The proportion of wine now left in the cask is 16 : 81. How much wine (in litres) did the cask hold originally?

(a) 18 (b) 24 (c) 32 (d) 42 (e) None of these

15. A vessel is filled with liquid, 3 parts of which are water and 5 parts are syrup. How much of the mixture must be drawn off and replaced with water so that the mixture is half water and half syrup?

(a) 1/3 (b) 1/4 (c) 1/5 (d) 1/7 (e) None of these

16. A container contains 40 litres of milk. From this container, 4 litres of milk were taken out and replaced by water. This process was repeated two more times. How much milk (in litres) is now contained by the container?

(a) 26.34 (b) 27.36 (c) 28 litres (d) 29.16 litres (e) None of these

17. A vessel is completely filled with a milk and water solution. The capacity of the vessel is 42 litres. 6 litres of this solution is replaced with pure water. The new concentration of milk in the milk and water solution is 30%. What was the concentration of milk in the original solution?

(a) 25% (b) 35% (c) 40% (d) 30% (e) None of these

18. A can contains a mixture of two liquids, A and B, in the ratio 7 : 5. When 9 litres of mixture are drawn off and the can is filled with liquid B, the ratio of A and B becomes 7 : 9. How many litres of liquid A were present in the can initially?

(a) 10 (b) 15 (c) 21 (d) 24 (e) 36

19. 100 employees at Grade I in an organization have an average salary of Rs. 42 per month while 150 employees at Grade II in the same organization have an average salary of Rs. 36 per month. What is the average salary (in Rs.) of one employee in the organization if these are the only grades in this organization?

(a) 36.2 (b) 40 (c) 38.4 (d) 37.8 (e) 39.2

20. A scientist mixes 80% sulphuric acid with water to get 60% sulphuric acid. If 9 litres of 80% sulphuric acid was mixed, what was the quantity of water mixed?

(a) 27 litres (b) 3 litres (c) 4.5 litres (d) 6 litres (e) None of the above

TEST 3

21. A shopkeeper has two different qualities (A and B) of cold drinks with A costing Rs. 24 per litre and B costing Rs. 21 per litre. If 3 litres of A are mixed with 5 litres of B, what is the price (per litre) of the resultant mixture?

(a) Rs. 21.95 (b) Rs. 23.05 (c) Rs. 22.50 (d) Rs. 22.13 (e) Rs. 22

22. A mixture of 3 litres in a vessel comprises milk and water in the ratio 4 : 1. What quantity of water should be added to the container so that the ratio of milk and water in the vessel becomes 3 : 2?

(a) 1000 ml (b) 980 ml (c) 975 ml (d) 1120 ml (e) None of these

23. In what ratio must sugar costing Rs. 44/kg be mixed with sugar costing Rs. 52/kg, so that the resultant mixture costs Rs. 49.50/kg?

(a) 5 : 9 (b) 7 : 11 (c)13 : 8 (d) 5 : 11 (e) 9 : 13

24. A mixture contains two types of rice mixed in the ratio 5:4 with the former type costing Rs. 45/kg. If the mixture so formed is worth Rs. 50/kg, what is the price (per kg) of the latter type of rice?

(a) Rs. 56 (b) Rs. 57.50 (c) Rs. 56.25 (d) Rs. 57 (e) Rs. 55.30

25. Mixture A has 3 kg gold and 1 kg silver. Mixture B has 5 kg gold and 3 kg silver. If A and B are mixed, what percentage of the new mixture is silver?

(a) 25% (b) 50% (c) 66.67% (d) 30% (e) 33.33%

26. Find the ratio in which rice at Rs. 32.60 per kg should be mixed with rice at Rs. 37 per kg to produce a mixture worth Rs. 34.80 per kg.

(a) 1 : 1 (b) 2 : 1 (c) 2 : 3 (d) 3 : 2 (e) None of these

27. How many kilograms of ordinary rice costing Rs. 31 per kg should be mixed with 20 kgs of basmati rice costing Rs. 47 per kg to make a profit of 25% by selling the mixture at Rs. 45 per kg?

(a) 39 kg (b) 37 kg (c) 44 kg (d) 41 kg (e) None of these

28. Find the ratio in which two varieties of coffee worth Rs. 350 per kg and 400 per kg should be mixed to make a profit of 8% on selling the mixture at Rs. 410.4 per kg.

(a) 4 : 3 (b) 3 : 2 (c) 2 : 1 (d) 1 : 4 (e) 2 : 3

29. Cake flour and pastry flour are mixed in the ratio 5:3 to form a cake batter. If the per unit cost price of the cake flour and cake batter is Rs. 360 and Rs. 350.625 respectively, what is the per unit cost price of pastry flour?

(a) Rs. 352 (b) Rs. 335 (c) Rs. 348 (d) Rs. 353 (e) Rs. 341

30. 16% concentrated mango juice syrup is mixed with 28% concentrated mango juice syrup to form a resultant mixture of 20.5% concentrated mango juice syrup. In what ratio is the 16% syrup mixed with the 28% syrup?

(a) 7 : 3 (b) 6 : 7 (c) 3 : 5 (d) 2 : 5 (e) 3 : 2

7 Variation

I. CONCEPT OF VARIATION

When one quantity changes with respect to another quantity, it is said that the two quantities vary with respect to each other or are proportional to ea,ch other.

1. DIRECT VARIATION

When an increase (or decrease) in one quantity (say y) results in a proportionate increase (or decrease) in another quantity (say x), then it is said that the two quantities are in **direct variation** or **directly proportional to each other** and the relation between them is denoted by:

$x \propto y$

and is read as "x is directly proportional to y" or "x varies directly as y".

In equation form, direct variation is written as:

$x = ky$

where k is known as the **constant of proportionality.**

Thus, if x_1 and y_1 are the original values; and x_2 and y_2 are the changed values.

$\therefore x_1 = ky_1$ and $x_2 = ky_2$

Dividing one equation by the other

$\frac{x_1}{x_2} = \frac{y_1}{y_2}$ and $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ or $x_1y_2 = x_2y_1$

If $k = 1$, change in x = change in y

If $k > 1$, change in x > change in y

If $k < 1$, change in x < change in y

Example 1: The cost of sugar is directly proportional to its quantity. If 10 kg sugar costs Rs. 200, what is the cost of 12 kg sugar?

Solution: Let the cost of the sugar be c and its quantity be q.

Since the cost of sugar is directly proportional to its quantity, their relationship can be expressed as follows,

$c = k \times q$

Substituting the first set of values of c and q,

$200 = k \times 10$

$\therefore k = 20$

Hence, the relation between c and q can now be expressed as, $c = 20 \times q$

$\therefore$ When $q = 12$,

$c = 20 \times 12 = 240$

Hence, the price of 12 kg sugar is Rs. 240.

Alternatively,

Cost of sugar is directly proportional to its quantity.

Since the quantity increases from 10 kg to 12 kg, i.e. by a factor of 1.2, the cost of sugar will also increase by the same factor i.e. it will be 1.2 times of its original value. Hence, the cost of sugar would be = 200 × 1.2 = Rs. 240

Example 2: The cost of a particular commodity is directly proportional to the square of its weight. If the cost for 5 kg of that commodity is Rs. 150, what will be the cost for 7 kg of the commodity?

Solution: Let W be the weight of the commodity and P be the cost corresponding to that weight.

$P \propto W^2$

or $P = kW^2$
For 5 kg,
$150 = k \times 5 \times 5$
$\therefore k = 6$
$\therefore P = 6W^2$
So, for 7 kg,
$P = 6 \times 7^2 = 294$
Thus, the price of 7 kg of the commodity will be Rs. 294.

☞ REMEMBER:

When one quantity (say x) varies directly as some power (say square) of another quantity (say y), then you cannot say that x is directly proportional to y; instead, x is directly proportional to the square of y. It is represented as
$x \propto y^2$

2. INVERSE VARIATION

When an increase in one quantity (say x) results in a decrease in another quantity (say y) or vice-versa, then the two quantities are said to be in **inverse proportion** or **inverse variation** and the relationship between then is expressed as,

$$x \propto \frac{1}{y}$$

which is read as "x is inversely proportional to y" or "x varies inversely as y".

In equation form, inverse variation is written as $x = \frac{k}{y}$

Where k is the constant of proportionality.
Thus, $xy = k$ Thus, if x_1 and y_1 are the original values; and x_2 and y_2 are the changed values.
$\therefore x_1y_1 = k$ and $x_2y_2 = k$

Dividing one equation by the other $\frac{x_1}{x_2} = \frac{y_2}{y_1}$ and $\frac{x_1}{y_2} = \frac{x_2}{y_1}$ or $x_1y_1 = x_2y_2$

Now, if y increases by some factor, x has to decrease by some factor.
$\therefore x \times (1/n) \times y \times n = k$
Thus, if y becomes three times its original value, x becomes one-third of its original value.

☞ REMEMBER:

When one quantity (say x) is inversely varying with some power (say square) of another quantity (say y), then you can say that x is inversely proportional to the square of y and it is represented as

$$x \propto \frac{1}{y^2}$$

Example 3: If 50 men are required to construct a bridge in 50 days, what is the number of men required to construct the same bridge in 10 days?

Solution: Here, even if you don't directly identify the question as a case of inverse proportion, it can be easily concluded using the fact that if the same work is to be done in less time, then the number of men required increases. Conversely, if there are fewer men than earlier, the work takes longer than earlier. Thus, this is clearly a case of inverse proportion.
Let the number of men required be m and the number of days taken be d.
Since these quantities are inversely proportional, their relationship can be expressed as follows,

$m = \frac{k}{d}$

Substituting the first set of values of *m* and *d*,
$k = m \times d = 50 \times 50 = 2500$

Hence, the relation between m and d can now be expressed as, $m = \frac{2500}{d}$

$\therefore$ When d = 10 the number of men required are, $m = \frac{2500}{10} = 250$

Hence, the number of men required is 250.
Alternatively,
Number of men and the days required are inversely proportional.
Since the number of days becomes 1/5 times (10/50 = 1/5), hence the number of men required will increase to 5 times, i.e. 5 × 50 or 250 men.

3. JOINT PROPORTION

Joint proportion is used to denote proportionality when one quantity varies as two or more other quantities. This could happen in three ways:
Sometimes a quantity may vary directly as one (or more quantities) or may vary inversely as one (or more quantities) or may simultaneously vary directly as well as inversely with two (or more) different quantities). Some common applications of such proportion are

Direct Variation with Multiple Quantities:
The Simple Interest (S.I.) accrued on a Principal of Rs. *P* placed for *n* years at a rate of *r* p.c.p.a is given by: $S.I. = \frac{1}{100} \times P \times n \times r$

A very common application of this is the concept of "work done".
The work done is directly proportional to the number of people working, the hourly (or daily) rate at which they work and the number of hours (or days) for which they work. Here, the constant of proportionality can be considered as 1.
Here, *W, N, r* and *h* correspond to work done, number of people, hourly (or daily) rate and number of hours (or days) respectively.

Inverse Variation with Multiple Quantities:
Consider the same example of "work done" seen earlier. If the work to be completed is the same in both cases (for instance, painting the same room), then the number of people required is inversely proportional to the product of the number of days that they work and the number of hours they work per day.

$\therefore N = \frac{W}{rh}$

Direct and Inverse Variation:
The law of gravitation between two bodies is directly proportional to the product of the mass of the two bodies and inversely proportional to the square of the distance between the two bodies. The constant proportionality in this case is the gravitational constant (denoted by G).
This relationship is expressed as:

$F = G \times \frac{m_1 \times m_2}{r^2}$

TEST 1

1. If y varies directly as x and inversely as z, and $y = 5$ when $x = 2$ and $z = 4$, find y when $x = 3$ and $z = 6$.

   (a) 1 (b) 5 (c) 10 (d) 2 (e) 4

2. P varies directly with Q, Z varies inversely with Q and A varies directly with P and inversely with Z. If $P = 27$, then $Q = 9$, $Z = 3$ and $A = 90$. What is A if $P = 81$?

   (a) 100 (b) 120 (c) 270 (d) 810 (e) None of these

3. If 200 men are required to construct a road of length 1 km in 5 days, what would be the length of the road constructed by 300 men in 5 days?

   (a) 2 km (b) 1.5 km (c) 3 km (d) 4 km

4. What is the ratio of the volume of two spheres having their radii in the ratio 1 : 2?

   (a) 1 : 2 (b) 1 : 4 (c) 1 : 8 (d) 1 : 16 (e) 1 : 32

5. The force between two charges is inversely proportional to the square of the distance between them. If the force is 20 Newtons when the distance between the two charges is 200 m, then what will be the force when the distance between the two charges increases to 2000 m?

   (a) 0.2 N (b) 20 N (c) 200 N (d) 2 N

6. The volume of a cone is directly proportional to the product of the square of the radius of its base and the height. If the radius and height of the cone are doubled, then what is the percentage change in the volume of the cone?

   (a) 100% (b) 800% (c) 300% (d) 400% (e) 700%

7. The area S of a trapezoid varies jointly as its height and the sum of its bases. If the area is 285 square metres when the height is 19 metres and the bases are 11 and 19 metres, then what is the area (in square metres) of another trapezoid whose height is 10 metres and whose bases are 10 and 15 metres respectively?

   (a) 250 (b) 150 (c) 125 (d) 100 (e) None of these

8. It takes 6 workers to lift 8 cars with 4 cranes. Also, the number of cranes (c) it takes for w workers to lift y cars varies directly as the number of cars and inversely as the number of workers. How many workers are required to lift 20 cars with 5 cranes?

   (a) 14 (b) 3 (c) 11 (d) 12 (e) 10

9. Because of an error in programming logic, a computer program calculates the area of a circle as inversely proportional to the cube of its radius, and the perimeter of the circle as inversely proportional to its area. (Constant of proportionality is the same in both cases). If the radius of the circle is 8 units, then what would be the perimeter of the circle according to the program?

   (a) 8 units (b) 16 units (c) 64 units (d) None of these (e) Data Insufficient

10. Ram found a relation between the marks of Mahesh, Ramesh and Durgesh. The relation was such that the marks of Mahesh vary jointly (and in direct proportion) with the square of the marks of Ramesh and the fourth power of the marks of Durgesh. By what percentage would the marks of Mahesh increase/decrease if the marks of Ramesh were doubled and those of Durgesh were halved?

    (a) 75% Increase (b) 75% Decrease (c) 25% Increase (d) 25% Decrease (e) None of these

TEST 2

11. 8 students working for 5 hours a day can solve a certain number of problems in 9 days. How many boys are needed to solve five times the original number of problems, if they work at 4 hours a day for 15 days?

(a) 12 (b) 30 (c) 45 (d) 10 (e) 24

12. At the rate of 28 lines per page, a book has 300 pages. If the book has to contain only 280 pages, how many lines should a page contain?

(a) 32 (b) 30 (c) 29 (d) 28 (e) 35

13. 24 people can construct a house in 15 days. But the owner would like to finish the work in 12 days. How many more workers should he employ?

(a) 6 (b) 7 (c) 8 (d) 9 (e) 10

14. It is known that current (I) in an electric circuit is inversely proportional to the resistance (R) in the circuit. When the resistance is 3 ohms, the current is 2 amperes. Find the resistance if the current is 5 amperes; and find the current when the resistance is 5 ohms.

(a) 1 ohms and 1 amperes respectively (b) 2 ohms and 1 amperes respectively
(c) 2 ohms and 1.2 amperes respectively (d) 1.2 ohms and 1.2 amperes respectively
(e) 1.2 ohms and 1 amperes respectively

15. A diamond weighing 20 decigram costs Rs. 3,600. Find the loss incurred when it breaks into three pieces whose weights are in the ratio 2 : 3 : 5. Note that the cost of the diamond varies as the square of its weight.

(a) Rs. 1,000 (b) Rs. 1,500 (c) Rs. 1,288.90 (d) Rs. 1,200 (e) Rs. 3,258

16. The length of the shadow of a 3 m high pole at a certain time of the day is 3.6 m. What is the height of another pole, whose shadow at the same time is 54 meters long?

(a) 45 (b) 50 (c) 54 (d) 60 (e) 40

17. Electric field strength is directly proportional to the charge and inversely proportional to square of the distance between the charge and the test charge. When the charge is 1C and the distance between the charge and the test charge is 1m, the field strength is 9×10^9 N/C. Find the field strength when charge is 2C and the distance is 2m.

(a) 18×10^9 N/C (b) 9×10^9 N/C (c) 13.5×10^9 N/C (d) 4.5×10^9 N/C (e) None of the above.

18. Gravitational force of attraction between two masses is directly proportional to product of their masses and inversely proportional to square of the distance between them. When the masses are 1kg each and the distance between them is 1m, the force of attraction is 5/3 N. Find the distance in meters between two masses of 2kg each and the force of attraction is 5/3 N.

(a) 2 (b) 4 (c) 5/3 (d) 1 (e) 3

19. The maximum load that a cylindrical column with a circular cross section can hold varies directly as the fourth power of the diameter and inversely as the square of the height. A 9 meter high column 2 meters in diameter will support 64 metric tons. How many metric tons can be supported by a column 9 meters high and 3 meters in diameter?

(a) 162 (b) 243 (c) 432 (d) 486 (e) 324

20. The monthly water bill of a firm has two components - a fixed charge and a variable charge that is directly proportional to the amount of water consumed in that month (in litres). In February, when 4000 litres were consumed, the bill was Rs.8,500. In March, the bill was Rs.11,000 against consumption of 6000 litres of water. What would be the bill in April if the water consumption in the firm was 6800 litres in April?

(a) Rs.12,000 (b) Rs.11,835 (c) Rs.11,750 (d) Rs.12,860

TEST 3

21. The weight of an iron bar is directly proportional to its length. If a 25 cm long iron bar weighs 287.5 gms, what is the weight (in gms) of a 36 cm long iron bar?

(a) 384.6 (b) 414 (c) 392 (d) 408.5 (e) None of these

22. 1 year ago, Akash's weight was 80 kg on earth and 6 kg on the moon. A person's weight on earth is directly proportional to his weight on the moon? What will be his weight on the moon if he now weighs 98 kgs on earth?

(a) 7.35 kg (b) 9.8 kg (c) 8.1 kg (d) 9.21 kg (e) 7 kg

23. Two sound waves are travelling in a uniform medium at a constant speed. The frequency of a sound wave varies inversely as its wavelength. If the ratio of their wavelengths is 3 : 2, what will be the ratio of their frequencies?

(a) 1 : 3 (b) 3 : 2 (c) 2 : 1 (d) 1 : 1 (e) None of these

24. q varies inversely as p and $p = 4$ when $q = 54$. Find p when $q = 8$.

(a) 42 (b) 39 (c) 27 (d) 31 (e) 22

25. If the current in a circuit is 65 amps when the resistance is 44 ohms, what is the current in a circuit when resistance is 52 ohms at a constant voltage? Current and resistance are inversely proportional to each other for constant voltage.

(a) 55 amps (b) 70 amps (c) 63 amps (d) 69 amps (e) 49 amps

26. If x varies inversely as the cube of y; and $x = 2$ when $y = 5$, find x when $y = 2$.

(a) 5 (b) 15.5 (c) 27 (d) 31.25 (e) 10.50

27. The length of a wire varies directly as its electrical resistance. If a wire 5 m long has a resistance of 12.5 ohms, what length of wire has a resistance of 35 ohms?

(a) 14.8 m (b) 14.52 m (c) 15 m (d) 14 m (e) 15.3 m

28. If Rohan covers a certain distance in 4 hours and 12 minutes at a speed of 60 kmph, how long will it take for Rohan to cover the same distance at 80 kmph?

(a) 2 hours and 58 minutes (b) 3 hours and 15 minutes
(c) 2 hours and 24 minutes (d) 3 hours and 9 minutes
(e) 2 hours and 35 minutes

29. If 5 men are required to construct a wall of 3 m in 2 days, what would be the length of the wall constructed by 8 men in 3 days?

(a) 6.5 m (b) 6 m (c) 6.8 m (d) 7 m (e) 7.2 m

30. Two points (4, 15) and (3, y) are present on the graph of a line having the equation $xy = k$. What is the value of y?

(a) 18 (b) 17 (c) 21 (d) 16 (e) 20

8 Time and Work

I. INTRODUCTION

When a person performs a certain activity, he/she does some work. The concept of time and work directly emanates from the principles of joint variation or joint proportionality. The basic equation involved in the concept of work done is:

Work Done = Number of people × Rate of working per unit time × Amount of time
i.e Work Done = number of men × Hours/Day × number of days.

II. CONCEPT OF UNIT WORK

Work is generally considered as 1 unit.
In this chapter, assume (unless otherwise explicitly mentioned) that if a person does some work in a certain number of days, he does equal amounts of work on each of those days.
In general, if a person takes *n* days to complete some work, then in one day he finishes $1/n^{th}$ of the work.

$$\therefore \text{The number of days to complete the work } = \frac{1}{\text{work done in one day}}$$

Example 1: If A completes $2/3^{rd}$ of some work in one day, in how many days can he finish it?

Solution: Work done in 1 day = 2/3

$$\text{Total days needed to complete the work} = \frac{1}{\text{work done in 1 day}} = \frac{3}{2} = 1.5 \text{ days}$$

Hence, A will complete the work in 1.5 days.

In general, if *n* persons complete a work in *d* days, 1 person will complete the same work in *nd* days, and *m* persons will do it in *nd*/*m* days.

Moreover, if two people do some work, and the first can complete it in *n* days, while the second takes *m* days to do the same; then in one day they can together do (1/*n* + 1/*m*) work; i.e. they can complete the work in [1/(1/*n* + 1/*m*)] days i.e. in (*nm*)/(*n* + *m*) days.

The same technique can also be extended to more than two people.

Example 2: If A alone completes a project in 6 days and B alone completes the same project in 4 days, then in how many days will they complete the project if **they both work** together?

Solution: A alone completes the project in 6 days.
Hence, work completed by A in 1 day = 1/6
B alone completes the project in 4 days.
Hence, work completed by B in 1 day = ¼

$$\text{Hence, work completed by A and B together in 1 day} = \frac{1}{6} + \frac{1}{4} = \frac{5}{12}$$

$$\text{Number of days to complete the work } = \frac{1}{\text{work done in one day}} = \frac{12}{5} = 2.4 \text{ days}$$

Hence, A and B working together can finish the work in 2.4 days.

Example 3: A and B together complete some work in 5 days, while A alone takes 15 days to complete it. How much time would B take to complete the work if he is working all alone?

Solution: Let B working alone take *b* days to complete the work.
Hence, work completed by B in one day = 1/*b*

A takes 15 days to complete the work alone.
Hence, work completed by A in one day = 1/15
Work done by A and B together in 1 day = Work done by A in one day + Work done by B in one day

$= \frac{1}{15} + \frac{1}{b} \quad ...(i)$

Also, A and B can together complete the work in 5 days.
∴Work done by A and B together in 1 day
= 1/5 ... (ii)
Equating (i) and (ii),

$\frac{1}{15} + \frac{1}{b} = \frac{1}{5}$

$\therefore \frac{1}{b} = \frac{1}{5} - \frac{1}{15}$

$\therefore \frac{1}{b} = \frac{2}{15}$

$\therefore b = \frac{15}{2} = 7.5$ days

Hence, B alone will take 7.5 days to finish the work.

Example 4: Dhruv can complete a piece of work in 8 days while Sameer can complete the same work in 12 days. They work together for 3 days. Then Dhruv quits the work. In how many days will Sameer now be able to finish the remaining work?

Solution: Dhruv alone completes the work in 8 days. So he does $1/8^{th}$ of the work in 1 day.
Sameer alone completes the work in 12 days. So he does $1/12^{th}$ of the work in 1 day.

Hence, total work completed together in 1 day $= \frac{1}{8} + \frac{1}{12} = \frac{5}{24}$

Sameer and Dhruv work together for only 3 days.

$\therefore$ Work completed together in 3 days $= 3 \times \frac{5}{24} = \frac{15}{24}$

$\therefore$ Amount of work that Sameer has to completealone $= 1 - \frac{15}{24} = \frac{9}{24}$

$\therefore$ Number of days that Sameer will take to complete the work $= \frac{9}{24} \div \frac{1}{12} = 4.5$ days

Hence, Sameer can finish the remaining work in 4.5 days.

III. USE OF JOINT PROPORTION TO SOLVE TIME-WORK PROBLEMS

Work done is a function of the number of people working on it, the time they spend on it on a unit basis (per hour, per day etc) and the total duration (number of hours, days etc). This joint proportionality is used to solve problems where either the number of workers or per unit time or the total duration or the total volume of work change. Please note that multiple quantities can change here. The equation governing this relationship is:

$$\frac{W_1}{W_2} = \frac{n_1 \times r_1 \times d_1}{n_2 \times r_2 \times d_2}$$

Here, *W*, *n*, *r* and *d* correspond to total work, number of workers, rate per day (or hour) and number of days (or hours respectively).
Thus, if the work to be done is building a wall, then work done is equivalent to the volume of the wall i.e. the product of the length, breadth and height of the wall, and so on.
The above formula could be altered and used for any other unit of time per day (not just number of hours per day).

Example 5: If a group of 15 stylists work for 6 hours a day, they can create a wedding collection in 30 days. If they have only 20 days, and they decide to put in 9 hours per day, then how many stylists are needed?

Solution: Here, the same wedding collection needs to be created in both cases. So, the work done is the same in both cases, say W units.
Let the number of stylists required be x.

$$\therefore \frac{W}{W} = \frac{15 \times 6 \times 30}{x \times 9 \times 20}$$

$$\therefore x = \frac{15 \times 6 \times 30}{9 \times 20} = 15$$

Thus, 15 stylists are needed.

Example 6: A swarm of 70 worker-bees are capable of building a hive in 60 hours. If, 20 hours after they start, 10 more bees join in, then how many hours will the bees take to build the rest of the hive?

Solution: Let the work done by 70 bees in 60 hours be W.
So, the work done by 70 bees in the first 20 hours is $W/3$ (as 20 hours is $1/3^{rd}$ of the total time required.
Now, work left = $2W/3$.
The number of bees working now = 70 + 10 = 80
The rate of working per hour is the same in both cases.
Let the number of hours now taken be h.

$$\therefore \frac{W/3}{2W/3} = \frac{70 \times 20}{80 \times h}$$

$$\therefore h = \frac{70 \times 20 \times 2}{80} = 35 \text{ hours}$$

Thus, it will take 35 hours for the bees to build the rest of the hive.

Example 7: Ravi and Kumar are working on an assignment. Ravi takes 6 hours to type 32 pages on a computer, while Kumar takes 5 hours to type 40 pages. How much time will they take, working together on two different computers to type an assignment of 110 pages?

Solution: Number of pages typed by Ravi in 1 hour = 32/6 =16/3
Number of pages typed by Kumar in 1 hour = 40/5 = 8
Number of pages typed by both in 1 hour

$$= \left(\frac{16}{3} + 8\right) = \frac{40}{3}$$

$$\therefore \text{Time taken by both to type 110 pages} = \frac{110}{40/3} = 8\frac{1}{4} \text{ hours}$$

Example 8: (CSAT 2011) A village having a population of 4000 requires 150 liters of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. The water of this tank will last for

(a) 2 days (b) 3 days (c) 4 days (d) 5 days

Solution: Note that 1000 litres = $1(\text{metre})^3$

$$\text{Thus, daily need of water} = \frac{4000 \times 150}{1000} \text{ m}^3$$

Total water in the tank = $(20 \times 15 \times 6)\ m^3 = 1800\ m^3$
Number of days water can last = 1800/600 = 3
Hence, **option b.**

IV. CONCEPT OF EFFICIENCY

If A is twice as efficient as B, it implies that A takes half the time as B.
So, if A is n times as efficient as B, then

$$\text{Number of days taken by } A = \frac{\text{Number of days taken by } B}{n}$$

However, if A is two times *more* efficient than B, it implies that A is thrice as efficient as B or A takes $1/3^{rd}$ the time that B takes.

Example 9: Sakshi can do a piece of work in 20 days. Tanya is 25% more efficient than Sakshi. The number of days taken by Tanya to do the same piece of work is:

Solution: Ratio of times taken by Sakshi and Tanya = 125: 100 = 5:4
Suppose Tanya takes x days to do the work.
5 : 4 :: 20 : x

$$\therefore x = \frac{20 \times 4}{5} = 16$$

Hence, Tanya takes 16 days to complete the work.

Example 10: If A takes 5 days to complete a job and B is twice as efficient as A, then in how many days can they finish the job together?

Solution: Number of days taken by A to finish the job = 5
B is twice as efficient as A.
So, B will take half the number of days as A.
∴Number of days taken by B to finish the job = 5/2

$$\text{Job completed together in 1 day} = \frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$

Number of days to complete the job = 5/3 = 1.667 days
Hence, A and B working together can finish the job in 1.667 days.

REMEMBER:

As efficiency increases, the number of days taken/required to complete the work decreases.

V. PIPES AND CISTERNS

The concept of pipes and cisterns is an extension of the concept of work. Pipes are of two types - inlet and outlet. Inlet pipes fill the tank, while outlet pipes empty the tank. Work done by an inlet pipe is treated as positive work and that done by an outlet pipe is treated as negative work.
Total tank filled = Portion of Tank filled by inlet pipes - Portion of Tank emptied by outlet pipes

Example 11: A tank has to be filled with pipes A and B. Pipe A can fill the tank in 6 hours and pipe B can fill it in 10 hours. In how much time will these pipes fill up the tank if both are opened simultaneously?

Solution: Pipe A can fill the tank in 6 hours. So, portion of tank filled by A in 1 hour = 1/6
Pipe B alone fills the tank in 10 hours. So, portion of tank filled by B in 1 hour = 1/10

$$\therefore \text{Portion of Tank filled by both A and B in 1 hour} = \frac{1}{6} + \frac{1}{10} = \frac{4}{15}$$

Number of hours taken to fill the tank = 15/4 = 3.75 hours
Hence, pipes A and B can fill the tank in 3.75 hours.

Example 12: An empty tank is connected to pipes A, B and C. Pipes A and B are inlet pipes and they fill the tank in 4 hours and 3 hours respectively while pipe C is an outlet pipe and it empties the tank in 2 hours. Find the time in which the tank will fill up if all the pipes are opened simultaneously.

Solution: Pipe A alone fills the tank in 4 hours. So, portion of tank filled by A in 1 hour = 1/4
Pipe B alone fills the tank in 3 hours. So, portion of tank filled by B in 1 hour = 1/3
Pipe C alone empties the tank in 2 hours. So, portion of tank emptied by C in 1 hour = 1/2
Portion of tank filled in 1 hour = Tank filled by

A + Tank filled by B − Tank emptied by C $= \frac{1}{4} + \frac{1}{3} - \frac{1}{2} = \frac{1}{12}$

∴ Number of hours to fill the tank = 12/1 = 12 hours
Thus, when all three pipes are opened simultaneously, the tank gets filled in 12 hours.

VI. CONCEPT OF TOTAL ASSUMED WORK – ALTERNATE METHOD

Problems on time and work done can also be solved by assuming the total amount of work done. This assumed work is derived from the time taken by each person/machine to complete the work. The following examples illustrate this concept and explain the steps involved.

Example 13: A tank has 3 pipes connected to it. There are 2 inlet pipes which can independently fill the tank in 12 hours and 10 hours respectively. There is also an output pipe which can independently empty the tank in 15 hours. The three tanks are simultaneously opened when the tank is empty. All three pipes are simultaneously closed after 7 hours. What percentage of the tank is filled?

Solution: Let the two inlet pipes be A and B while the output pipe be C respectively.
Hence, A, B and C can fill or empty the tank in 12, 10 and 15 hours respectively.
Step I: Assume the total work to be done. Here, the capacity of the tank is the total work to be done. The total capacity of the tank can be assumed to be some number such that the three pipes can fill or empty it in the required time.
The total capacity of the tank can be assumed to be the L.C.M of the time taken by each pipe to fill or empty the tank.
Hence, total capacity of the tank = L.C.M (12, 10, 15) = 60 litres.
Hence, A can independently fill 60/12 = 5 litres/hour.
Similarly, B can independently fill 60/10 = 6 litres/hour.
Finally, C can independently empty 60/15 = 4 litres/hour.
Step II: Apply these values in the given conditions.
As seen above, if all 3 pipes are opened together, they can fill A + B − C litres/hour.
Hence, the 3 pipes fill 5 + 6 − 4 = 7 litres/hour.
All 3 pipes are opened for 7 hours.
Hence, in 7 hours, the total amount filled = 7 × 7 = 49 litres.

Hence, percentage of tank filled $= \frac{49}{60} \times 100 = 81.77\%$

Example 14: Professor Khadooschand punished 3 naughty students Abba, Dabba and Jabba by giving them some homework. He gave them a maximum of 120 minutes to complete the problems. He also allowed a break of 20 minutes, on the condition that this break would be counted as part of the 120 minute session. Abba, Dabba and Jabba figured out that they could have completed the problems by themselves in 3 hours, 5 hours and 2 hours respectively. Abba being the most sincere student among the 3 started solving the problems, while the other two students rested. He did so for half an hour before feeling tired. He then called out for help. Hence, Dabba and Jabba replaced him and started working. They worked hard for 40 minutes after which all 3 students took a break. At this point, 1 student got a call from his girlfriend and started chatting away. The other two students worked together and completed the work before the deadline elapsed. Which student got the call?

Solution: This problem involves a lot of extraneous information. First, filter the data required to assume the total work.

Abba, Dabba and Jabba could independently solve all the problems in 3, 5 and 2 hours respectively.

Assume the total number of problems to be a multiple of 3, 5 and 2, say 30.

Hence, there were 30 problems to be solved.

Hence, Abba, Dabba and Jabba can respectively solve 10, 6 and 15 problems per hour.

The basic assumption in such problems is that the rate of working of each person is constant.

Abba worked alone for half an hour.

Since he could solve 10 problems in an hour, he completed 5 problems in 30 minutes.

Now, Dabba and Jabba replaced him and worked for 40 minutes.

Hence, only Dabba and Jabba were working for the next 40 minutes.

40 minutes is 2/3rd of an hour.

Hence, the total number of problems solved in the next 40 minutes is

$$\left(\frac{2}{3} \times 6\right) + \left(\frac{2}{3} \times 15\right) = 4 + 10 = 14 \text{ problems.}$$

Thus, at the end of 30 + 40 = 70 minutes, 5 + 14 = 19 problems were solved.

Now, all three took a break of 20 minutes.

Thus, at the end of 90 minutes, 19 problems were solved.

Now, for the next 30 minutes, only 2 students worked, while the third was on the phone.

The entire sequence can be represented as shown in the figure below.

30 min	**40 min**	**20 min**	**30 min**
Abba	**Dabba + Jabba**	**Break**	**Jabba + Abba**
10/2 = 5 prob	**(40/60) × (6 + 15) = 14 prob**		**(1/2) × (10 + 15) = 12.5 prob**

Hence, the two students must be such that they could have completed 11 problems in 30 minutes.

Abba, Dabba and Jabba could have completed 5, 3 and 7.5 problems respectively in 30 minutes.

Consider each combination.

Abba + Dabba together: 8 problems in half an hour

Dabba + Jabba together: 10.5 problems in half an hour

Jabba + Abba together: 12.5 problems in half an hour.

Thus, only the combination of Jabba and Abba could have completed the remaining 11 problems within half an hour.

Hence, it was Dabba who was on the phone during this time.

TEST 1

1. If A and B can complete a job in 4 and 5 days respectively when working alone, then how many days will they take to complete the job together?

(a) 50/9 (b) 20/9 (c) 1/20 (d) 9/20

2. If A and B together complete a job in 10 hours and A takes 25 hours to do the job alone, in how many hours can B alone complete the job?

(a) 30 (b) 15 (c) 16.66 (d) 6.16

3. Two outlet pipes together empty a 1300 litre tank in 7.2 minutes. What is the rate at which the tank gets empty?

(a) 325 litres/min (b) 394 litres/min (c) 180.5 litres/min (d) 135.4 litres/min

4. If a woman completes 2/3rd of a task in 1 day, then find the time taken by a man to complete the task if he is half as efficient as the woman.

(a) 0.75 (b) 1.5 (c) 3.33 (d) 3

5. If 7/8th of some work is done in 1 day, then how much work will be left after half a day?

(a) 9/16 (b) 7/4 (c) 7/8 (d) 9/7

6. It takes 16 days for 10 workers working 7 hours a day to build a room 60 m long, 5 m high and 14 m wide. How many men will be required to build a room 50 m long, 6 m high and 28 m wide if the number of days is halved but the working hours remain the same?

(a) 25 (b) 30 (c) 40 (d) 20 (e) None of these

7. Ajay and Vijay together have to complete a project. Ajay alone can complete it in 8 days while Vijay alone requires 16 days. If both of them decide to work on alternate days, starting with Ajay, then in how many days will they be able to finish the project?

(a) 10.25 days (b) 11 days (c) 10 days (d) 10.5 days (e) 11.5 days

8. Two inlet pipes A and B fill a tank completely in 4 hours and 12 hours respectively. An outlet pipe C empties the tank in 3 hours. The tank is initially full. If all pipes are opened simultaneously, then what happens to the tank after 5 hours?

(a) The tank overspills. (b) The tank remains full. (c) The tank is empty
(d) The tank empties in 2 hours. (e) None of these

9. A tank is initially full. Pipe A can empty it in 3 hours while pipes B and C fill it in 9 and 12 hours respectively. If all the pipes are opened simultaneously, then after how many hours will the tank be empty?

(a) 7.2 (b) 4 (c) 2.7 (d) 9.6 (e) 3.3

10. A man is thrice as efficient as a woman and a woman is twice as efficient as a child. If all of them, working together, complete a task in 6 days, then find the number of days that the child will take to complete the task alone.

(a) 9 (b) 18 (c) 36 (d) 54 (e) 45

TEST 2

11. A and B together complete a task in 14 days. B and C complete the same task in 8 days while A and C together complete it in 7 days. Who is the most efficient of them all?

(a) A (b) B (c) C (d) B and C

12. A and B together complete a task in 14 days. B and C together complete the same task in 8 days while A and C together complete it in 7 days. Find the number of days taken by the least efficient person to complete the task.

(a) 112 (b) 112/3 (c) 112/5 (d) 112/7 (e) 112/13

13. Arjun can do a piece of work in 7 days and Karan can do in 11 days. How long will they take to do the same work working together?

(a) 3.78 days (b) 4 days (c) 4.28 days (d) 4.5 days (e) 4.78 days

14. Pipe A can fill a tank in 4 minutes whereas pipes A and B together can fill the same tank in 3 minutes. How much time (in minutes) will pipe B take to fill the tank?

(a) 10 (b) 12 (c) 9 (d) 7 (e) 16

15. X alone can do a piece of work in 15 days and Y alone can do it in 10 days. X and Y undertook to do it for Rs. 720. With the help of Z they finished it in 5 days. How much is paid to Z?

(a) Rs. 120 (b) Rs. 75 (c) Rs. 240 (d) Rs. 90 (e) Rs. 360

16. A can complete a project in 20 days and B can complete the same project in 30 days. If A and B start working on the project together and A quits 10 days before the project is completed, in how many days will the project be completed?

(a) 21 (b) 15 (c) 11 (d) 7 (e) 18

17. A and B can together do some work in 12 days while B and C together can do it in 15 days and A and C together can do it in 20 days. Find the number of days taken by A alone to finish it.

(a) 25 (b) 30 (c) 35 (d) 40 (e) 60

18. A is twice as good a workman as B. Together they finish a piece of work in 18 days. In how many days can A alone finish the work?

(a) 24 (b) 25 (c) 27 (d) 30 (e) 36

19. 3 men can complete a piece of work in 6 days. Two days after they started the work, 3 more men joined them. How many days will they take to complete the remaining work, if all the men work at the same rate?

(a) 1 (b) 2 (b) 3 (d) 4 (e) 5

20. Two persons, A and B, working together can dig a trench in 8 hours while A alone can dig it in 12 hours. In how many hours B alone can dig such a trench?

(a) 8 (b) 12 (c) 16 (d) 18 (e) 24

TEST 3

21. A can finish a work alone in 18 days while B can finish it alone in 15 days. B worked alone for 10 days and then left the job. In how many days can A alone finish the remaining work?

(a) 5 (b) 4 (c) 7 (d) 6 (e) None of these

22. A is 30% more efficient than B. How much time will they, working together, take to complete a job which B alone could have done in 23 days?

(a) 13 (b) 15 (c) 10 (d) 17 (e) None of these

23. A can do a work in 20 days and B can do the same work in 40 days. If both A and B work with 80% efficiency, find the number of days required for both of them to finish the work, working together.

(a) 20 (b) 30 (c) 18 (d) 50/3 (e) 15

24. A does 80% of a work in 20 days. He then calls in B and they together finish the remaining work in 3 days. How many days would B alone take to do the whole work?

(a) 27 (b) 37 (c) 37.5 (d) 40 (e) None of these

25. A tank is filled in 5 hours by three pipes A, B and C. Pipe C is twice as fast as B and pipe B is twice as fast as A. How much time will pipe A alone take to fill the tank?

(a) 20 (b) 25 (c) 35 (d) None of these (e) Cannot be determined

26. Two pipes A and B can individually fill a tank in 15 minutes and 20 minutes respectively. Both the pipes are opened together but after 4 minutes, pipe A is turned off. What is the total time required to fill the tank?

(a) 10 min. 20 sec. (b) 11 min. 45 sec. (c) 12 min. 30 sec.
(d) 14 min. 40 sec. (e) None of these

27. One pipe can fill a tank three times as fast as another pipe. If together the two pipes can fill the tank in 36 minutes, then the slower pipe will be able to fill the tank alone in how many minutes?

(a) 81 (b) 108 (c) 144 (d) 192 (e) None of these

28. Three pipes A, B and C can fill a tank in 6 hours. After filling the tank together for 2 hours, C is closed and A and B can fill the remaining part in 7 hours. The number of hours taken by C alone to fill the tank is:

(a) 10 (b) 12 (c) 14 (d) 16 (e) 20

29. A does a work in 15 days and B does the same work in 10 days. What is the efficiency of the work if both of them are working together?

(a) 1/6 (b) 2/3 (c) 1/2 (d) 5/9 (e) None of these

30. A group of 10 people can complete a project in 20 hours. If, after 10 hours, 6 new people join the group, how many hours will the group take to complete the rest of the project?

(a) 8 hours (b) 6.25 hours (c) 6 hours (d) 7.5 hours

TEST 4

31. Shreya is twice as efficient as Ramya. If Ramya alone takes 6 days to complete some work, in how many days can both of them finish the work together?

(a) 3 days (b) 2.5 days (c) 3.5 days (d) 2 days

32. 4 boys and 6 girls can complete a piece of work in 10 days while 12 boys and 8 girls can do the same work in 4 days. What is the time taken by 20 boys and 30 girls to do the same work?

(a) 2 days (b) 3 days (c) 1.8 days (d) 2.2 days

33. Omkar can do a piece of work alone in 8 days. Kushal and Sanket can together do it in 4 days while Omkar and Sanket can together do it in 3 days. How long will Kushal alone take to complete the piece of work?

(a) 22 days (b) 28 days (c) 25 days (d) 24 days

34. A and B can complete a piece of work together in 3 days whereas C and D together take 4 days to complete the same piece of work. E alone takes 4 days to complete this work. In how much time will they finish the work, working together?

(a) $4\frac{1}{3}$ days (b) $\frac{9}{2}$ days (c) $1\frac{1}{5}$ days (d) $3\frac{1}{5}$ days

35. A pump can fill a tank in 2 hours. Because of a leak in the tank, it took 2 hours and 30 minutes to fill the tank. If the tank is full and the pump is closed, how much time does the leak take to empty the tank?

(a) 9 hours (b) 10 hours (c) 7 hours (d) 8 hours

36. A person gets Rs. 500 if he works for 10 hours. How much money does he earn by working for 14 hours, if he gets paid at the same rate?

(a) Rs. 700 (b) Rs. 750 (c) Rs. 775 (d) Rs. 725

37. Two inlet pipes A and B can fill a tank in 16 minutes and 20 minutes respectively. An outlet pipe C can empty the tank in 8 minutes. In how much time does an empty tank get filled if all three pipes are opened simultaneously?

(a) 13 minutes (b) 15 minutes (c) 14 minutes (d) The tank never gets filled

38. Two inlet pipes A and B can fill a tank in 10 minutes and 25 minutes respectively. Both pipes are simultaneously opened but pipe A is closed after 5 minutes. What is the time now required to fill the tank?

(a) 9.2 minutes (b) 7.5 minutes (c) 13.4 minutes (d) 11 minutes

39. 20 electric pumps, working 6 hours a day, can raise 4800 gallons of water in 4 days. What is the time taken by 16 electric pumps, working 8 hours a day, to raise 3840 gallons of water?

(a) 3 days (b) 4 days (c) 3.5 days (d) 4.2 days

40. 12 carpenters, working 6 hours a day, can make 240 tables in 20 days. How many tables will 16 carpenters, working 10 hours a day, make in 24 days?

(a) 640 (b) 560 (c) 480 (d) 520

41. In a garrison, there was food for 1000 soldiers for one month. After 10 days, 1000 more soldiers joined the garrison. How long would the soldiers be able to carry on with the remaining food? **[UPSC 2013]**

(a) 25 days (b) 20 days (c) 15 days (d) 10 days

42. The tank-full petrol in Arun's motor-cycle lasts for 10 days. If he starts using 25% more everyday, how many days will the tank-full petrol last? **[UPSC 2013]**

(a) 5 (b) 6 (c) 7 (d) 8

9 Time and Distance

I. INTRODUCTION

Physical bodies can be either stationary or in motion. Motion occurs when a body of any shape or size changes its position with respect to an external stationary point. In motion, the body can either move at a constant speed or a variable speed which includes the case of acceleration and deceleration.

The mathematical equation that describes motion has three variables namely **speed, time** and **distance** and the relationship is:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

The units have to be consistent in this relationship. This formula can be used to tackle various practical applications of time, speed and distance such as trains, boats and streams, clocks and races, circular motion and straight line motion.

A. VARIATIONS OF TIME, SPEED AND DISTANCE

1. SPEED AND DISTANCE ARE DIRECTLY PROPORTIONAL WHEN TIME IS CONSTANT.

i.e. Speed $\propto$ Distance, when Time = Constant

Example: Consider that two racers start running simultaneously from the same point in the same direction on a straight race course. The first racer runs at a speed of 10 kmph, while the second runs at a speed of 12 kmph. One hour later, a whistle is blown and both racers stop moving. Then, the ratio of the distance covered by the two racers is:

$$\frac{d_1}{d_2} = \frac{s_1}{s_2} = \frac{10}{12} = \frac{5}{6}$$

(Here, it is obvious that the time is constant; i.e. both racers run for 1 hour.)

Example: Two people, Anna and Bob, start walking simultaneously towards each other on a straight road, which is 10 km long. They meet each other 3 km from Anna's starting point. Then, the ratio of their speeds will be:

$$\frac{s_{Anna}}{s_{Bob}} = \frac{d_{Anna}}{d_{Bob}} = \frac{3}{7}$$

(Again, the time of travel of both Anna and Bob is equal. This is because they start walking simultaneously. Hence, when they meet, they have walked for the same time.)

2. DISTANCE AND TIME ARE DIRECTLY PROPORTIONAL WHEN SPEED IS CONSTANT.

i.e. Time $\propto$ Distance, when Speed = Constant

Example: Consider that a man walks for 1 hour at a speed of 5 kmph and another man walks for 2 hours, also at 5 kmph. Then, the ratio of the distance covered by the first man to that covered by the second man is calculated as:

$$\frac{d_1}{d_2} = \frac{t_1}{t_2} = \frac{1}{2}$$

(Here, the speed of both men is equal to 5 kmph.)

3. TIME IS INVERSELY PROPORTIONAL TO SPEED WHEN DISTANCE IS CONSTANT.

i.e. $\text{Time} \propto \frac{1}{\text{Speed}}$ when Distance = Constant

Example: Tarzan accidentally stepped on some thorns and could walk at only 4/5th of his normal speed. Owing to this, he takes half an hour longer than usual to reach his destination. Then, his original time period can be calculated as follows:

$$\frac{s_{original}}{s_{hurt}} = \frac{t_{hurt}}{t_{original}}$$

$$\therefore \frac{1}{4/5} = \frac{t_{original} + 30}{t_{original}}$$

$$\therefore \frac{5}{4} = 1 + \frac{30}{t_{original}}$$

$\therefore t_{original} = 120 \text{ minutes} = 2 \text{ hours}$

(The distance covered by Tarzan in both cases is equal.)

Example: A girl cycles from her house to her school at 6 kmph and reaches there 10 minutes late. Had she gone at 7 kmph, she would have made it 2 minutes early. Then, the distance from her house to her school is calculated as:

$$\frac{s_{late}}{s_{early}} = \frac{t_{early}}{t_{late}}$$

$$\therefore \frac{6}{7} = \frac{t_{late} - 12}{t_{late}}$$

$\therefore t_{late} = 84 \text{ minutes}$

Hence, Distance = 6 × 84/60 = 8.4 km

(Again, the distance covered by the girl in both cases is equal to the distance between her house and school.)

Example 1: A car covers a distance of 100 km in 3 hours and when it returns it covers the same distance in 5 hours. Find the ratio of the speed of the car in both the directions.

Solution: The distance covered in each case is the same i.e. 100 km.

Since the distance is constant, the speed is inversely proportional to the time taken.

$\therefore$ Ratio of the speeds $= \frac{S_1}{S_2} = \frac{T_2}{T_1}$

$\therefore$ Ratio of the speeds = 5 : 3

Example 2: Two cyclists start moving towards each other from their houses at the same time. Their speeds are 10 km/hr and 11 km/hr respectively. Find the meeting point if the distance between their houses is 28 km.

Solution: Since both cyclists start at the same time and also meet at the same time, the time taken by both the cyclists is constant. Hence, the distance travelled by them is proportional to their speeds.

Ratio of the distances = Ratio of the speeds = 10 : 11

Distance covered by the first cyclist $= \frac{10}{10 + 11}$ of 28 kms $= \frac{40}{3}$ km

Distance covered by the second cyclist $= \frac{11}{10 + 11}$ of 28 kms $= \frac{44}{3}$ km

Hence, the two cyclists meet 40/3 km from the first cyclist's house and 44/3 km from the second cyclist's house.

Example 3: (CSAT 2011) If a bus travels 160 km in 4 hours and a train travels 320 km in 5 hours at uniform speeds, then what is the ratio of the distances travelled by them in one hour?

(a) 8 : 5 (b) 5 : 8 (c) 4 : 5 (d) 1 : 2

Solution: Velocity of the first bus $v_1 = 160/4 = 40$ km/hr
Velocity of second bus $v_2 = 320/5 = 64$ km/hr

$$\therefore \frac{d_1}{d_2} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2} = \frac{40}{64} = \frac{5}{8}$$

Hence, **option b.**

B. CONVERSION OF UNITS

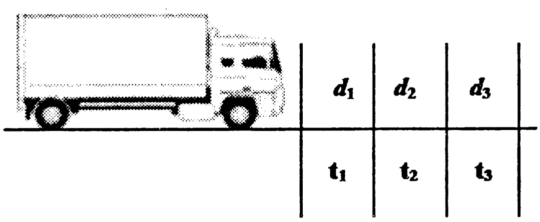
While solving problems on time, speed and distance it is important to ensure that the units of these parameters are consistent with each other. Speed is generally expressed in terms of metres/seconds (m/s) or kilometers/hour (kmph or km/hr). However, occasionally non-conventional units such as km/min or m/min may also be used. In such a case, express time in terms of minutes instead of hours or seconds. Based on the units of speed, the units for distance and time have to be used. The converse is also true.
Note that occasionally the problem may give all the units in terms of metres and seconds (or minutes), but the answer options may be in terms of km/hr. In such a case, either solve the entire problem in m/s and finally convert it to km/hr or do the conversion initially and then proceed. This judgement can also be taken based on the numerical values. Often the values given are such that initial conversion may make calculations easier.
The following conversion factors will be useful for solving problems on time and distance.

1. 1 hour = 60 minutes = 3600 seconds
2. 1 kilometre = 1000 meters
3. 1 km/hr = 5/18 m/sec
4. 1 m/sec = 18/5 km/hr

II. AVERAGE SPEED

If an object travels a particular distance at different speeds during different time-intervals, then its average speed is calculated by dividing the total distance it covers by the total time it takes to cover the total distance.
Thus, if d_1, d_2 and d_3 are the distances covered by an object in time intervals t_1, t_2 and t_3 respectively, then the average speed is given by:

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{d_1 + d_2 + d_3}{t_1 + t_2 + t_3}$$



Corollary

1. If the distance is constant, then the average speed is given by the harmonic mean of the individual speeds. If a and b are the respective individual speeds, then the average speed is given by:

$$S_{avg} = \frac{2ab}{a + b}$$

To find the average speed when more than two different speeds are involved (and the distance is constant), use the formula:

$$S_{avg} = \frac{n}{\frac{1}{s_1} + \frac{1}{s_2} + \frac{1}{s_3} + \dots + \frac{1}{s_n}}$$

where, n is the total number of individual speeds, and $s_1, s_2, \dots, s_n$ are the speeds.

2. If the time is constant, then the average speed is given by the arithmetic mean of the given speeds. If a and b are the respective speeds, then the average speed is given by,

$$S_{avg} = \frac{a+b}{2}$$

To find the average speed when more than two different speeds are involved (and time is constant), use the formula:

$$S_{avg} = \frac{s_1 + s_2 + s_3 + \dots + s_n}{n}$$

where, n is the total number of individual speeds, and $S_1, S_2, \dots, S_n$ are the speeds.

Example 4: (CSAT 2012) Mr. Kumar drives to work at an average speed of 48 km per hour. The time taken to cover the 1st 60% of the distance is 10 minutes more than the time taken to cover the remaining distance. How far is his office?

(a) 30 km (b) 40 km (c) 45 km (d) 48 km

Solution: Let Mr. Kumar take t minutes to cover 40% of the distance.
Hence, he takes $t + 10$ minutes to cover 60% of the distance.

Now, average speed is constant. Hence, we have, $\frac{t+10}{t} = \frac{3}{2}$

Hence, $t = 20$ minutes
Hence, time required to complete the journey = $2t + 10 = 50$ minutes

Hence, total distance travelled is; $\frac{50}{60} \times 48 = 40$ km

Hence, **option b.**

Example 5: If Mike covers a distance of 300 km in three stretches of 100 km each with speeds of 30 km/hr, 60 km/hr and 80 km/hr respectively, then what is the average speed of Mike throughout the journey?

Solution: Mike covers equal distances with different speeds each time. Let the speeds be a, b and c respectively. Hence, the average speed of the journey is the harmonic mean of these three speeds. Hence, the average speed is given by,

$$S_{avg} = \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}} = \frac{3abc}{ab + ac + bc}$$

$$\therefore S_{avg} = \frac{3 \times 30 \times 60 \times 80}{(30 \times 60) + (30 \times 80) + (60 \times 80)} = 48$$

Hence, S_{avg} = 48km/hr

Example 6: Siddhartha arrives at the office late every day by half an hour. On a particular day, he reduced his speed by 20% and hence arrived 45 minutes late instead. Now, if he has to arrive on time, by what percentage should he increase his speed?

Solution: Let Siddhartha's daily speed and time be s and t respectively and the distance be d.
$\therefore d = s \times t$ ... (i)

On reducing the speed by 20% he arrives 15 minutes late as compared to his usual time.
$\therefore d = 0.8s \times (t + 15)$ ... (ii)
Equating (i) and (ii),
$s \times t = 0.8s \times (t + 15)$
$\therefore t = 0.8t + 12$
$\therefore 0.2t = 12$
$\therefore t = 60$ minutes
Hence, Siddhartha takes 1 hour daily to reach his office. But even then, he is 30 minutes late. Hence, if he has to reach office in 30 minutes i.e. half of his current time then he should double his speed ($d = s \times t = 2s \times 0.5t$)
Hence, he should increase his speed by 100% in order to reach office in time.

Example 7: Travelling at 5/6th of his original speed, Manish reaches his office from home late by 5 minutes. Find the original time taken to reach office from home.

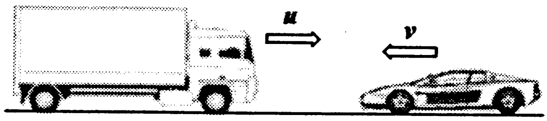
Solution: Let s be the original speed and t be the original time taken to reach office.
Hence,
$d = s \times t$ ... (i)
Travelling at 5/6th of his original speed he takes 5 minutes more. Hence,
$d = (5/6)s \times (t + 5)$ ... (ii)
Since distance is constant, equate (i) and (ii),
$s \times t = (5/6)s \times (t + 5)$
$\therefore t = (5/6) \times (t + 5)$
$\therefore t = 25$ min
Hence, he takes 25 minutes originally to reach his office.
Alternatively,
Here, since distance is constant; speed is inversely proportional to time.
So, if t is the original time and the new speed is 5/6 times the original speed, then the new time is 6/5 times of the original time. Also, it is given that the new time is 5 minutes more than the original time.
Hence, $(6/5)t = t + 5$
Hence, $t = 25$ minutes

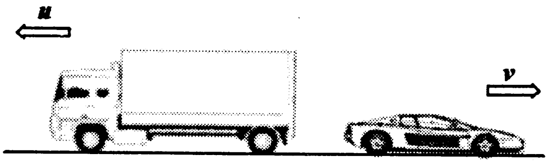
III. RELATIVE SPEED

The word **relative** means with respect to or compared to one another. Relative speed refers to the speed of an object with respect to another which may be **stationary** or **moving** in the **same** or **opposite direction**. It should be noted here that in order for the speed to be relative, the frame of reference should be in either of the two objects i.e. speed of one object with respect to the other. This can be better explained using a phenomenon that is observed every day.
Suppose you are travelling in a train (so, the frame of reference is inside this train) and observe another train coming towards you from the opposite direction on a parallel track. The speed of the second train will seem much faster than what it actually is. On the other hand, if the second train were moving at the same speed, in the same direction as your train and on a parallel track, then it will appear to be stationary when seen from your train. So, what you observe is actually the speed of the second train 'relative' to your own speed (which is equal to the speed of the first train).

i. Travelling in opposite directions

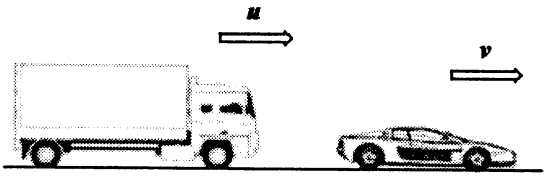
When two objects are moving in opposite directions, towards each other or away from each other (as shown in the following two figures) on a straight line with speeds u and v, then their **relative speed** = $u + v$. This applies when two objects move towards each other, cross each other and then go in opposite directions.





ii. Travelling in the same direction

When two objects are moving in the same direction on a straight line at speeds u and v (as shown in the following figure), where the faster one is either drawing closer to the slower object or moving away from the slower object, then the relative speed of one object with respect to the other = $|u - v|$.



An important point to note in relative speeds is that the starting time of all the objects should be the same. If it is not the same, the distance between these objects should be considered at some common time and then the formulae for relative speed should be applied. For instance, if Car A starts at 8:00 a.m. with a speed of 25 km/hr and Car B starts at 10.00 a.m., the start times are different. Since Car B starts after Car A, Car A would be in motion (or would have covered some distance at 10.00 a.m.). Hence, consider 10.00 a.m. as the common time. At this time, the distance covered by Car A is 2 × 25 = 50 km. Hence, the relative distance between the two cars is 50 km. Now, proceed with the relevant formulae.

Example 8: A thief escaped from a prison with a speed of 20 km/hr and after 2 hours the police followed the thief with a speed of 30 km/hr. When will the police catch the thief?

Solution: Since the police started 2 hours after the thief, the start time of the police is taken as the reference time.

In two hours, the thief would have covered 40 km.

Thus, the relative distance between the thief and the police is 40 km.

Also, because the police are chasing the thief, both are running in the same direction.

Thus, time taken by the police to catch the thief $= \frac{\text{Relative distance}}{\text{Relative speed}} = \frac{40}{30 - 20} = 4$ hours

Alternatively,

Let the speed of the thief be s_t and that of the police be s_p. Also, let the time duration of the entire chase (i.e. time from when the police started chasing to when the thief was caught) be t. Since the distance covered by both parties is the same, the ratio of the speeds will be inversely proportional to the ratio of the time periods. Thus, $\frac{s_p}{s_t} = \frac{t+2}{t}$

$\therefore \frac{30}{20} = 1 + \frac{2}{t}$

$\therefore t = 4$ hours

☞ REMEMBER:

In the above example, the first method used the concept of relative speed. Here, the frame of reference was assumed to be either the thief or the police. In the alternative method, the frame of reference was assumed to be some external stationary point and the speeds of each object was considered individually. The importance of selecting the appropriate frame of reference is clear from the above example; it was much more convenient solving it using the former method.

IV. APPLICATIONS OF RELATIVE SPEED

A. TRAINS

The concept of relative speed can be used to solve problems based on trains. When two trains are moving in the same direction or in opposite directions, the total distance required to be travelled before they cross each other completely is equal to the sum of the lengths of the two trains. This is because when you say 'cross', it means that the end of one train must pass the end of the other train, in case of trains moving in opposite directions. If the trains are moving in the same direction, then in order for them to 'cross' each other, the end of the faster train must pass the start of the slower one. This distance is covered at the relative speed of the two trains.

Let S_1 be the speed of a train of length d_1, S_2 be the speed of another train of length d_2, S_{obj} be the speed of a moving object of negligible length, and let t be the time taken for crossing.

If the train of length d_1 crosses a stationary object of negligible length, then the Time-Speed-Distance formula can be modified to: $t = \dfrac{d_1}{S_1}$

If the train of length d_1 crosses the moving object (at S_{obj}) of negligible length, then the Time-Speed – Distance formula can be modified to: $t = \frac{d_1}{S_1 \pm S_{obj}}$

The speeds are subtracted in the above case if the train overtakes the object and they are added in the object comes from the direction opposite to that of the train.

If the train of length d_1 crosses the train of length d_2 (meaning $S_1 > S_2$), then the Time-Speed-Distance formula can be modified to: $t = \dfrac{d_1 + d_2}{S_1 + S_2}$, if the trains are travelling in opposite directions

$t = \dfrac{d_1 + d_2}{S_1 - S_2}$, if the trains are travelling in the same direction

If a train of length d1 crosses a stationary object having a length d_2 (such as a railway platform), the Time – Speed – Distance formula can be modified to: $t = \dfrac{d_1 + d_2}{S_1}$

Example 9: A train crosses a man travelling in another train in the opposite direction in 10 seconds. But the train requires 30 seconds to cross the same man if the train were travelling in the same direction. If the length of the first train is 180 meters and that of the other train in which the man is sitting is 120 meters, then find the speed of the first train.

Solution: Note that the problem states that the first train takes 10 seconds to cross the man. In such a case, the train starts crossing the man when the engine of the first train is parallel to the man and it completes crossing the man when the end of the last bogey is parallel to the man.

This applies irrespective of the position of the man within the second train.

Consequently, this becomes a case of a train crossing a moving body of negligible length.

Here the length of the man's train is redundant.

However, the speed of the man becomes equal to the speed of the train in which he is travelling.

Let a be the speed of the train and b be the speed of the man (which is the speed of his train).

Thus, when the two trains are running in opposite directions, their relative speed would be the sum of their speeds. Hence,

$a + b = 180/10 = 18$ m/sec ... (i)

Similarly, when the two trains are moving in the same direction, their relative speed would be the difference of their speeds. Hence,

$a - b = 180/30 = 6$ m/sec ... (ii)

Solving (i) and (ii) simultaneously,

$a = 12$ m/sec

Hence, speed of the first train = 12 m/sec

Example 10: Two super-fast trains, which travel at speeds of 120 km/hr and 150km/hr, left the Beijing Railway Station at exactly the same time, and were headed in the same direction. Thirty minutes later, another super-fast train left Beijing Station in the same direction as the previous two. It crossed the faster train an hour and a half after it crossed the slower one. Find the third train's speed. (Note: Assume all trains to be of negligible length.)

Solution: While the start time of the first two trains is the same, the third train starts 30 minutes after the first two. Hence, consider the reference time to be the time when the third train starts.

The speeds of the first two trains are 120 km/hr and 150 km/hr respectively. Hence, in 30 minutes they will each travel 60 km and 75 km respectively. Hence, in 30 minutes, the relative distance between the third train and the slower train will be 60 km; and that between the faster train and the third train will be 75 km. Since,

$$\text{Relative Speed} = \frac{\text{Relative Distance}}{\text{Time}}$$

Hence, if the speed of the third train is u km/hr and the time taken by the third train to cross the slower train is x hours, then,

$$u - 120 = \frac{60}{x}$$

$$\therefore x = \frac{60}{u - 120}$$

$$\text{Also, } u - 150 = \frac{75}{x + 1.5}$$

$$\therefore u - 150$$

$$= \frac{75}{\frac{60}{u - 120} + 1.5}$$

$$= \frac{75(u - 120)}{60 + 1.5(u - 120)}$$

$$= \frac{75u - 9000}{1.5u - 120}$$

$$\therefore (u - 150)(1.5u - 120) = 75u - 9000$$

$$\therefore 1.5u^2 - 420u + 27000 = 0$$

$$\therefore u^2 - 280u + 18000 = 0$$

$$\therefore (u - 100)(u - 180) = 0$$

$$\therefore u = 100 \text{ or } u = 180$$

Since the third train is faster than both the other trains, its speed must be 180 km/hr.

B. BOATS

The basic equation of time, speed and distance is also useful to solve problems of boats and streams.

Downstream motion of a boat is its motion in the same direction as the flow of the stream. **Upstream** motion of a boat is its motion in the opposite direction to the flow of the stream.
If the speed of the boat in still water is S_b and the speed of the stream or river or current is S_s then,
Effective speed of the boat when it is moving downstream is $S_d = S_b + S_s$
Effective speed of the boat when it is moving upstream is, $S_u = S_b - S_s$

Do not confuse this case with the formulae for trains. Here, in downstream motion, even though the boat and stream go in the same direction, the speeds are added. This is because the stream aids the motion of the boat and thus increases the effective speed of the boat. On the other hand, in upstream motion, the speeds are subtracted even though the boat and stream move in opposite directions. This is because the stream obstructs the movement of the boat and so reduces its effective speed.

REMEMBER:

If speed of the boat upstream (S_u) and speed of the boat downstream (S_d) are given, then speed of boat (S_b) and speed of stream (S_s) can be found by using the following formula.

(Speed of boat) $S_b = \frac{S_u + S_d}{2}$

(Speed of stream) $S_s = \frac{S_d - S_u}{2}$

Example 11: If the speed of a boat in still water is 4.5 km/hr and speed of the stream is 2.5 km/hr, find the total time required by the boat to travel 28 km downstream and 12 km upstream.

Solution: $S_d = S_b + S_s = 4.5 + 2.5 = 7$ km/hr
$S_u = S_b - S_s = 4.5 - 2.5 = 2$ km/hr
Time taken for downstream travel = 28/7 = 4 hours
Time taken for upstream travel = 12/2 = 6 hours
Total time required = 4 + 6 = 10 hours

Example 12: A boat covers 18 km downstream in 4 hours and 18 km upstream in 12 hours. Find the speed of the boat in still water and speed of the stream.

Solution: Downstream speed $S_d = \frac{18}{4} = 4.5$ km/hr

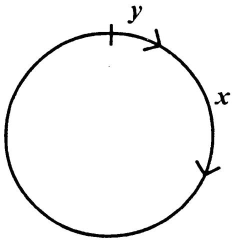
Upstream speed $S_u = \frac{18}{12} = 1.5$ km/hr

∴ Speed of boat in still water $S_b = \frac{S_d + S_u}{2} = \frac{4.5 + 1.5}{2} = 3$ km/hr

Speed of stream $S_s = \frac{S_d - S_u}{2} = \frac{4.5 - 1.5}{2} = 1.5$ km/hr

C. CIRCULAR TRACKS

In circular motion, instead of a straight track, there is a circular track where two objects can run in the same direction or in opposite directions with different speeds. In case of objects running in the **same direction**, the faster object overtakes the slower one. Whenever the faster object comes in contact with the slower object, this is known as *overlapping* or *lapping* of the slower object by the faster object.
The relative speed of **two objects** moving around a circle in the same direction, starting at the same point is taken as ($x - y$), where x and y are the speeds of the faster and slower objects respectively.



So, the time taken by the two objects, starting from the same initial point, to meet each other for the first time if they are running on a circular track in the same direction

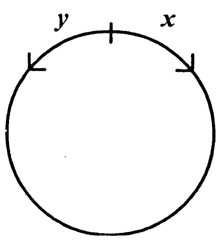
$$= \frac{\text{Circumference of the track}}{(x - y)}$$

This is because, when moving in the same direction, the distance by which the faster object would be ahead of the slower one, at the time of their first meeting, will be equal to the circumference of the track. Hence, their relative distance will be the circumference of the track.

If **more than two objects** start moving simultaneously around a circular track from the same point, in the same direction, then they will meet again for the first time at a time which is given by the LCM of the times that the fastest runner takes in totally overlapping (defined earlier) each of the slower runners.

For example, if A, B and C start moving in the clockwise direction from the same initial point on the circle where C is the fastest runner, and if we define T_{AC} as the time in which C completely overtakes A and T_{BC} as the time in which C completely overtakes B, then the LCM of T_{AC} and T_{BC} will be the time in which A, B and C will meet again for the first time.

The relative speed of two objects moving around a circle in **opposite directions**, starting from the same point is taken as $(x + y)$, where x and y are the speeds of the two objects.



Hence, the time taken by them to meet each other for the first time $= \frac{\text{Circumference of the track}}{(x + y)}$

Moreover, if two (or more) objects start moving simultaneously around a circular track in the **same or opposite directions**, starting from the same initial point, then they will meet again for the first time *at the starting point* at a time which is given by the LCM of the times taken by each object to complete one round.

Example 13: Amit and Sunil start running together on a circular track in opposite directions with speeds 30 m/sec and 50 m/sec. The length of the circular track is 600 m. When will they meet for the first time and when will they meet at the starting point for the first time?

Solution: Time taken to meet each other for the first time
= Circumference/Relative speed = 600/(30 + 50) = 7.5 seconds
Time taken by Amit to complete a round = 600/30 = 20 seconds
Time taken by Sunil to complete a round = 600/50 = 12 seconds
Time when they meet at the starting point for the first time = LCM (20, 12) = 60 seconds

Example 14: Two runners are running on a circular track of radius 14 metres. When the two runners start running simultaneously in the same direction, they meet each other every 22 seconds. When they start running simultaneously in opposite directions, they meet each other every 10 seconds. Now, the two runners run on a straight 100 metre track in the same direction. If the runners run at the same speed as before, then how much head-start (in metres) must the faster runner give the slower one, so that they both reach the finish line together?

Solution: Since the radius of the circular track is 14 metres,

Its circumference $= 2 \times \frac{22}{7} \times 14 = 88$

Let the speeds of the faster and slower runners be S_F and S_S respectively.

Now, when they run in the same direction, they meet every 22 seconds. Hence,

Relative speed, $S_F - S_S = \frac{88}{22} = 4$ ... (i)

When they run in opposite directions, they meet every 10 seconds. Hence,

Relative speed, $S_F + S_S = \frac{88}{10} = 8.8$ ... (ii)

Solving equations (i) and (ii) simultaneously,

$S_F = 12.8/2 = 6.4$ m/s

$\therefore S_S = 8.8 - 6.4 = 2.4$ m/s

Now, there are two runners, with speeds 6.4 m/s and 2.4 m/s, running in the same direction on a 100 m track. Let x be the head-start that the faster runner gives the slower one. Once the slower runner is ahead of the faster one by x metres, the time for which the two runners run before reaching the finish will be the same. Hence,

$$\frac{S_S}{S_F} = \frac{\text{Distance covered by the slower runner after he is given the head start}}{\text{Distance covered by the faster runner}}$$

$$\therefore \frac{2.4}{6.4} = \frac{100 - x}{100}$$

$$\therefore \frac{2.4}{6.4} = 1 - \frac{x}{100}$$

$$\therefore \frac{x}{100} = 1 - \frac{2.4}{6.4} = \frac{4}{6.4}$$

$$\therefore x = \frac{4}{6.4} \times 100 = 62.5 \text{ metres}$$

Hence, the faster runner gives the slower one a head-start of 62.5 metres.

Example 15: Two aeroplanes were circling each other in mid-air. When they moved in opposite directions, they met each other every 3 hours; and when they moved in the same direction, they met each other every 15 hours. They realized that when moving in opposite directions, the distance between the two planes reduced from 95 km to 10 km every 12 minutes. Determine the speed of the faster aeroplane (in km/hr).

Solution: Let the speeds of the two planes be S_1 and S_2 respectively.

It is given that when the planes moved in opposite directions, the distance between the planes reduced from 95 to 10 km in 12 minutes. So, in a time period of 12 minutes, the planes covered a relative distance of 95 – 10 = 85 km with a relative speed of $S_1 + S_2$

$$\therefore S_1 + S_2 = \frac{85}{12/60} = 425 \text{ km/hr}$$

When the aeroplanes move in opposite directions, they meet every 3 hours. Hence,

$$S_1 + S_2 = \frac{\text{Circumference of the path}}{3}$$

= 425 km/hr ... (i)
Hence, Circumference of the path = 3 × 425 = 1275 km
When the aeroplanes move in the same direction, they meet every 15 hours. Hence,

$$S_1 - S_2 = \frac{1275}{15} = 85 \text{ km/hr} \quad \text{... (ii)}$$

From (i) and (ii),
S_1 = (425 + 85)/2 = 510/2 = 255 km/hr
Hence, speed of the faster aeroplane is 255km/hr.

V. RACES

A contest of speed between contestants is called a **race**. If all the contestants reach the finishing line at the same time, then the race is called a **dead heat**. The distance run by the winner is equal to the length of the race.

A beats B by *x* meters implies that in a race of *L* meters B is *x* meters behind A, who is at the finishing line; which means that when A covers *L* meters, B has covered (*L* – *x*) meters in the same time. Here, time is constant; hence the speeds of the runners are directly proportional to the distances covered i.e .*L* and *L* – *x*

A gives B a start of *x* meters means that in a race of *L* meters, A starts the race only when B has covered *x* meters, which implies that if both A and B run at the same speeds, then when A covers *L* meters, B would have covered (*L* + *x*) meters. However, if the speeds of A and B are not equal and are unknown, it is not possible to predict the winner.

A beats B by *t* seconds implies that when A and B start together from the starting point, A reaches the finishing point *t* seconds before B finishes. It also means that if A takes *T* seconds to complete the race, B takes *T* + *t* seconds to complete the race. Hence, both cover the same distance in different times; consequently their speeds are inversely proportional to the times taken to complete the race.

A gives B a start of *t* seconds implies that A starts the race *t* seconds after B starts from the starting point. If the speeds of A and B are unknown, it is not possible to predict the winner.

A beats B by *x* meters or *t* seconds means B runs *x* meters in *t* seconds.

A can give B *x* meters (or *t* seconds) implies that even if A starts after B has covered *x* metres (or *t* seconds after B has started), A and B will reach the end point at the same time. The result of these two cases is a dead heat.

Example 16: In a 1200 m race, A can beat B by 120 m and in a 1000 m race, B can beat C by 50 m. Find the distance by which A beats C in a 800 m race.

Solution: A can beat B by 120 m in a 1200 m race.
Hence, when A covers 1200 m, B covers 1080 m.
So, when A covers 800 m, B will cover only (1080 × 800)/1200 = 720 m.
Similarly, when B covers 1000 m, C covers 950 m.
So, when B covers 720 m, C will cover (950 × 720)/1000 = 684 m.
Hence, when A covers 800 m, C will cover 684 m.
So, A will beat C by 116 m in a race of 800 m.

Example 17: A hare and a turtle decided to race each other, the race beginning at an oak tree and finishing at a pine tree. The turtle's speed was only half of that of the hare. However, the hare got caught cheating and was forced to hop back to the oak tree and start again, after it had already covered 2/3rd of the total distance. The turtle, slow but steady and honest, won the race by 8 minutes. How long did the hare take to complete the race?

Solution: Let the total distance from the oak tree to the pine tree be d kilometres. Let the hare's speed be x km/min; so the turtle's speed = $x/2$ km/min.
Hence, the hare ran $2d/3$ kilometres, got caught cheating, then ran back another $2d/3$ km to the oak tree; then finally ran d km from the oak to the pine tree.

$\therefore$ Total distance covered by the hare $= \frac{2d}{3} + \frac{2d}{3} + d = \frac{7d}{3}$

$\therefore$ Time taken by the hare to complete the race $= \frac{\frac{7d}{3}}{x} = \frac{7d}{3x}$ minutes

Time taken by the turtle to complete the race $= \frac{d}{x/2} = \frac{2d}{x}$

Since the turtle beat the hare by 8 minutes, hence
(Time taken by the hare to complete the race) – (Time taken by the turtle to complete the race) = 8

$$\therefore \frac{7d}{3x} - \frac{2d}{x} = \frac{d}{x}\left(\frac{7}{3} - 2\right) = \frac{d}{x} \times \frac{1}{3} = 8$$

$$\therefore \frac{d}{x} = 24 \text{ minutes}$$

$\therefore$ Time taken by the hare to complete the race $= \frac{7}{3} \times \frac{d}{x} = \frac{7}{3} \times 24 = 56$ minutes

TEST 1

1. A bus travels from City A to City B at a speed of 55 km/hr, from City B to City C at 110 km/hr and from City C to City A at 55 km/hr. What would be the time taken for the bus to travel from City A to City B and then to City C if the distance between each of the cities is 220 km?

   (a) 10 (b) 8 (c) 6 (d) 5

2. In a particular race, the time durations taken by three contestants to complete the race is in the ratio 8 : 3 : 6. Find the ratio of their speeds.

   (a) 6 : 3 : 8 (b) 3 : 8 : 4 (c) 1 : 2 : 3 (d) 4 : 3 : 1 (e) 3 : 8 : 6

3. Meher travelled 20% of the time walking at a speed of 10 km/hr, 50% of the time in a bus at 40 km/hr and rest of the journey in a cab at 50 km/hr. What is the average speed of Meher over the entire journey?

   (a) 37 km/hr (b) 25.97 km/hr (c) 42 km/hr (d) 40 km/hr (e) 20.68 km/hr

4. When Arun drives at a speed of 40 km/hr towards his office, he reaches late by 15 minutes; but if he drives at a speed of 60 km/hr, he reaches early by 10 minutes. Find the usual time he takes to reach his office exactly on time.

   (a) 40 min (b) 45 min (c) 50 min (d) 60 min (e) None of these

5. Two trains start simultaneously from Mumbai and Ahmedabad towards each other with speeds 80 km/hr and 100 km/hr respectively. When the two trains meet each other, it was observed that one of the trains has covered 320 km more than the other. Find the distance between Mumbai and Ahmedabad?

   (a) 2400 (b) 2560 (c) 2880 (d) 2890 (e) 1280

6. In a 100 m race, Usha beats Parvati by 10 m and Parvati beats Anuja by 5 m in the same race. By how many meters does Usha beat Anuja in the same race?

(a) 12 m (b) 15 m (c) 14.5 m (d) 14 m (e) 12.5 m

7. A particular goods train runs at a speed of 108 km/hr. It crosses a stationary pole on the way in 13 seconds. Find the length of the goods train in meters.

(a) 390 (b) 290 (c) 216 (d) 324 (e) 130

8. An airplane can travel at a speed of 1100 km/hr when it does not face resistance from air. Air currents are flowing from east to west at a speed of 100 km/hr. How many hours will the airplane take to complete a return journey from Mumbai to Delhi, if the distance between the two cities is approximately 4800 kms? (Mumbai lies in the west, whereas Delhi is in the east.)

(a) 4.5 (b) 4 (c) 8 (d) 8.8 (e) None of these

9. A ship can cover 40 miles upstream and 90 miles downstream in 10 hours. It can also cover 60 miles upstream and 60 miles downstream in 10 hours. Find the speed of the ship in still water.

(a) 10 miles/hr (b) 12.5 miles/hr (c) 15 miles/hr (d) 20 miles/hr (e) Data insufficient

10. Two trains are running in opposite directions and they are 120 m and 210 m long respectively. Find the time taken (approximately) for the two trains to cross each other if they are running with speeds 50 km/hr and 70 km/hr respectively?

(a) 5 sec (b) 8 sec (c) 10 sec (d) 12 sec (e) 20 sec

TEST 2

11. Gurmeet starts walking from point A at an uniform speed of 4 km/hr. Forty-five minutes later, Deepika starts walking in the same direction as Gurmeet from the same point. Deepika overtakes Gurmeet after 36 minutes. Find Deepika's speed.

(a) 4 km/hr (b) 5 km/hr (c) 4.5 km/hr (d) 6 km/hr (e) 9 km/hr

12. Two cyclists start from the same point at the same time in opposite directions on a circular track 1080 m long. The first cyclist travels at a speed of 43.2 km/hr and the second cyclist travels at a speed of 54 km/hr. When they meet at the starting point for the first time, how many times have they met each other including their meeting at the starting point for the first time?

(a) 4 (b) 5 (c) 8 (d) 9 (e) 10

13. An airplane covers a certain distance at a speed of 240 km/hr in 5 hours. To cover the same distance in 1(2/3) hours, it must travel at a speed (in km/hr) of:

(a) 300 (b) 360 (c) 480 (d) 600 (e) 720

14. In covering a distance of 30 km, Abhay takes 2 hours more than Sameer. If Abhay doubles his speed, then he would take 1 hour less than Sameer. Abhay's speed is:

(a) 5 km/hr (b) 6 km/hr (c) 6.25 km/hr (d) 7.5 km/hr (e) None of these

15. Robert is travelling on his cycle and has calculated to reach point A at 2 P.M. if he travels at 10 kmph. He will reach there at 12 noon if he travels at 15 kmph. At what speed (in km/h) must he travel to reach A at 1 P.M.?

(a) 8 (b) 11 (c) 12 (d) 12.5 (e) 15

16. It takes eight hours for a 600 km journey, if 120 km is travelled by train and the rest by car. It takes 20 minutes more, if 200 km is travelled by train and the rest by car. The ratio of the speed of the train to that of the car is:

(a) 2 : 3 (b) 3 : 4 (c) 3 : 2 (d) 4 : 3 (e) None of these

17. John drove for 3 hours at a rate of 50 km per hour and for 2 hours at 60 km per hour. What was his average speed for the whole journey?

(a) 52 km/hr (b) 54 km/hr (c) 56km/hr (d) 58 km/hr (e) 55 km/hr

18. A train is moving at speed of 132 kmph. If the length of the train is 110 meters, how long will it take to cross a railway platform 165 m long?

(a) 8s (b) 7s (c) 7.5s (d) 9s (e) 10s

19. Ramesh travels from Andheri to Bandra with a speed of 10 km/h and returns back with a speed of 15km/h. Find the average speed of Ramesh during the whole journey?

(a) 12 km/hr (b) 12.5 km/hr (c) 13km/ hr (d) None of these (e) Data insufficient

20. A train passes a station platform in 36 seconds and a man standing on the platform in 20 seconds. If the speed of the train is 54 km/hr, what is the length of the platform?

(a) 120 m (b) 240 m (c) 200 m (d) 300 m (e) 250 m

TEST 3

21. A car drove from Town A to Town B without stopping. The car travelled the first 40 miles of its journey at an average speed of 25 miles per hour. What was the car's average speed, in miles per hour, for the remaining 120 miles if the car's average speed (in miles per hour) for the entire trip was 40 miles per hour?

(a) 28 (b) 40 (c) 50 (d) 60 (e) 70

22. A goods train runs at the speed of 72 km/hr and crosses a 250 m long platform in 26 seconds. What is the length of the goods train?

(a) 230 m (b) 240 m (c) 250 m (d) 260 m (e) 270 m

23. The speed of a motor boat itself is 20 km/hr and the rate of flow of the river is 4 km/hr. Moving with the stream, the boat went 120 km. What distance will the boat cover in the same time if it goes against the stream?

(a) 80 km (b) 180 km (c) 60 km (d) 100 km (e) None of these

24. A man goes from city A to city B situated 60 km apart by a boat. His onward journey was with the stream while the return journey was an upstream journey. It took him four and half hours to complete the round trip. If the speed of the stream is 10 km/hr, how long did it take him to complete the onward journey?

(a) 3 hours (b) 3.5 hours (c) 2.25 hours (d) 1.5 hours (e) None of these

25. On a straight highway, 2 cars start from the same point in opposite directions. Each travels for 8 km, takes a left turn and then travels for 6 km. What is the distance between them now?

(a) 16 km (b) 20 km (c) 25 km (d) 10 km (e) Data insufficient

26. If a boat is moving upstream with a speed of 14 km/hr and goes downstream with a speed of 40 km/hr, then what is the speed of the stream?

(a) 13 km/hr (b) 14 km/hr (c) 27 km/hr (d) 20 km/hr (e) Data insufficient

27. A and B start moving in opposite directions from the same point on a circular track with speeds of 5 km/h and 10 km/h. Find the time after which they first meet if the length of the circular track is 15 km.

(a) 1 hour (b) 2 hours (c) 1.5 hours (d) 3 hours (e) None of these

28. There is a circular track with perimeter 300 meters. Two joggers, A and B, start from the same point and run at 5 m/s and 10 m/s respectively in the same direction. After what time will they meet again for the first time?

(a) 4 min (b) 3 min (c) 2 min (d) 1 min (e) None of these

29. A, B, C participate in a 200 m race, where A beats B by 20 m and C by 40 m. If B beats C by 24 m, what is the ratio of speeds of B and C?

(a) 5 : 4 (b) 4 : 3 (c) 4 : 5 (d) 9 : 8 (e) Data inconsistent

30. A, B, C participate in a 100m race, in which A beats B by 20m and B beats C by 25m. What is the ratio of speeds of A and C if all three run with a constant speed throughout the race?

(a) 5 : 4 (b) 6 : 5 (c) 5 : 2 (d) 5 : 3 (e) Cannot be determined

TEST 4

31. A car going at three-fourth of its usual speed, reaches its destination 15 minutes late. What is its usual time to cover the same journey?

(a) 48 minutes (b) 45 minutes (c) 43 minutes (d) 41 minutes

32. Aniket covers a distance of 50 km in 30 minutes. He travels at a speed of s kmph for the first 20 minutes and at 3s kmph for the remaining time. What is the value of s?

(a) 88 kmph (b) 75 kmph (c) 80 kmph (d) 60 kmph

33. Mohini covers a distance of 10 km at 30 kmph and the next 10 km at 50 kmph. What is the average speed over the entire journey of 20 km?

(a) 41.1 kmph (b) 37.5 kmph (c) 36.8 kmph (d) 39.5 kmph

34. A man takes 3 hours to row a boat 6 kms upstream and 4 hours to cover 20 kms downstream in a river. What is the speed of the river?

(a) 1.2 kmph (b) 1.5 kmph (c) 1.4 kmph (d) 1 kmph

35. If a car moves at an average speed of 60 kmph, it reaches its destination on time. When it travels at an average speed of 54 kmph, it reaches 20 minutes late. What is the distance between the source and destination?

(a) 210 km (b) 180 km (c) 195 km (d) 200 km

36. A boat covers 24 kms upstream in 12 hours and 24 kms downstream in 4 hours. What is the speed of the boat in still water and the speed of the stream?

(a) 5 kmph, 2 kmph (b) 3 kmph, 4 kmph (c) 4 kmph, 5 kmph (d) 4 kmph, 2 kmph

37. Chirag is twice as fast as Piyush and Piyush is four times as fast as Bhavesh. If Bhavesh can cover a certain distance in 48 minutes, what is the time taken by Chirag to cover the same distance?

(a) 4 minutes (b) 6 minutes (c) 6.4 minutes (d) 5.6 minutes

38. Varun covers a certain distance in 40 minutes if he walks one way and rides back the other way. He gains 12 minutes if he rides both ways. How much time (in minutes) does he take if he walks both ways?

(a) 52 (b) 50 (c) 53 (d) 51

39. Mohan has to cover 10 kms in 40 minutes. If he covers half the distance in three-fourth of the total time, what must be his speed to cover the remaining distance in the remaining time?

(a) 26 kmph (b) 29 kmph (c) 28 kmph (d) 30 kmph

40. A car is running alongside a railway track, at 30 kmph, and is 320 meters ahead of a train of length 160 meters running at 66 kmph in the same direction. In how much time will the train overtake the car?

(a) 48 seconds (b) 36 seconds (c) 5 seconds (d) 40 minutes

41. Two trains are travelling at 84 kmph and 52 kmph in the same direction. The fast train completely passes a man sitting in the slow train in 18 seconds. What is the length of the fast train?

(a) 160 m (b) 190 m (c) 140 m (d) None of the above

42. A car covers a distance of 300 km in 12 hours and a bus covers the same distance in 16 hours. What is the ratio of the speed of the car to the bus?

(a) 7 : 4 (b) 5 : 6 (c) 3 : 5 (d) 4 : 3

43. Two trains of lengths 120 m and 140 m take 12 seconds and 10 seconds respectively to cross a signal post. In what time (in seconds) will these trains cross each other when they travel in opposite directions?

(a) 9.9 (b) 11.3 (c) 10.83 (d) 8.71

44. Two runners start running together on a circular track of length 800 m, in opposite directions, with speeds of 20 m/s and 30 m/s respectively. When will they meet for the first time on the track?

(a) 15 s (b) 13 s (c) 16 s (d) 14.2 s

45. Two trains A and B start at the same time from Mumbai and Pune respectively and on the same track. The distance between the two cities is 150 km. If the two trains travel in the same direction, train A meets train B after 5 hours. If they travel in opposite directions, they meet after 3 hours. What are the speeds of A and B (in kmph)?

(a) 45, 25 (b) 40, 10 (c) 55, 30 (d) 50, 20

46. A thief running at 8 km/hr is chased by a policeman whose speed is 10 km/hr. If the thief is 100 m ahead of the policeman, then the time required for the policeman to catch the thief will be **[UPSC 2013]**

(a) 2 min (b) 3 min (c) 4 min (d) 6 min

47. A train travels at a certain average speed for distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete the total journey, what is the original speed of the train in km/hr? **[UPSC 2013]**

(a) 24 (b) 33 (c) 42 (d) 66

48. Four cars are hired at the rate of Rs. 6 per km plus the cost of diesel at Rs. 40 a litre. In this context, consider the details given in the following table: **[UPSC 2013]**

Car	Mileage (km/*l*)	Hours	Total Payment (Rs)
A	8	20	2120
B	10	25	1950
C	9	24	2064
D	11	22	1812

Which car maintained the maximum average speed?

(a) Car A (b) Car B (c) Car C (d) Car D

49. A person can walk a certain distance and drive back in six hours. He can also walk both in 10 hours. How much time will he take to drive both ways? **[UPSC 2013]**

(a) Two hours (b) Two and a half hours (c) Five and a half hours (d) Four hours

10 Number Systems

I. INTRODUCTION

Numbers are used for counting, comparing, measuring and thereby understanding quantities.
The symbols used to represent numbers are called **numerals**.
Currently, the number system most commonly in use is the **decimal number system**. The name is derived from the fact that in this system 10 different numerals are used viz. 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 to represent any quantity. Thus, any quantity can be represented by using the numerals 0 to 9 in some order. This quantity which is being represented is called a number.

II. FACE VALUE AND PLACE VALUE

Consider the number 956.

It can be written as 900 + 50 + 6. Thus the digit 9 represents the number 900, 5 represents 50 and 6 represents 6.

The digits' value is called the face value and the number it gets multiplied by represents the place value. The rightmost place is called the units place and has the place value 1. The digit to the immediate left of it is called the tens place and has the place value 10 and so on. The place value increases by a factor of 10 as one goes from right to left.

The place value of different digits in a decimal number system is as follows:

.....	Thousands	Hundreds	Tens	Units
.....	1000	100	10	1

For example, the number 5104 can be represented as follows:

5	1	0	4
1000	100	10	1

So, the value of this number is given as
$(5 \times 1000) + (1 \times 100) + (0 \times 10) + (4 \times 1) = 5000 + 100 + 0 + 4 = 5104$

III. CLASSIFICATION OF NUMBERS

A. NATURAL NUMBERS

Counting numbers start from 1. These numbers are called **natural numbers**.
Natural numbers are 1, 2, 3, 4, 5, 6...
The numbers −7, 0, 4.5 etc. are NOT natural numbers.
Natural numbers which are multiples of 2 are called **even numbers**. They are 2, 4, 6, 8...
Natural Numbers which are not divisible by 2 are called **odd numbers**. They are 1, 3, 5, 7...

B. WHOLE NUMBERS

Natural numbers combined with zero are termed as whole numbers.
Whole numbers are 0, 1, 2, 3, 4, 5, 6, 7...

C. INTEGERS

Positive and negative natural numbers, along with zero, together represent the set of integers.
Integers are ... −3, −2, −1, 0, 1, 2, 3 ...

D. FRACTIONS

To indicate that you have half a stick, it can be said that if you divide a stick into two equal parts, then you will have one part. This means you have 1 out of 2 parts of a stick. The part with you can be indicated as ½. This is known as a **fraction.**

a. PROPER FRACTION

A fraction, whose absolute value is less than 1, is called a **proper** fraction (i.e. their numerator is less than the denominator).

For example, $\frac{1}{2}, \frac{3}{4}, \frac{7}{9}$

b. IMPROPER FRACTION

A fraction, whose absolute value is greater than 1, is called an **improper** fraction (i.e. their numerator is greater than the denominator).

c. MIXED FRACTION

A **mixed** fraction is represented as $p\frac{a}{b}$

Here $a < b$ and $b \neq 0$.
Thus, a mixed fraction consists of an integer p and a
Any improper fraction can be represented as a mixed fraction and vice versa.
To convert an improper fraction into a mixed fraction, divide the numerator by the denominator. The quotient thus obtained becomes the integral part (p) of the mixed fraction, the remainder becomes the new numerator (a) and the original denominator remains the denominator (b).
For instance, convert 17/5 to a mixed fraction.
The quotient when 17 is divided by 5 is 3 and the remainder is 2.

Conversely, to convert a mixed fraction to an improper fraction, the denominator remains the same and the numerator becomes (integral part × denominator) + numerator.
Thus, in the above case, we get $(3 \times 5) + 2 = 17$
So, 17 becomes the numerator and 5 remains the denominator.

E. REAL NUMBERS

Any number which has no imaginary part in it is called a real number.

a. Rational Number

A number that can be expressed as: $\frac{p}{q}$, where $q \neq 0$, is called a rational number.

b. Irrational Number

If we consider a square plot of area 2 square metres, then each side of the square plot is $\sqrt{2}$ metres.
$\sqrt{2} = 1.4142135...$

This is neither a recurring decimal nor a terminating decimal. It cannot be expressed as a fraction either (all recurring and terminating decimals can be expressed as a fraction). Such a number, which exhibits irrational behaviour and yet exists, is called an **irrational number**

F. PRIME NUMBERS

Any number which has only 2 factors – one and the number itself is called a **prime number.**
∴ 13 is a prime number.

Primality Test

The primality test is generally used to check whether the given number is prime or not. If a number n is divisible by any number m where m can have any value from 2 to $(n - 1)$ then n is composite, otherwise it is prime.

However, instead of testing all values of m from 2 to $(n - 1)$, you need to only test for the values of m up to $\sqrt{n}$.

If you find it difficult to find all prime numbers $\leq \sqrt{n}$, then simply test if n is divisible by 2 or 3, and if not, run through all the numbers of the form $6k \pm 1 \leq \sqrt{n}(k \neq 0)$.
This works because all prime numbers (except 2 and 3) are of the form $(6k \pm 1)$.

For example, to check whether 179 is prime or not:

Step 1: Check divisibility by 2 and 3 (179 is not divisible by 2 or 3.)

Step 2: If n is not a perfect square, find a suitable value of n' which is a perfect square closest to n and greater than n.

(Here, 179 is not a perfect square. 196 is greater than 179 and is the closest perfect square.
$\therefore n' = 196$)

Step 3: Check divisibility by all numbers of the form $6k \pm 1 \leq \sqrt{(n')}$. (In this case, $\sqrt{(n')} = 14$. Therefore, $6k \pm 1$ will take the values 5, 7, 11, and 13. Now, check the divisibility of 179 by all these numbers.)

Step 4: If the number is not divisible by any of the above numbers then the given number is prime, otherwise it is composite.

G. COMPOSITE NUMBERS

Any number which has more than 2 factors is a **composite number**.
$\therefore$ 12 is a composite number.

H. DECIMALS

A collection of digits after a period (called the decimal point) is called a **decimal** fraction.
Every decimal number represents a fraction. These fractions have denominators with powers of 10.
For example, 0.45 = 45/100

I. MIXED NUMBER

A **mixed** number consists of a whole number and a fraction.

For example, $5\frac{3}{4}$ is a mixed number.

J. MODULUS

The absolute value (or **modulus**) $|a|$ of a real number 'a' is the numerical value of a without regard to its sign.
For example, the absolute value of 3 is 3, and the absolute value of -3 is also 3.

K. PERFECT NUMBERS

If the sum of all the factors of a natural number excluding the number itself, is equal to the number, then the number is known as a **perfect number**.

For example, 6 is a perfect number, because
6 = 1 + 2 + 3 and the factors of 6 are 1, 2, 3 and 6. (6 is the smallest perfect number.)

L. COMPLEX NUMBERS

Any number which is of the form $A + Bi$, where A, B are real numbers, is called a **complex** number. A is called the real part and Bi is called the imaginary part.

M. IMAGINARY NUMBERS

An **imaginary** number is a quantity of the form ix, where x is a real number and 'i' is the positive square root of -1.

Example 1: Rakesh wants to distribute a pair of chocolates to each of his students whose roll numbers are prime numbers. In all, there are 109 students in his class with roll numbers from 1 to 109. How many chocolates must he buy for this purpose?

Solution: Rakesh needs to distribute chocolates based on the number of prime numbers between 1 and 109.
From 1 to 100 there are 25 prime numbers. Also between the numbers 101 to 109, there are four prime numbers viz. 101, 103, 107 and 109. Therefore, in all he would distribute, 29 × 2 viz. 58 chocolates.

Example 2: Find out whether 337 and 343 are prime numbers.

Solution: Use the primality test. Neither 337 nor 343 are divisible by either 2 or 3. The perfect squares closest to the given numbers are 324 and 361. Here, consider larger of the two numbers.
$\therefore\ n' = 361$ and $\sqrt{(n')} = 19$
So test divisibility of the given numbers with numbers of the form $6k \pm 1$ up to 19 i.e. 5, 7, 11, 13, 17, 19.
343 is a perfect cube of 7, whereas 337 is not divisible by any of the numbers.
∴ 337 is a prime number, while 343 is not.

Example 3: What is the least number that should be added to 259 to make it a perfect square?

Solution: The perfect squares closest to 259 are $16^2 = 256$ and $17^2 = 289$
Since we need to find a number to be added, we will consider 289 as the required perfect square.
∴ Number to be added = 289 − 259 = 30

IV. BASIC MATHEMATICAL OPERATIONS

Addition, Subtraction, Multiplication and Division are some of the basic operations on numbers. When there are multiple operations involved, the BODMAS rule is followed. This implies that you first need to solve the expression within brackets, then apply division or multiplication (these are considered to be at the same level and so multiplication can also be done before division) and finally apply addition or subtraction (these are considered to be at the same level and so subtraction can be done before addition).

A. DIVISION

In case of division, the number which is being divided is called the **dividend** and the number that is dividing it, is called the **divisor**. The number of times the divisor divides the dividend is called the **quotient**.
For example, divide 7 by 3. The number 7 can be represented as 3 + 3 + 1 i.e. it includes the number 3 twice and after that it leaves the number 1, which is called the **remainder**.
Here, 7 is the dividend, 3 is the divisor, 2 is the quotient and 1 is the remainder.
Dividend = Divisor × Quotient + Remainder
If the remainder is 0, then the dividend is perfectly divisible by the divisor, and it is said to be a **multiple** of the divisor. Also, in such a case, the divisor is said to be a **factor** of the dividend.

Example 4: If 233 gives a remainder of 5 when divided by a certain natural number n, then what will be the remainder when 466 is divided by $2n$ assuming that the quotient in both cases is the same?

Solution: Let the common quotient in both cases be q.
Using the first statement,
$233 = nq + 5$
$\therefore nq = 228$
Then,
$466 = (2n)q + r = 2(nq) + r$
$\therefore\ 466 = 2(228) + r$

$\therefore r = 466 - 456 = 10$

Example 5: When a certain number is divided by 12, the remainder left is 1. However, when the same number is divided by 14, the remainder left is 5. Find the first such number.

Solution: A number which gives a remainder of 1 on dividing by 12 can be represented as $(12n + 1)$.
Further, when the same number is divided by 14, the remainder is 5. So, the same number can also be represented as $(14m + 5)$

$\therefore 12n + 1 = 14m + 5$

$\therefore 12n = 14m + 4$

$\therefore 6n = 7m + 2$

$\therefore n = \frac{7m + 2}{6}$

Since n and m are both integers, look for values of m that give an integral value of n.
This condition is satisfied for $m = 4$ and $n = 5$
Thus, the first number that satisfies this condition is:
$12n + 1 = (12 \times 5) + 1 = 61$
Or, $14m + 5 = (14 \times 4) + 1 = 61$

B. POWER

This is another function to represent numbers. If you multiply 2 by 2, you can write it as, $2^2 = 2 \times 2$
If you multiply 2 by 2 by 2, you can write it as, $2^3 = 2 \times 2 \times 2$
Similarly, If you multiply 2 by 2 n times, you can write it as, $2^n = 2 \times 2 \times 2 \times 2 \ldots n$ times
This is referred to as '2 raised to n'. It is also called as '2 to the power of n' and n is known as the index or exponent.

TEST 1

1. What is the value of the following? $1.5 + 1.8 \div 0.9 + (3 - 4) - 1.2 \times 2$

   (a) 1.2 (b) 2.4 (c) 1.5 (d) 2 (e) None of these

2. What is the value of the following? $[3 + (2 - 4) \times 7 + 3 - (8 \times 2 - 12) \div 4]$

   (a) -9 (b) -21 (c) -32 (d) 1 (e) None of these

3. What is the value of the following? $\left(5 + \left[\frac{1}{3} + \frac{1}{4}\right] \times 12 - 3\right) - 3 \times \frac{1}{3}$

   (a) 8 (b) 3 (c) 6 (d) 5 (e) None of these

4. What is the value of the following? $4\frac{1}{3} - \left\{\frac{1}{6} \times \left(3 + 2\frac{1}{5} + 5 - 4\frac{1}{5}\right)\right\}$

   (a) $\frac{11}{3}$ (b) $3\frac{1}{3}$ (c) $\frac{1}{3}$ (d) $4\frac{1}{3}$ (e) None of these

5. What is the value of $3 + 6 \div 3 \times 2$?

   (a) 7 (b) 6 (c) 4 (d) 1.5 (e) None of these

6. Find the value of: $\frac{[2^4 + (16 - 3 \times 4)]}{[(6 + 3^2) \div (7 - 4)]}$

   (a) 2.4 (b) 4 (c) 5 (d) 13.6 (e) 6.8

7. What is the value of $(7 - \sqrt{9}) \times (4^2 - 3 + 1)$?

(a) 62 (b) 60 (c) 56 (d) 48 (e) 42

8. What is the value of $(33 - 2 \times 7) + (5 \times 3 - 22)$?

(a) 8 (b) 24 (c) 186 (d) 536 (e) None of these

9. What is the value of $(15 \div 3 + 4) - (3^2 - 7 \times 2)$?

(a) 4 (b) 14 (c) 5 (d) 15 (e) 25

10. What is the value of $(3 + 2)^2 - 5 \times 3 + 2^3$?

(a) 2 (b) 6 (c) 18 (d) 38 (e) None of these

TEST 2

11. A certain number when divided by 13 leaves a remainder of 2. What will be the remainder when the square of that number is divided by 13?

(a) 2 (b) 4 (c) 5 (d) 6 (e) 7

12. A teacher asked a student to multiply a number by 4/7. Instead, he multiplied by 7/4 and obtained an answer greater than the correct answer by 99. What was the original number?

(a) 42 (b) 77 (c) 63 (d) 84 (e) None of these

13. The smallest natural number n, for which $2n + 1$ is not a prime number is,

(a) 3 (b) 5 (c) 4 (d) 6 (e) None of these

14. Find the sum of all prime numbers between 60 and 75.

(a) 199 (b) 201 (c) 211 (d) 272 (e) 276

15. If 123 yields a remainder of 13 when divided by a certain natural number n, what will be the remainder when 492 is divided by $4n$ assuming that the quotient in both cases is the same?

(a) 13 (b) 26 (c) 39 (d) 52 (e) 65

16. The largest 3-digit prime number is:

(a) 991 (b) 999 (c) 993 (d) 997

17. If x is an odd number and y is an even number, which of the following is odd?

(a) $x + y$ (b) $x + y + 1$ (c) xy (d) $xy + 2$

18. The difference between two numbers is 12238. On dividing the larger number by the smaller, we get 76 as the quotient and 13 as the remainder. What is the smaller number?

(a) 170 (b) 160 (c) 163 (d) None of these

19. For a positive fraction less than 1, the difference between the fraction and its reciprocal is 11/30. What is the value of the fraction?

(a) $\frac{5}{6}$ (b) $\frac{6}{5}$ (c) $\frac{30}{11}$ (d) $\frac{2}{3}$

20. What is the least number that should be added to 755 to make it a perfect square?

(a) 20 (b) 27 (c) 34 (d) 29

TEST 3

21. $1\frac{2}{3}-\frac{1}{6}+\frac{1}{2}-\frac{1}{3}+x=\frac{8}{3}$. What is the value of x?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

22. $\frac{\sqrt{0.81}\times\sqrt{0.25}}{\sqrt{x}}=\sqrt{2.25}$ What is the value of x?

(a) 0.03 (b) 0.09 (c) 0.05 (d) 0.04 (e) None of these

23. $438 \div 73 \times 424 \div 53 = x$. What is the value of x?

(a) 52 (b) 67 (c) 48 (d) 39 (e) 58

24. $76 \div 19 \div 3 \times 6 + 104 \div 13 \times 2 = x$. What is the value of x?

(a) 8 (b) 16 (c) 24 (d) 28 (e) 18

25. Convert $31\frac{3}{8}$ to an improper fraction.

(a) $\frac{251}{8}$ (b) $\frac{131}{154}$ (c) $\frac{5}{7}$ (d) $\frac{51}{64}$ (e) $\frac{1}{9}$

26. $\left[1+\left(\frac{1}{2}+\frac{1}{3}\right)\times 18-4\right]\times\frac{1}{2}=x$. What is the value of x?

(a) 3 (b) 5 (c) 4 (d) 2 (e) 6

27. Which among the following options is a proper fraction?

(a) $1\frac{2}{5}$ (b) $1\frac{1}{2}$ (c) $\frac{11}{9}$ (d) $\frac{15}{17}$ (e) $\frac{13}{12}$

28. $[(16)^2 \div 8 \times 14] \div 2 = 16 \times x$. What is the value of x?

(a) 11 (b) 12 (c) 18 (d) 14 (e) 16

29. $3.3 + 33.03 + 333.003 + 0.33 + 3.03 = x$. What is the value of x?

(a) 336.639 (b) 381.369 (c) 372.693 (d) 333.063 (e) 363.396

30. If the sum of the squares of two numbers is 970 and the product of those two numbers is 483, what is the absolute difference between the two numbers?

(a) 2 (b) 8 (c) 5 (d) 9 (e) 7

11 Number Theory

I. INTRODUCTION

Number theory is a branch of pure mathematics which deals with the properties of numbers. It is an important concept for most entrance examinations.

II. FACTORS AND MULTIPLES

If a natural number y completely divides a natural number x (without leaving any remainder or decimal portion), then y is called a **factor** of x. On the other hand, x is called a **multiple** of y.
Factors are also known as **divisors.**
So, if one considers the number 9, it can be divided completely by 1, 3 and 9. Thus, 1, 3 and 9 are factors of 9. Also, 9 is said to be a multiple of 1, 3 and 9. Similarly, 5 is not a factor of 9 because when 9 is divided by 5, the remainder left is 4 ($\neq 0$)

⚠ IMPORTANT:

9 can be completely divided by 4.5 but it does not mean that 4.5 is a factor of 9, because 4.5 is not a natural number.
Similarly, 2 can completely divide 9 without leaving any remainder, but 2 is not a factor of 9, because the result of the division $9 \div 2$ i.e. 4.5 is not a natural number.
Every number has at least two factors, 1 and the number itself.
If the given number is a prime number then 1 and the number itself are the only two factors of the given number whereas composite numbers always have more than two factors.

Example 1: Find the factors of 28.

Solution: Factors of 28 are the integral numbers which can completely divide 28 without leaving any remainder or decimal portion.
$\therefore$ 1, 2, 4, 7, 14 and 28 are the factors of 28.

III. HIGHEST COMMON FACTOR

If there are 2 natural numbers x and y, the highest natural number which divides both x and y completely is called the highest common factor (HCF) or greatest common divisor (GCD) of x and y.
In general, the largest natural number which completely divides the given natural numbers is the HCF of the given numbers.

Consider the numbers 6 and 8 to understand the concept of HCF.
The factors of 6 are 1, 2, 3 and 6.
The factors of 8 are 1, 2, 4 and 8.
The factors common to both 6 and 8 are 1, 2.
The highest of the common factors (1 and 2) is 2.
Thus, HCF of 6 and 8 is 2.
This can be written as "HCF (6, 8) = 2".

A. PROCESS TO FIND HCF

Step 1: Factorize all the given numbers into their prime factors.
Step 2: Collect all the common prime factors.
Step 3: Raise each of the prime factors to its minimum available power and multiply.
You can also use this method to find HCF of three numbers.

Example 2: Find the GCD of 300 and 450.

Solution: Factorize all the given numbers into their prime factors,
$300 = 2 \times 2 \times 3 \times 5 \times 5 = 2^2 \times 3 \times 5^2$
$450 = 2 \times 3 \times 3 \times 5 \times 5 = 2 \times 3^2 \times 5^2$
So, looking at the common factors, observe that 2, 3 and 5 are a part of both numbers.
We now raise each of these common prime factors to their minimum available power. The minimum available power of 2, 3 and 5 in the two numbers is 1, 1 and 2 respectively.
Hence, the GCD (300, 450) = $2 \times 3 \times 5^2 = 150$

Example 3: Find the HCF of 100, 200 and 250.

Solution: On factorizing the 3 numbers into their prime factors,
$100 = 2^2 \times 5^2$
$200 = 2^3 \times 5^2$
$250 = 2^1 \times 5^3$
Raising the common prime factors 2 and 5 to the minimum available powers gives,
HCF (100, 200, 250) = $2^1 \times 5^2 = 50$

B. ALTERNATE METHOD TO FIND HCF

Step 1: Divide the larger of the two numbers by the smaller number and check the remainder obtained.
Step 2: Divide the smaller number by this remainder and check the remainder now obtained.
Step 3: Continue till remainder becomes 0
Step 4: When remainder is 0, the divisor for that division is the HCF.

Example 4: Find the HCF of 300 and 840

Solution: Step 1: Divide 840 by 300 and check the remainder. When 840 is divided by 300, the quotient is 2 and the remainder is 240
Step 2: Now, 240 becomes the new divisor and the smaller number i.e. 300 becomes the new dividend. So, divide 300 by 240 and check the remainder. When 300 is divided by 240, the remainder is 60
Step 3: Continue the process. So, 240 gets divided by 60. When 240 is divided by 60, the remainder is 0.
Step 4: Since the remainder becomes zero, the process stops here and the last divisor i.e. 60 becomes the HCF.
Therefore, the HCF of 300 and 840 is 60.
This concept can also be used to find the HCF of 3 or more numbers. For 3 numbers, select two numbers initially and find the HCF of the two. Then apply this process again on this newly found HCF and the third number.

Example 5: Find the HCF of 7920, 540 and 1650

Solution: Consider any two of these three numbers, say 540 and 1650
Divide 1650 by 540.
When 1650 is divided by 540, the remainder is 30.
Now, divide 540 by 30.
When 540 is divided by 30, the remainder is 0.
Thus, the HCF of 1650 and 540 is 30.
Now, consider 30 with the third number i.e. 7920 and divide 7920 by 30.
When 7920 is divided by 30, the remainder is 0.
Therefore, the HCF of 7920, 540 and 1650 is 30.
This method is especially useful when the numbers whose HCF is to be found are large enough.

C. APPLICATION OF HCF

Example 6: Find the greatest number which when dividing 49 and 35 leaves a remainder 4 and 5 respectively.

Solution: Let the required number be n.
When n divides 49, the remainder is 4.
$\therefore$ n will completely divide 49 – 4 i.e. 45.
Similarly, when n divides 35, the remainder is 5.
$\therefore$ n will completely divide 35 – 5 i.e. 30.
Hence, n is a common factor of 45 and 30.
Since, n is the greatest possible number such that it satisfies the given conditions, n has to be the HCF of 30 and 45.
$30 = 2 \times 3 \times 5$
$45 = 3^2 \times 5$
So, the HCF of 30 and 15 is 3×5 i.e. 15
Thus, n is 15.
The above concept can be generalized as given below:
"The greatest natural number that will divide x, y and z leaving remainders r_1, r_2 and r_3, respectively, is the HCF of $(x - r_1)$, $(y - r_2)$ and $(z - r_3)$".

Example 7: Find the greatest number which when divides 148, 635 and 762 leaves remainders 4, 5 and 6 respectively.

Solution: As shown in the previous example,
The required number = HCF of (148 – 4), (635 – 5) and (762 – 6) = HCF (144, 630, 756)
$144 = 2^4 \times 3^2$
$630 = 2 \times 3^2 \times 5 \times 7$
$756 = 2^2 \times 3^3 \times 7$
Hence, HCF = $2 \times 3^2 = 18$

D. HCF OF TWO PRIME AND CO-PRIME NUMBERS

If you take the prime factors of any 2 prime numbers, say 7 and 13, you get,
$7 = 7^1$
$13 = 13^1$

Thus HCF (7, 13) = 1
Thus for any two different prime numbers, the HCF is always 1.

Two numbers are said to be co-prime when their only common factor is 1. It is not necessary for both these numbers to be prime.
For instance, consider 4 and 9. Neither 4 nor 9 is prime. The factors of 4 are 1, 2 and 4. Similarly, the factors of 9 are 1, 3 and 9. The only common factor in this case is 1 and it is also the HCF.
Thus, for any two co-prime numbers, the HCF is always 1.

IV. LEAST COMMON MULTIPLE

If there are 2 natural numbers x and y, the least natural number which can be divided by both x and y completely is called the least common multiple (LCM) of x and y. In general, the smallest natural number which is completely divisible by the given natural numbers is the LCM of the given numbers.

Consider the numbers 6 and 8 to understand the concept of LCM.

The multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66...
The multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64...
The multiples common to both 6 and 8 are 24, 48...

The least of the common multiples (24, 48...) is 24.
Thus, LCM of 6 and 8 is 24.
This can be written as "LCM (6, 8) = 24".

Example 8: Find the LCM of 96 and 36.

Solution: The multiples of 96 are 96, 192, 288...
The multiples of 36 are 36, 72, 108, 144, 180, 216, 252, 288...
The least common multiple for 96 and 36 is 288.
Thus, LCM (96, 36) = 288

A. PROCESS TO FIND LCM

Step 1: Factorize all the given numbers into their prime factors.
Step 2: Collect all the distinct prime factors occurring in either of the numbers.
Step 3: Raise all the prime factors to their maximum available powers and multiply.

Example 9: Find the LCM of 42 and 36.

Solution: On factorizing all numbers into their prime factors,
$42 = 2 \times 3 \times 7$
$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$
The distinct prime factors that occur in either of the given numbers are 2, 3 and 7.
The highest power of 2 is 2^2, the highest power of 3 is 3^2, while the highest power of 7 is 7^1.
Thus, LCM (42, 36) = $2^2 \times 3^2 \times 7^1 = 4 \times 9 \times 7 = 252$
This method can be used to find L.C.M. of more than two numbers as well.

Example 10: Find the LCM of 8, 12 and 15.

Solution: $8 = 2^3$
$12 = 2^2 \times 3$
$15 = 3 \times 5$
Hence, LCM (8, 12, 15) = $2^3 \times 3 \times 5 = 120$

B. APPLICATION OF LCM

Example 11: Find the least number which when divided by 42 and 70 leaves remainder 4 in each case.

Solution: Let the required number be n.
When n is divided by 42, the remainder is 4.
$\therefore$ $(n - 4)$ will be completely divisible by 42.
Similarly, when n is divided by 70, the remainder is 4.
$\therefore$ $(n - 4)$ will be completely divisible by 70.
Thus, $(n - 4)$ is the multiple common to both 42 and 70.
Since n is the least possible number, one can say that $(n - 4)$ is the LCM of 42 and 70
$42 = 2 \times 3 \times 7$
$70 = 2 \times 5 \times 7$
So, LCM of 42 and 70 = $2 \times 3 \times 5 \times 7 = 210$
$\because n - 4 = 210$
$\therefore n = 210 + 4 = 214$

One can generalize the above concept as given below:
"The smallest natural number that is divisible by x, y and z leaving the same remainder r in each case = LCM of (x, y and z) + r".

Example 12: Find the least number which when divided by 12, 14 and 20 leaves a remainder 3 in each case.

Solution: The required number = LCM (12, 14, 20) + 3
$12 = 2^2 \times 3$
$14 = 2 \times 7$
$20 = 2^2 \times 5$
Hence, LCM (12, 14, 20) = $2^2 \times 3 \times 5 \times 7 = 420$.
Hence, the required number is
420 + 3 = 423

Example 13: Find the least number which when divided by 11, 15 and 20 gives the remainders 7, 11 and 16 respectively.

Solution: Let the required number be x.
Then, the remainder when x is divided by 11 is 7. So, $(x - 7)$ is completely divisible by 11. Since, 11 is obviously divisible by 11, it implies that
$[(x - 7) + 11]$ is also divisible by 11.
$\therefore$ $(x + 4)$ is completely divisible by 11.
Similarly, $(x + 4)$ is also completely divisible by 15 and 20.
Hence, $(x + 4)$ is the LCM (11, 15, 20)
$\therefore x$ = LCM (11, 15, 20) − 4 = 660 − 4 = 656
This can be generalized as follows:
"The smallest natural number that on division by x, y and z leaves remainders a, b and c such that $x - a = y - b = z - c = d$ (some constant difference) is given as:
LCM $(x, y, z) - d$".

Example 14: (CSAT 2011) Three persons start walking together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

(a) 25 m, 20 cm (b) 50 m, 40 cm (c) 75 m, 60 cm (d) 100 m, 80 cm

Solution: Let the minimum distance be d cm.
Hence, d is the minimum number such that it is an integral multiple of 40, 42 and 45.
$\therefore$ d = LCM(40, 42, 45) = 2520
$\therefore$ d = 2520 cm = 25 m 20 cm
Hence, **option a.**

C. LCM OF PRIME AND RELATIVELY PRIME NUMBERS

If you take the prime factors of any 2 prime numbers, say 7 and 13, you get
$7 = 7^1$
$13 = 13^1$
Thus LCM (7, 13) = $7^1 \times 13^1 = 91$

Thus for any two different prime numbers, the LCM is the product of the two numbers.
For example, LCM (3, 11) = 33
Similarly, for any two co-prime numbers, the LCM is the product of the two numbers.
For example, LCM (4, 9) = 36

V. <u>HCF AND LCM</u>

A. HCF AND LCM OF FRACTIONS

The following formula is useful to find the HCF of fractions:

$$\text{HCF of fractions} = \frac{\text{HCF of numerators of all fractions}}{\text{LCM of denominators of all fractions}}$$

The following formula is useful to find the LCM of fractions:

$$\text{LCM of fractions} = \frac{\text{LCM of numerators of all fractions}}{\text{HCF of denominators of all fractions}}$$

REMEMBER:

While taking LCM and HCF of fractions, convert the fractions in the standard form i.e. reduce the fractions in its lowest terms.

Example 15: Find the LCM of 1/2 and 3/4.

Solution: Here the LCM of the numerators, i.e. 1 and 3, is 3 and the HCF of the denominators, i.e. 2 and 4, is 2.
Thus, using the above formula:

$$\text{LCM of } \frac{1}{2} \text{ and } \frac{3}{4} = \frac{\text{LCM of 1 and 3}}{\text{HCF of 2 and 4}} = \frac{3}{2}$$

Example 16: Find the product of HCF and LCM of 7/13 and 3/5.

Solution: HCF of numerators = HCF (7, 3) = 1
LCM of denominators = LCM (13, 5) = 13 × 5 = 65
Thus HCF of the fractions = 1/65
Similarly,
LCM of numerators = LCM (7, 3) = 7 × 3 = 21
HCF of denominators = HCF (13, 5) = 1
Thus LCM of fractions = 21/1 = 21
Thus, the required product = HCF × LCM = 1/65 × 21/1 = 21/65

REMEMBER:

In the above example, product of the two fractions = 7/13 × 3/5 = 21/65
Thus, HCF × LCM of two numbers = Product of the two numbers
Consider one more case. HCF (6, 8) × LCM (6, 8) = 2 × 24 = 48 = Product of two numbers 6 and 8
This is true for two numbers only. However, if there are more than two co-prime numbers, then this formula can be applied to any number of numbers.

IMPORTANT:

It must be kept in mind that HCF and LCM are concepts defined only for positive numbers, be it an integer or a fraction. HCF and LCM are not defined for negative numbers or zero.

B. APPLICATIONS OF HCF AND LCM

Example 17: How many pairs of natural numbers exist such that their HCF is 19 and their LCM 5985?

Solution: Let the two numbers be *a* and *b* respectively. Now, HCF of the two numbers is 19. This means that both *a* and *b* are multiples of 19. That is, the prime factorization of *a* and *b* would look something like this:
a = 19 × ______________
b = 19 × ______________
Since HCF = 19, therefore 19 will be the only prime factor that is common to both *a* and *b*.
Now, LCM of *a* and *b* = 5985 = $3^2 \times 5 \times 7 \times 19$

This means that between the two numbers, only the primes 3^2, 5, 7 and 19 should be used (no other number, prime or otherwise, can be used).
Combining this with the condition that 19 should be the only prime factor that is common to both numbers, there exist the following possibilities:

1. $(19, 19 \times 3^2 \times 5 \times 7) = (19, 5985)$
2. $(19 \times 7, 19 \times 3^2 \times 5) = (133, 855)$
3. $(19 \times 5, 19 \times 3^2 \times 7) = (95, 1197)$
4. $(19 \times 3^2, 19 \times 5 \times 7) = (171, 665)$

Hence, **FOUR** pairs of natural numbers exist that such that their HCF is 19 and their LCM is 5985.

VI. DIVISIBILITY TESTS

Sometimes you may have to determine whether a given number is divisible by some other number; but the quotient is not needed, so it would be a waste of time to actually divide the number. Instead, there are simple tests to do this. These are called divisibility tests and they help in finding out factors of any number.

A. DIVISIBILITY TEST OF 2

To find out if a number is divisible by 2, just check the last digit of that number. If the last digit of a number is 2, 4, 6, 8 or 0, then it is divisible by 2; otherwise it is not.
For example, 5132 is divisible by 2 but 26119 is not divisible by 2.

B. DIVISIBILITY TEST OF 3

Add all the digits of the given number. If the sum obtained is divisible by 3, only then is the number divisible by 3.
For example, 143 is not divisible by 3, because sum of digits = 1 + 4 + 3 = 8 and 8 is not divisible by 3.
Similarly, 186 is divisible by 3, because sum of digits = 1 + 8 + 6 = 15 and 15 is divisible by 3.

C. DIVISIBILITY TEST OF 4

If the number formed by the last two digits of a number is divisible by 4 or if the last two digits of a number are 0, then the number is divisible by 4.
For example, 6124 is divisible by 4, because the number formed by last two digits i.e. 24 is divisible by 4. Similarly, 3700 is divisible by 4 as the last two digits are 0. This also means that all multiples of 100 are divisible by 4.
But, 3842 is not divisible by 4, because the number formed by last two digits i.e. 42 is not divisible by 4.

D. DIVISIBILITY TEST OF 5

Only if the last digit of a number is 5 or 0, is it divisible by 5.
For example, 2365890 and 455152125 are divisible by 5 but 22445644 is not divisible by 5.

E. DIVISIBILITY TEST OF 6

If a number is divisible by 2 and 3 both, then it is divisible by 6. (Use divisibility tests of 2 and 3 individually on the given number; if it passes both, then it is divisible by 6).
For example, the last digit of 258 is 8; thus, it is divisible by 2. The sum of digits of 258 = 2 + 5 + 8 = 15, which is divisible by 3. Thus, 258 is divisible by both 2 and 3; hence it is divisible by 6.

F. DIVISIBILITY TEST OF 7

Double the last digit and subtract it from the number left with the remaining digits. If the result is divisible by 7, then the number is divisible by 7; otherwise it is not.
For example, 161 is divisible by 7, because the last digit of 161 is 1. Doubling it, you get 2. The remaining digits give the number 16. On subtracting 2 from 16 you get 14, which is divisible by 7.
The process can be repeated for a number with more than 3 digits. For example, to test the number 109543:

10954 - (3 × 2) = 10948
1094 - (8 × 2) = 1078
107 - (8 × 2) = 91
9 - (1 × 2) = 7, which is divisible by 7.
Hence, 109543 is divisible by 7.

☞ REMEMBER:

0 is divisible by 7. So, if the result turns out to be 0, then the number is divisible by 7. For example, check for the divisibility of 1932 by 7 using the above method.

G. DIVISIBILITY TEST OF 8

If the number formed by last three digits of a number is divisible by 8 or if the last 3 digits of a number are 0, only then is the number divisible by 8.
For example, 6124 is not divisible by 8, because the number formed by last three digits, i.e. 124, is not divisible by 8.

H. DIVISIBILITY TEST OF 9

Add all the digits of the number. If the sum obtained is divisible by 9, then the number is divisible by 9; otherwise, it is not.
For example, 51363 is divisible by 9, because the sum of the digits = 5 + 1 + 3 + 6 + 3 = 18 is divisible by 9.

☞ REMEMBER:

The difference between two numbers *ab* and *ba*, i.e. $|ab - ba|$ is always divisible by 9.

I. DIVISIBILITY TEST OF 10

Only if the last digit of a number is 0, is it divisible by 10.

J. DIVISIBILITY TEST OF 11

Add up all the digits at odd positions in the given number. Then add up all the digits at even positions. If the difference of these two additions is a multiple of 11, then the number is divisible by 11; otherwise, it is not. The positions of the digits are taken from left to right; i.e. the first digit will have position 1 (odd position), the second digit will have position 2 (even position), and so on.
For example, consider the number 13475:
Digits in odd positions are 1 (position 1),
4 (position 3) and 5 (position 5).
Their sum is 1 + 4 + 5 = 10
Digits in even positions are 3 (position 2) and 7 (position 4).
Their sum is 3 + 7 = 10
The difference in the two sums = 10 - 10 = 0
Since 0 is divisible by 11, 13475 is divisible by 11.

K. DIVISIBILITY TEST OF 12

If the number is divisible by both 3 and 4, then the number is divisible by 12.
Here use the divisibility tests of 3 and 4 individually on the given number, as they are co-prime numbers.

⚠ IMPORTANT:

You cannot use the divisibility tests of 6 and 2 (for divisibility test of 12) as 6 is a multiple of 2.

Example 18: What should be the values of *a* and *b* such that 1*a*8*b* is divisible by 2, 3, 4, 6, 7 and 8?

Solution: In order for 1*a*8*b* to be divisible by 2, *b* should be 2, 4, 6, 8 or 0.

Also, since 1a8b is divisible by 4, the number formed by the last two digits (i.e. 8b) should be divisible by 4. Hence, b must be 0, 4 or 8.

Since the number is divisible by 3, the sum of all its digits should be divisible by 3; i.e. $(9 + a + b)$ should be divisible by 3. As 9 is already divisible by 3, you can say that $(a + b)$ ought to be divisible by 3. So, the possible pairs of (a, b) are (3, 0), (6, 0), (9, 0), (2, 4), (5, 4), (8, 4), (1, 8), (4, 8) and (7, 8).

There is no need to check whether the number is divisible by 6, since any number divisible by both 2 and 3 will be divisible by 6.

To be divisible by 8, the number formed by the last 3 digits (i.e. a8b) should by divisible by 8. Of the above possible pairs, only the pairs (6, 0), (5, 4) and (4, 8) satisfy this criterion.

($\because$ 680, 584 and 488 are divisible by 8).

For a number to be divisible by 7, $(x - 2y)$ must be divisible by 7, where y is the unit's digit and x is the number formed by the remaining digits.

Now, substitute the above 3 pairs of a and b and see which of those satisfy these criteria:

1488 $\rightarrow$ 148 − 16 = 132 $\rightarrow$ 13 − 4 = 9, which is not divisible by 7

1584 $\rightarrow$ 158 − 8 = 150 $\rightarrow$ 15 − 0 = 15, which is also not divisible by 7

1680 $\rightarrow$ 168 − 0 = 168 $\rightarrow$ 16 − 16 = 0, which is divisible by 7

Hence, $a = 6$ and $b = 0$

TEST 1

1. What is the HCF of the following set of numbers?
12, 24, 16, 32, 8

(a) 2 (b) 4 (c) 6 (d) 8 (e) None of these

2. Four signals in a straight line turn red in 8s, 12s, 16s and 20s seconds respectively. If at time $t = 0$, all four signals are red, then after how many minutes will all four become red again?

(a) 1 (b) 2 (c) 3 (d) 4 (e) None of these

3. Let X be the LCM of 32, 128, 512 & 1024. What is the sum of digits of X?

(a) 6 (b) 7 (c) 8 (d) 9 (e) None of these

4. There are three classes of 91, 143 and 208 students respectively who go out together for physical training. All the students are divided into groups of equal sizes and each group contains students only from one of the classes. What would be the largest possible group size?

(a) 4 (b) 7 (c) 11 (d) 13 (e) 16

5. What is the largest number which divides 98, 147, 268 and 365 to leave a remainder of 2, 3, 4 and 5 respectively?

(a) 16 (b) 24 (c) 45 (d) 48 (e) None of these

6. The LCM and HCF of two numbers is 18 and 7200 respectively. If one of the numbers is 450, find the other number.

(a) 290 (b) 300 (c) 288 (d) 320 (e) 180

7. If N is the highest 4 digit number divisible by 12, 40, 32 & 72, what is the value of N?

(a) 9990 (b) 9020 (c) 8640 (d) 8900 (e) None of these

8. The product of two numbers a and b $(a > b)$ is 1924. If the H.C.F. of these numbers is 2, then what is the value of a?

(a) 26 (b) 39 (c) 48 (d) 74 (e) None of these

9. Which is the greatest number that leaves an equal remainder when 1086, 946 and 995 are divided by it?

(a) 4 (b) 5 (c) 6 (d) 7 (e) Data inadequate

10. The H.C.F. of two numbers is 36 and their LCM is 1950. If one of the numbers is 234, what is the other number?

(a) 250 (b) 300 (c) 350 (d) 275 (e) 325

TEST 2

11. Which is the least number, which when divided by 3, 4 and 5, leaves remainder 1, 2 and 3 respectively?

(a) 58 (b) 59 (c) 67 (d) 116 (e) None of these

12. The least four digit number which is exactly divisible by 3, 4, 5 or 8 is :-

(a) 9000 (b) 6400 (c) 3600 (d) 4900 (e) None of these

13. There are three springs S_1, S_2, S_3, which are stretched and let go. Each spring is 100% elastic and comes to its original position every time after a fixed interval. S_1 comes back to its original position in 72 seconds. Similarly, S_2 and S_3 come back to their original position in 84 and 96 seconds respectively. When will they come back together for the second time?

(a) $60\frac{1}{5}$ mins (b) $67\frac{1}{5}$ mins (c) 60 mins (d) 66mins

14. There are 3 bells which ring at different intervals. For the first time, they ring together 15525 seconds after ringing together at the start. Each bell rings at different time intervals. It is also known that one of the intervals is a prime number, one is the square of a natural number and one is the cube of a natural number. Also, if all three intervals are co-prime, what is the sum of the intervals at which the bells ring?

(a) 75 seconds (b) 77 seconds (c) 79 seconds (d) 81 seconds

15. Find the greatest number that divides 43, 91 and 183 so as to leave the same remainder in each case.

(a) 4 (b) 7 (c) 9 (d) 13 (e) 14

16. The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is:

(a) 74 (b) 94 (c) 184 (d) 364 (e) None of these

17. The least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3 is:

(a) 3 (b) 13 (c) 23 (d) 33 (e) 43

18. A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and C in 198 seconds, all starting at the same point. After what time will they meet again at the starting point?

(a) 26 minutes and 18 seconds (b) 42 minutes and 36 seconds (c) 45 minute
(d) 46 minutes and 12 seconds (e) None of these

19. Three numbers are in the ratio of 3 : 4 : 5 and their L.C.M. is 2400. Their H.C.F. is:

(a) 40 (b) 80 (c) 120 (d) 160 (e) 200

20. The greatest number which on, dividing 1657 and 2037, leaves a remainder of 6 and 5 respectively is:

(a) 123 (b) 127 (c) 235 (d) 305 (e) None of these

TEST 3

21. The L.C.M. of two numbers is 48. The numbers are in the ratio 2 : 3. So, the sum of the numbers is:

(a) 28 (b) 32 (c) 40 (d) 64 (e) 72

22. If the sum of two numbers is 55 and the H.C.F. and L.C.M. of these numbers are 5 and 120 respectively, then the sum of the reciprocals of the numbers is equal to:

(a) $\frac{55}{601}$ (b) $\frac{601}{55}$ (c) $\frac{11}{120}$ (d) $\frac{120}{11}$ (e) None of these

23. Find the highest common factor of 36 and 84.

(a) 2 (b) 3 (c) 6 (d) 12 (e) 24

24. The product of two numbers is 2028 and their H.C.F. is 13. The number of such pairs is:

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

25. The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is:

(a) 101 (b) 107 (c) 111 (d) 185 (e) 193

26. The G.C.D. of 1.08, 0.36 and 0.9 is:

(a) 0.03 (b) 0.9 (c) 0.18 (d) 0.36 (e) 0.09

27. If an integer n is divisible by 3, 5 and 12, what is the next larger integer divisible by all these numbers?

(a) $n + 3$ (b) $n + 5$ (c) $n + 12$ (d) $n + 60$ (e) $n + 15$

28. Mr. Brackett works in a factory with his two sons. He is allowed to take a break every 140 minutes while his two sons are allowed to take breaks every 210 minutes and 280 minutes. How many minutes will they have to wait after their first break together to get together again?

(a) 12 hours (b) 13 hours (c) 14 hours (d) 15 hours (e) 16 hours

29. The product of the L.C.M and H.C.F of two natural numbers is 54. The difference of these two numbers is 3. Find the numbers.

(a) 6, 9 (b) 18, 15 (c) 2, 27 (d) 9, 12

30. 3 strings of lengths 7 m, 3 m 85 cm and 12 m 95 cm are to be cut into pieces of equal lengths. What is the greatest possible length of each piece (in cm)?

(a) 35 (b) 23.8 (c) 23 (d) 36

31. 294 blue balls, 252 pink balls and 210 yellow balls are to be distributed equally among some students such that each student gets balls of exactly one colour and no balls are left over. What is the maximum number of balls that each student gets?

(a) 40 (b) 42 (c) 52 (d) 50

32. The least number which, when divided by 12, 15, 20 and 54, leaves a remainder of 8 in each case is:

(a) 504 (b) 532 (c) 544 (d) 548

33. How many pairs of numbers lying between 35 and 90 have their H.C.F as 20?

(a) 1 (b) 2 (c) 3 (d) 4

34. The H.C.F of 900, 1800 and a third number is 100 and their LCM is 24 × 32 × 53. What is the third number?

(a) 2700 (b) 2000 (c) 100 (d) 18000

TEST 4

35. A number 123x is divisible by 7. Find out the value of the digit x if the number is also divisible by 3.

(a) 2 (b) 4 (c) 6 (d) 9 (e) More than 1 value of x is possible

36. When the integer n is divided by 8, the remainder is 3. What is the remainder when 6n is divided by 8?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

37. If 7513a4821b is divisible by 10 and 11, what is the respective value of a and b?

(a) 3, 5 (b) 8, 0 (c) 4, 0 (d) 7, 0

38. A ticket seller keeps a record of all his tickets in tabular form. The table has 58 rows and 58 columns. Each cell indicates a ticket number. The seller has the same number of tickets as the number of cells in the table. If he has already sold 3128 tickets, how many tickets does he still have?

(a) 236 (b) 512 (c) 354 (d) 128

39. Arjun is renovating his kitchen and has decided to cover one wall with designer tiles. He arranges the tiles in a row-column grid on the wall such that each row and column has 14 tiles. Arjun bought 200 tiles and had some tiles left after covering the wall. How many tiles were left with him?

(a) 8 (b) 16 (c) 4 (d) 32

40. If 4584193ab is divisible by both 8 and 5, what is the value of $a + b$?

(a) 13 (b) 11 (c) 8 (d) None of the above

41. How many 4-digit numbers are completely divisible by 4?

(a) 2000 (b) 2250 (c) 2500 (d) None of these

42. What is the sum of all the 3-digit numbers divisible by 3?

(a) 165000 (b) 161550 (c) 165150 (d) None of these

43. How many of the following numbers are divisible by 4 as well as 8?
9376, 3822, 5417, 6484, 9352, 6138.

(a) 1 (b) 3 (c) 2 (d) 0

44. On dividing a number by 5, 3 is obtained as a remainder. What is the remainder when the square of the same number is divided by 5?

(a) 3 (b) 9 (c) 1 (d) 4

45. On subtracting a 3-digit number 26a from another 3-digit number 968; a 3-digit number 70b is obtained. If 70b is divisible by 11, what is the value of $(a - b)$? a and b are the units digits of 26a and 70b respectively.

(a) 4 (b) 0 (c) 7 (d) 11

46. There are five hobby clubs in a college viz. , photography, yachting, chess, electronics and gardening. The gardening group meets every second day, the electronics group meets every third day, the chess group meets every fourth day, the yachting group meets every fifth day and the photography group meets every sixth day. How many times do all the five groups meet on the same day within 180 days? **[UPSC 2013]**

(a) 3 (b) 5 (c) 10 (d) 18

47. A gardener has 1000 plants. He wants to plant them in such a way that the number of rows and the number of columns remains the same. What is the minimum number of plants that he needs more for this purpose? **[UPSC 2013]**

(a) 14 (b) 24 (c) 32 (d) 34

12 Linear Equations

I. INTRODUCTION

An equation is a mathematical statement which implies that two quantities separated by the symbol '=' are equal.
For example, $3x + 6 = 0$,
Here, x is called the variable, 3 is called the coefficient of the variable and 6 is called the constant.

II. TYPES OF EQUATIONS

Linear equations exist in many forms depending upon the kind of equality they use.

A. LINEAR EQUATIONS IN ONE VARIABLE

To solve an equation is to find a numerical value that the variable can take so that the equation holds true. Such a value of the variable is called the **solution** of the equation. The solution of an equation is said to satisfy that equation. When the value of the solution is entered in the equation, the LHS becomes numerically equal to the RHS.

For instance, consider $x + 4 = 8$. This equation can hold true only if $x = 4$. Thus, $x = 4$ is a solution of the equation.
To solve a linear equation in one variable the following steps should be followed:

Step 1: Simplify the LHS and the RHS by removing brackets.

Step 2: Shift all the terms containing the variable to the LHS and the constant terms to the RHS with appropriate change(s) of sign and then simplify.

While shifting terms to the opposite side, a positive sign changes to negative (and vice versa) and a multiplication changes to a division (and vice versa).

Example 1: Find the value of x, if $7x + 5(2 - x) + 10 = 4x - 6$

Solution: Step 1:
$7x + 5(2 - x) + 10 = 4x - 6$
$\therefore\ 7x + 10 - 5x + 10 = 4x - 6$
$\therefore\ 2x + 20 = 4x - 6$
Step 2:
$\therefore\ 2x - 4x = -20 - 6$
$\therefore\ -2x = -26$
$\therefore\ x = 13$

Example 2: Find the value of y, if $\frac{9y - 2(y + 2)}{y + 2} = 5$

Solution: Step 1:

$$\frac{9y - 2(y + 2)}{y + 2} = 5$$

$$\frac{9y - 2y - 4}{y + 2} = 5$$

$\therefore\ 7y - 4 = 5(y + 2)$
$\therefore\ 7y - 4 = 5y + 10$

Step 2:
$\therefore 7y - 5y = 4 + 10$
$\therefore 2y = 14$
$\therefore y = 7$

B. SIMULTANEOUS LINEAR EQUATIONS

An equation of the form $ax + by = c$, where the highest power of the variables x and y is unity is called a linear equation in two variables. Here $a \neq 0$, $b \neq 0$ and a, b and c are real numbers. The values of x and y for which the equation holds are called the solution of the equation.
Linear equations in two variables, which are both satisfied by the same unique solution, are called **simultaneous equations.**
Two general methods of solving simultaneous equations are described below:

SUBSTITUTION METHOD

Consider the equations $3y - x = 1$ and $7y - 2x = 4$ to be solved simultaneously. Thus,
$3y - x = 1$ ... (i)
$7y - 2x = 4$ ... (ii)

From equation (i),
$3y = 1 + x$

$y = \frac{(1 + x)}{3}$ ... (iii)

Substituting this value of y in equation (ii),

$\frac{7(1 + x)}{3} - 2x = 4$

$\therefore \frac{(7 + 7x)}{3} - 2x = 4$

$\therefore 7 + 7x - 6x = 12$
$\therefore x = 5$

On substituting $x = 5$ in equation (iii), we have

$y = \frac{(1 + 5)}{3}$

$\therefore y = 2$

$\therefore x = 5$ and $y = 2$ satisfy the two equations and (5, 2) is a solution of the two equations.

ELIMINATION METHOD

Consider the equations $2x + 3y = 4$ and $3x + 4y = 5$ to be solved simultaneously. Thus we have,

$2x + 3y = 4$ ... (i)
$3x + 4y = 5$ ... (ii)

Multiply each equation by the coefficient of x (or y) in the other equation.

Multiplying equation (i) by 3 and equation (ii) by 2

$6x + 9y = 12$ ... (iii)
$6x + 8y = 10$ ... (iv)

Subtracting equation (iv) from equation (iii),
$y = 2$
Substituting $y = 2$ in equation (i)
$2x + (3 \times 2) = 4$
$\therefore x = -2/2 = -1$

$\therefore$ $x = -1$ and $y = 2$ satisfy both the equations and $(-1, 2)$ is a solution of the two equations.
Consider two linear equations $ax + by = c$ and $px + qy = r$.
These two equations are said to have a unique solution only if $(a/p) \neq (b/q)$.
For instance, in the above case, $(2/3) \neq (3/4)$. So, the equations have a unique solution $(-1, 2)$.
If $(a/p) = (b/q)$, then the two equations have infinite solutions.
For instance, if the two equations are $2x + 3y = 4$ and $6x + 9y = 12$, then $(2/6) = (3/9) = (4/12)$.
Thus, we actually end up with only one equation in two variables i.e. $2x + 3y = 4$
This can have infinite solutions. So, when $x = 1, y = 2/3$ and when $x = 2, y = 0$ and so on.

C. LINEAR INEQUATIONS

A statement which contains the "greater than" or "less than" symbols is called an inequation.
E.g. $3x + 5 < 8$

Example 3: The sum of three consecutive numbers is 84. Find the numbers.

Solution: Let x be the first number. Then the other two consecutive numbers are $x + 1$ and $x + 2$.
Hence we have,
$x + (x + 1) + (x + 2) = 84$
$\therefore 3x + 3 = 84$
$\therefore 3x = 81$
$\therefore x = 27$
The numbers are 27, 28 and 29.

Example 4: Tickets for a concert were sold at Rs. 5, Rs. 3 and Re. 1 each. Thirty more tickets were sold at Rs. 5 than at Rs. 3, and twice as many at Re. 1 as at Rs. 3. If total receipts from the sale of tickets were Rs. 950, then how many tickets of each kind were sold?

Solution: Let the number of Rs. 3 tickets sold = x
Hence, number of Rs. 5 tickets sold = $30 + x$
and number of Re. 1 tickets sold = $2x$
Total amount collected = Rs. 950
$\therefore 3x + 5(30 + x) + 2x = 950$
$\therefore 10x = 950 - 150$
$\therefore x = 80$
$\therefore$The number of tickets sold for Rs. 3 is 80, the number of tickets sold for Rs. 5 is 110 and the number of tickets sold for Re. 1 is 160.

Example 5: In a particular jungle which only had deer and human visitors, there were 70 heads and 188 legs. How many deer and visitors were there?

Solution: Let the number of deer be x and the number of human visitors be y.
The number of heads is 70.
Hence,
$x + y = 70$ ... (i)
Since each deer has 4 legs and each human visitor has 2 legs,
$\therefore 4x + 2y = 188$ ... (ii)
Multiplying equation (i) by 2 and subtracting the same from equation (ii),
$2x = 48$
Hence, $x = 24$ and $y = 46$
So, there were 24 deer and 46 visitors.

Example 6: The sum of the digits of a two digit number is 7. If the digits are reversed, the number so obtained when increased by 3 equals 4 times the original number. Find the original number.

Solution: Let the digit in the ten's place = x
Let the digit in the unit's place = y
So, the original number = $10x + y$
If the digits are reversed, the new number = $10y + x$
According to given conditions,
$x + y = 7$ ... (i)
$(10y + x) + 3 = 4(10x + y)$
$\therefore 10y + x + 3 = 40x + 4y$
$\therefore -39x + 6y = -3$
Dividing the above equation by -3
$\therefore 13x - 2y = 1$ ... (ii)
Multiplying equation (i) by 2 and adding equation (ii)
$15x = 15$
$\therefore x = 1$
Substituting the value of x in equation (i), we get $y = 6$
$\therefore$ The original number is $(10 \times 1) + (1 \times 6) = 16$.

Example 7: (CSAT 2011) A person has only Rs. 1 and Rs. 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs. 75, then the number of Rs. 1 and Rs. 2 coins are, respectively

(a) 15 and 35 (b) 35 and 15 (c) 30 and 20 (d) 25 and 25

Solution: Let x be the number of Rs.1 coins and y be the number of Rs.2 coins.
Total number of coins = $x + y = 50$ ... (i)
Total amount of money = $x + 2y = 75$ ... (ii)
From (i) and (ii),
$y = 25$
$\therefore x = 25$
Hence, **option d.**

Example 8: One fourth of a number exceeds one-fifth of the same number by 3. Find the number.

Solution: Let the number be x.
According to the question,

$$\frac{x}{4} = \frac{x}{5} + 3$$

$$\frac{x}{4} - \frac{x}{5} = 3$$

$$\frac{5x - 4x}{20} = 3$$

$\therefore (5x - 4x) = 60$
$\therefore x = 60$
$\therefore$ The number is 60.

Example 9: Solve $\frac{2}{x} + \frac{3}{y} = 10$ and $\frac{3}{x} - \frac{4}{y} = -2$

Solution: In such a case, rather than taking the LCM and trying to solve the problem, it is a better idea to replace the given variables with some suitable variables, find the values of the replacements and then re-substitute the new variables with the given variables to get the required values.

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$

Hence,

$2a + 3b = 10$ ... (1)

$3a - 4b = -2$ ... (2)

Multiply (1) by 4 and (2) by 3

Hence,

$8a + 12b = 40$ ... (3)

$9a - 12b = -6$ ... (4)

Add (3) and (4)

$\therefore 17a = 34$

$\therefore a = 2$

Substituting in (1), $b = 2$

$\therefore x = \frac{1}{a} = \frac{1}{2}$ and $y = \frac{1}{b} = \frac{1}{2}$

$\therefore x = y = \frac{1}{2}$

D. LINEAR EQUATIONS IN THREE OR MORE VARIABLES

A set of 3 unique equations (those which cannot be algebraically derived from one another) in 3 variables also can be solved simultaneously for a common solution.

For example,

We have the following set of equations,

$x + y + 2z = 9$ ... (i)

$2x + 4y + 6z = 28$ ... (ii)

$3x + 4y + 5z = 26$ ... (iii)

The above set of equations is a system of three equations with three variables.

Both substitution and elimination methods can be used to solve these equations.

If we multiply (i) by 2 and subtract it from (ii), we get

$y + z = 5$ ... (iv)

If we multiply (i) by 3 and subtract it from (iii), we get

$z - y = 1$ ... (v)

Equation (iv) and equation (v) can be added to get $z = 3$. Using this value of z we can easily find y and x.

REMEMBER:

To solve a system of simultaneous equations, the number of independent equations must be at least equal to the number of variables.

Example 10: Solve the system of equations for x, y and z.

$x - y + z = 6$

$2x + 2y + 3z = 11$

$2x - 3y - 2z = 1$

Solution: $x - y + z = 6$ ... (i)

$2x + 2y + 3z = 11$ ... (ii)

$2x - 3y - 2z = 1$ ... (iii)

Multiplying equation (i) by 2 and subtracting from equation (ii),

$4y + z = -1$ ... (iv)

Subtracting (iii) from (ii),

$5y + 5z = 10$ or $y + z = 2$ ... (v)

Now equation (iv) and (v) are simultaneous equations in y and z.
Solving, $y = -1$ and $z = 3$
Substituting the value of y and z in (i), $x = 2$
Thus $x = 2, y = -1$ and $z = 3$.

Example 11: Find the values of x, y and z.
$z + 4y + 3x = 33$
$4x + y - z = 6$
$2y + 8x - 2z = 12$

Solution: $z + 4y + 3x = 33$ ... (i)
$4x + y - z = 6$ ... (ii)
$2y + 8x - 2z = 12$ ... (iii)
Dividing equation (iii) by 2,
$y + 4x - z = 6$, which is the same as equation (ii). This is because the ratio of the coefficients is equal i.e. $(1/2) = (4/8) = (-1/-2) = (6/12)$
Thus there are only two equations but three variables.
Hence, the values of x, y and z cannot be found.

TEST 1

1. Find the value of m, from the following simultaneous equations.
$15m + 17n = 21$
$17m + 15n = 11$
(a) 3 (b) −3 (c) 2 (d) −2 (e) −1

2. Carla has Rs. 2,750 in her purse in denominations of hundred and fifty. She has 32 notes in all counting both hundred and fifty. How many hundred rupee notes does she have in her purse?
(a) 23 (b) 9 (c) 24 (d) 8 (e) 25

3. If 1 is added to the numerator of a certain fraction, its value becomes 7/19 and if 1 is added to the denominator of the original fraction, its value becomes 1/3. Find the original fraction.
(a) $\frac{20}{57}$ (b) $\frac{13}{39}$ (c) $\frac{34}{96}$ (d) $\frac{13}{38}$ (e) None of these

4. Aishwarya's age 10 years hence will be twice Deepika's present age. Six years back, Aishwarya's age was 5/3 times Deepika's age at that time. Find the present age of Aishwarya and Deepika respectively.
(a) 36, 18 (b) 26, 18 (c) 36, 12 (d) 48, 36 (e) 18, 26

5. Find the value of $(x + y)$, from the given set of equations.
$$\frac{7}{x} + \frac{13}{y} = 27$$
$$\frac{13}{x} + \frac{7}{y} = 33$$
(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{5}{2}$ (d) 2

6. Amar brought bananas to school. He gave one-fourth of the bananas to the Physics teacher and one-sixth of the bananas to his Chemistry teacher. The Chemistry teacher gave the head-master 2 bananas and now has 4 bananas left. How many bananas did Amar give to the Physics teacher?
(a) 12 (b) 36 (c) 5 (d) 9 (e) 23

7. There are three cities: A, B and C. Three friends are discussing the population (in millions) of the three cities. One says: 'A has 9 million people'. The second says: 'B has as many people as A and C combined'. The third says: 'The number of people in A added to half of the number of people in B is the number of people in C'. What is the total number of people (in millions) in all three cities combined?

(a) 48 (b) 54 (c) 63 (d) 72 (e) 81

8. Sam, Harry and Jake had some candies each. Together Sam and Harry had 19 candies. Even after giving three candies to Jake, Sam had two more candies than him. Then Harry gave two of his candies to Jake and was also left with two more candies than him. How many candies does Jake have now?

(a) 1 (b) 4 (c) 5 (d) 6 (e) 8

9. Students were standing in rows for exercise. Each row had an equal number of students. If 5 students less were to stand in each row, 6 more rows would be required and if 5 students more were to stand in each row then the number of rows required would be reduced by 2. Find the total number of students.

(a) 10 (b) 40 (c) 50 (d) 70 (e) None of these

10. Hermione purchases 3 apples, 7 mangoes and 1 orange for a total of Rs. 120. Ron buys 4 apples, 5 mangoes and an orange for Rs. 164.50 from the same shop. If Harry picks 1 apple, 11 mangoes and an orange from the same shop, then how much does he have to pay?

(a) Rs. 29 (b) Rs. 31 (c) Rs. 35 (d) Rs. 40 (e) Cannot be determined

TEST 2

11. When a two digit number is divided by the sum of the digits, the quotient is 4. If the digits are reversed, the new number is 6 less than twice the original. Find the number.

(a) 24 (b) 42 (c) 16 (d) 28 (e) None of these

12. A man earns Rs. 800/- more than his wife. One- fourth of the man's salary and one-eighth of the wife's salary amount to Rs. 500/- which is saved every month. Find their monthly expenditure.

(a) Rs. 1,600 (b) Rs. 1,700 (c) Rs. 1,800 (d) Rs. 1,900 (e) Rs. 2,000

13. 15 pens, 10 pencils together cost Rs. 90 and the sum of cost of 1 pencil and 1 pen is Rs. 7. What will the total cost if the prices of pens and pencils are exchanged and same amount of pens and pencils remain as given in the 1st case?

(a) 85 (b) 90 (c) 95 (d)100

14. Find the value of x, if $7x + 8(2 - x) + 10 = 4x - 4$

(a) 5 (b) 6 (c) 7 (d) 8 (e) 9

15. Suresh wins one million in a lottery. He spends half the money to buy a house, half of the remaining amount to buy a car and 20% of the remaining amount to buy a motorcycle. Find the amount left with Suresh.

(a) 1.5 lakhs (b) 2.5 lakhs (c) 2 lakhs (d) 1 lakhs (e) 4 lakhs

16. A two digit number when reversed becomes one less than thrice the original number. Find the original number.

(a) 15 (b) 27 (c) 39 (d) 14 (e) More than one number is possible

17. Solve the following linear equation;

$11a + 17b = 73$
$17a + 11b = 67$

(a) $a = 3, b = 2$ (b) $a = 3, b = 3$ (c) $a = 2, b = 3$ (d) $a = 2, b = 2$ (e) None of these

18. In a management test, 3 marks are awarded for a correct answer and 1 mark is deducted for an incorrect one. There is no negative marking. Suresh attempted 70 out of 100 questions and managed to score 170 marks. Find the number of questions correctly answered by Suresh.

(a) 60 (b) 50 (c) 55 (d) 65 (e) 59

19. How many two digit numbers are 72 less than the number obtained by reversing the digits of the original number?

(a) 1 (b) 5 (c) 3 (d) 2 (e) None of these.

20. On a certain island, there are coins available in only two denominations – Rs. 2 and Rs. 5. Suresh has 100 coins with him, such that the total amount is Rs. 350. How many Rs. 2 coins does Suresh have?

(a) 50 (b) 55 (c) 60 (d) 65 (e) 70

TEST 3

21. A two digit number when reversed becomes three less than the four times the original value. Find the original number.

(a) 15 (b) 17 (c) 16 (d) 19 (e) None of these

22. The units digit of a certain two digit number is three more than the tens digit. Find the difference between the number and the number obtained by reversing the number.

(a) 18 (b) 9 (c) 3 (d) 27 (e) None of these

23. Solve the following linear equation;

$3a + 4b = 40,$
$7a + 3b = 49$

(a) $a = 4, b = 6$ (b) $a = 5, b = 6$ (c) $a = 4, b = 7$
(d) $a = 3, b = 8$ (e) More than one solution is possible

24. Ramesh had twice as many 2 rupee coins as 5 rupee coins. Had the number of coins been interchanged, he would have had 30 rupees extra. How many coins did Ramesh have in all?

(a) 12 (b) 6 (c) 18 (d) 21 (e) 30

25. Find a if a, b and c satisfy the following equations;

$a - 3b + 3c = -4$
$2a + 3b - c = 15$
$4a - 3b - c = 19$

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

26. Find x if x, y and z satisfy the following equations;

$4x + y - 2z = 0$
$3x - 3y + 3z = 9$
$6x - 2y + z = 0$

(a) 1 (b) 6 (c) 9 (d) 3 (e) Not defined

27. Eight years from now Anuradha will be twice as old as she was 6 years ago. What is her present age?

(a) 14 years (b) 8 years (c) 12 years (d) 20 years (e) None of these

28. In a particular jungle where only tigers and human visitors are present, there are 84 heads. If the total number of legs in this jungle is 272, what is the respective number of tigers and human visitors present?

(a) 49, 35 (b) 52, 32 (c) 47, 37 (d) 50, 34

29. A person has two sons. Two years ago, the elder son was twice as old as the younger one and two years hence, the father will be twice as old as the elder son. If the younger son is now 14 years old, what is the person's present age (in years)?

(a) 52 (b) 56 (c) 58 (d) 54 (e) None of these

30. A mother is seven times older than her daughter now, but two years hence she will be only five times older. What is the mother's present age (in years)?

(a) 28 (b) 30 (c) 31 (d) Data insufficient (e) None of these

TEST 4

31. Symphony tickets cost Rs. 16 for adults and Rs. 8 for students. A total of 14 tickets worth Rs. 160 were sold. How many adult and student tickets were sold respectively?

(a) 6 and 10 (b) 7 and 7 (c) 8 and 6 (d) 10 and 6 (e) 6 and 8

32. Bottles of three different colours are placed on a shelf. There are 12 red bottles. The number of blue bottles equals half the number of green bottles. The number of green bottles is equal to the number of blue bottles added to one third of the red bottles. What is the total of number of blue bottles?

(a) 4 (b) 8 (c) 6 (d) 10

33. The population of 3 villages is found. Village A has a population of 4000. The total population of village B is twice that of village C and that of village C equals the population of village A added to one-fourth the population of village B. What is the population of village B?

(a) 16000 (b) 8000 (c) 14000 (d) 7000

34. Riya's salary is more than Jiya's salary by Rs.1,200. One-third of Riya's salary and one-sixth of Jiya's salary add up to Rs.800. What is the salary of Jiya?

(a) Rs.850 (b) Rs.650 (c) Rs.900 (d) Rs.800

35. 20 pencils and 8 erasers together cost Rs.116 while 1 pencil and 1 eraser together cost Rs.7. What is the total cost of 20 pencils and 8 erasers if the cost of 1 pencil is interchanged with that of 1 eraser?

(a) Rs.80 (b) Rs.115 (c) Rs.105 (d) Rs.95

36. If $\dfrac{3x}{1+\dfrac{1}{1+\dfrac{x}{1-x}}} = 1$ then find the value of x?

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) 1

37. The sum of 4 numbers is 170. If you add 10 to the first number, subtract 15 from the second number, multiply the third number by 2 and divide the fourth number by 3, all four results obtained are equal. What is the sum of the smallest and the largest original numbers?

(a) 100 (b) 110 (c) 105 (d) 70

38. Roshni was asked her age. She said: "If you take my age 5 years ago, multiply it by 2 and then subtract two-thirds of my current age, you will get my current age".
What is Roshni's current age?

(a) 30 (b) 50 (c) 20 (d) 40

39. There are 5 members in a family, out of which 3 are working. Now, one-fourth the salary of the third member, one-fifth the salary of the second member and one-tenth the salary of the first member add up to Rs. 9,000 (which are the total savings of the family). Find the monthly expenditure of the family if the second member earns Rs.5,000 more than the first while the third member earns Rs.5,000 more than the second.

(a) Rs.26,000 (b) Rs.35,000 (c) Rs.44,000 (d) Rs.36,000

40. Rahul and Rohit have some money. If Rahul gives Rs.50 to Rohit, then Rohit has twice the amount now left with Rahul. If Rohit gives Rs.40 to Rahul, Rahul has fourteen times the amount now left with Rohit. How much money (in Rs) do Rahul and Rohit respectively have in the beginning?

(a) 50, 100 (b) 100, 50 (c) 40, 50 (d) 50, 90

41. Four friends A, B, C and D distribute some money among themselves in such a manner that A gets one less than B, C gets 5 more than D, D gets 3 more than B. Who gets the smallest amount? **[UPSC 2013]**

(a) A (b) B (c) C (d) D

13 Sequences, Progressions and Series

I. INTRODUCTION

Terms arranged in a definite order form a **sequence**. The terms in a sequence may be numbers, letters, symbols or even words.

II. SEQUENCES

A sequence is a logically ordered list of elements related to each other by some relationship. Identify the pattern followed by the terms in a sequence and use the pattern to find the terms of the sequence, sum of the terms in the sequence or to identify properties of the sequence.
Terms of a sequence are generally denoted by $T_1, T_2, T_3, \ldots, T_n$.
A lot of sequences either display a difference based pattern or a multiplicative pattern. However, there are infinite ways in which sequences can be formed. Only a couple of patterns are discussed below to show how to analyze a sequence and mathematically represent it.

A. DIFFERENCE

In these sequences, consecutive terms are related to each other in terms of the difference between the two. This difference can be constant or may follow a logical pattern itself.

For example, consider the sequence 1, 3, 7, 13, 21, 31 .....
Observe that the difference between successive terms is 2, 4, 6, 8 and 10. Hence, the pattern followed by the difference is that they are all multiples of 2. Hence, the term after 31 should be 31 + 12 = 43. Mathematically, each term of the sequence can be expressed as:

$T_1 = 1 + 2 \times 0 = 1$
$T_2 = 1 + 2 \times 1 = 3$
$T_3 = 3 + 2 \times 2 = 7$
$T_4 = 7 + 2 \times 3 = 13$
$T_5 = 13 + 2 \times 4 = 21$
$T_6 = 21 + 2 \times 5 = 31$

Thus the n^{th} term of the sequence can be written as
$T_n = T_{n-1} + 2(n - 1)$

We can see that the n^{th} term depends on the previous term as well as its position in the sequence.
Hence
$T_7 = 31 + 2 \times 6 = 43$

The advantage of representing a sequence mathematically is that later terms of the sequence can be found easily without the need to write the entire sequence. For instance, in the above case, if T_{38} is known and the value of T_{40} is to be found, there is no need to write the entire sequence up to 40 terms. Instead, using the value of T_{38}, the value of T_{39} and consequently, that of T_{40} can be found in just 2 steps.

Example 1: Find the 7^{th} term of the sequence 1, 2, 4, 7, 11, 16 ...

Solution: $T_1 = 1$
$T_2 = 1 + 1 = 2$
$T_3 = 2 + 2 = 4$
$T_4 = 4 + 3 = 7$
$T_5 = 7 + 4 = 11$
$T_6 = 11 + 5 = 16$

The n^{th} term of this sequence can be expressed as
$T_n = T_{n-1} + (n - 1)$
$\therefore T_7 = T_6 + (7 - 1) = 16 + 6 = 22$
Hence the 7^{th} term of this sequence would be
$16 + 6 = 22$.

B. CUMULATIVE SEQUENCE

Consider the sequence 1, 1, 2, 3, 5, 8, 13, 21.....
From the sequence, observe that after the second term, the next term is the sum of the previous two terms. Hence the sequence can be mathematically represented in the following manner.

$T_1 = 1$
$T_2 = 1$
$T_3 = 1 + 1 = 2$
$T_4 = 2 + 1 = 3$
$T_5 = 3 + 2 = 5$
$T_6 = 5 + 3 = 8$
$T_7 = 8 + 5 = 13$
$T_8 = 13 + 8 = 21$
and so on.

As can be seen, the next term is the sum of its previous two terms, hence
$T_n = T_{n-1} + T_{n-2}$
$\therefore T_9 = T_8 + T_7$
$\therefore T_9 = 21 + 13 = 34$

In these types of sequences the pattern is formed with the help of its previous terms.

Example 2: Find the next term of the series 3, 4, 11, 24, 43, ...

Solution: Difference between the 1^{st} and 2^{nd} term = 1
Difference between the 2^{nd} and 3^{rd} term = 7 = 1 + 6
Difference between the 3^{rd} and 4^{th} term = 13 = 7 + 6
Difference between the 4^{th} and 5^{th} term = 19 = 13 + 6
Thus the difference between the 5^{th} and 6^{th} terms = 19 + 6 = 25.
Hence the next term is 43 + 25 = 68.

III. PROGRESSIONS

There are 3 specific types of sequences which show a specific mathematical relationship among their terms. These 3 types, also known as progressions, are named **Arithmetic Progression, Geometric Progression** and **Harmonic Progression.**

IV. ARITHMETIC PROGRESSION

The terms of a sequence are said to be in **Arithmetic Progression (A.P.)** when they differ by a constant value known as their **common difference**, denoted by ***d***. In other words, the difference between any two consecutive terms in an A.P. is constant. The first term of an A.P. is generally denoted by ***a***.

A. THE n^{th} TERM OF AN A.P.

The n^{th} term of an A.P. is,
$T_n = a + (n - 1)d$
Example 3: Find the fifteenth term of the A.P. −3, −9, −15, ...

Solution: $T_n = a + (n - 1)d$
Here, $a = -3$, $d = -9 - (-3) = -6$ and $n = 15$

$\therefore T_{15} = -3 + (14)(-6)$
$\therefore T_{15} = -87$

Example 4: The ninth term exceeds the fifth term of an A.P. by 32. The sum of the ninth and fifth terms is 114. Find the eighth term of the A.P.

Solution: $T_9 = a + (9 - 1)d = a + 8d$
$T_5 = a + (5 - 1)d = a + 4d$
$T_9 - T_5 = 32$
$\therefore (a + 8d) - (a + 4d) = 32$
$\therefore 4d = 32$
$\therefore d = 8$
$\therefore T_9 + T_5 = (a + 8d) + (a + 4d) = 2a + 12d$
$T_9 + T_5 = 114$
$\therefore 114 = 2(a + 6d)$
$\therefore a + 6d = 57$
$T_8 = a + 7d = a + 6d + d = 57 + 8 = 65$

Example 5: The 54th and the 4th terms of an A.P. are −61 and 64 respectively. Find the 23rd term.

Solution: The 54th and 4th term of the given A.P. can be represented as shown below.
$a + 53d = -61$ ... (i)
$a + 3d = 64$ ... (ii)
Subtracting, we get
$50d = -125$
$d = -5/2$

$$a = 64 - 3d = \frac{143}{2}$$

Hence the 23^{rd} term $= a + 22d$

$$= \frac{143}{2} + 22\left(-\frac{5}{2}\right) = \frac{33}{2}$$

B. SUM OF *n* TERMS OF AN A.P.

Let the first term and common difference of an A.P. containing n terms be a and d respectively. Let T_n be the last term of the A.P. Then, the sum of n terms of the A.P. is

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + T_n]$$

Example 6: The sixth and eighth terms of an A.P. are 38 and 52 respectively. Find the sum of the first twelve terms of the A.P.

Solution: $T_6 = a + 5d = 38$
$T_8 = a + 7d = 52$
Solving the two equations, $d = 7$ and $a = 3$
$\therefore$ The sum of 12 terms of the A.P. is

$$S_{12} = \frac{12}{2}[2 \times 3 + (11)7] = 498$$

Example 7: The fourteenth and fifteenth terms of an A.P. are 25 and 32 respectively. Find the 30th term, sum of the first 30 terms and the first term of the A.P.

Solution: $T_{14} = a + 13d = 25$
$T_{15} = a + 14d = 32$
$\therefore d = 7$ and $a = -66$

$T_{30} = -66 + 29 \times 7 = 137$

The sum of the first n terms $= \frac{n}{2}(a + T_n)$

$\therefore$ The sum of the first 30 terms $= \frac{30}{2}(-66 + 137) = 1065$

Example 8: The sum of the first 16 terms of an A.P. is equal to the sum of the first 24 terms of the A.P. Find the sum of the first 40 terms of the A.P.

Solution: $\because S_{16} = S_{24}$
$\therefore (16/2) \times [2a + 15d] = (24/2) \times (2a + 23d)$
$\therefore 4a + 30d = 6a + 69d$
$\therefore 2a + 39d = 0$
$S_{40} = (40/2) \times (2a + 39d) = 20 \times 0 = 0$

Example 9: (CSAT 2011) A contract on construction job specifies a penalty for delay in completion of the work beyond a certain date is as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day etc., the penalty for each succeeding day being Rs. 50 more than that of the preceding day. How much penalty should the contractor pay if he delays the work by 10 days?

(a) Rs. 4950 (b) Rs. 4250 (c) Rs. 3600 (d) Rs. 650

Solution: The penalty pattern is an AP.
First term a = Rs.200
Common difference d = Rs.50
$n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{10} = \frac{10}{2}[2 \times 200 + (10-1) \times 50] = \text{Rs.} 4250$

Hence, **option b.**

V. GEOMETRIC PROGRESSION

The terms of a sequence are said to be in **Geometric Progression (G.P.)** when they increase or decrease by a constant factor. This constant factor is called the **common ratio**, denoted by r, and can be found by dividing any term of the sequence by the preceding term.

If all terms of a G.P. are greater than the preceding terms, the G.P. is an increasing G.P. In such a case, $r > 1$.

If all terms of a G.P. are less than the preceding terms, the G.P. is a decreasing G.P. Here, $r < 1$.

If $r = 1$, all terms of the G.P. are equal.
Such a sequence is both, an A.P. and a G.P.
If the first term is a, the terms of the progression are $a, ar, ar^2, ar^3 \ldots$

A. THE n^{TH} TERM OF A G.P.

If $T_1, T_2, T_3 \ldots, T_n$ denote consecutive terms of a G.P., then
The n^{th} term of the geometric progression is given by

$T_n = ar^{n-1}$

Example 10: Find the fifth term of the G.P. whose first term is 3 and the common ratio is 1/3.

Solution: $a = 3$ and $r = (1/3)$

$= 3\left(\frac{1}{3}\right)^{5-1}$

= 3/81 = 1/27

Example 11: The product of the first five terms of a G.P. is 28. Find the third term.

Solution: $a \times ar \times ar^2 \times ar^3 \times ar^4 = 28$
$\therefore a^5r^{10} = 28$
The third term of this G.P. will be ar^2
So, try to express the above equation in terms of the third term.
$\therefore (ar^2)^5 = 28$
$\therefore ar^2 = \sqrt[5]{28}$
$\therefore$ The third term $= \sqrt[5]{28}$

Alternatively,
In such a case, assume the central term to be a and find the other terms from this point onwards.
In this case, there are 5 terms in all. Hence, let the third term of this G.P. be a.
Hence, the 1st, 2nd, 4th and 5th terms will be a/r^2, a/r, ar and ar^2 respectively.

$\therefore \frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 = 28$

$\therefore a^5 = 28$
$\therefore$ The third term $= a = \sqrt[5]{28}$
In case the G.P. has an even number of terms, one can take the two central terms to be a/r and ar and proceed from there.

Example 12: The product of the first three consecutive terms of an increasing G.P. is 216 and their sum is 21. Find the fourth term of this G.P.

Solution: Let the three terms of the G.P. be a/r, a and ar respectively.
Then,
$a/r \times a \times ar = 216$
$\therefore a^3 = 216$
$\therefore$ The second term $= a = 6$
Also, $a/r + a + ar = 21$
$\therefore 6r^2 + 6r + 6 = 21r$
$\therefore 6r^2 - 15r + 6 = 0$
$\therefore (2r - 1)(r - 2) = 0$
$\therefore r = 2$ or $r = ½$
$\because$ the G.P. is an increasing G.P.
$\therefore r = 2$
The fourth term of this G.P. is $ar^2 = 6 \times 2^2 = 24$

B. SUM OF *n* TERMS OF A G.P.

The sum of n terms of a G.P. with

$r < 1$ is $\frac{a(1-r^n)}{1-r}$

The sum of n terms of a G. P. with $r > 1$ is $\frac{a(r^n - 1)}{r-1}$

The sum of an infinite number of terms of a decreasing G. P. $= \frac{a}{1-r}$

Example 13: Find the sum of 5 terms of the series $\frac{1}{5}, \frac{1}{2}, \frac{5}{4}, \ldots$

Solution: The given series is a G.P. with $a = 1/5$ and $r = 5/2$
Since $r > 1$

$$\therefore S_5 = \frac{\frac{1}{5}\left[\left(\frac{5}{2}\right)^5 - 1\right]}{\frac{5}{2} - 1}$$

$\therefore S_n = 2993/240 \approx 12.88$

Example 14: Find the sum of n terms of the series 1, 1.5, 3, 2.25, 5, 3.375, 7,...

Solution: The given series can be broken into two different series:
S_1 = 1, 3, 5, 7, ...
S_2 = 1.5, 2.25, 3.375, ...
S_1 is an A.P. with first term (a) = 1 and common difference (d) = 2

$\therefore$ Sum of n terms of $S_1 = \frac{n}{2}[2 + 2(n-1)] = n^2$

S_2 is a G.P. with first term (a) = 1.5 and common ratio (r) = 1.5
Since $r > 1$

$\therefore$ Sum of n terms of $S_2 = \frac{1.5(1.5^n - 1)}{1.5 - 1} = \frac{1.5(1.5^n - 1)}{0.5}$

$= 3(1.5^n - 1)$
$\therefore$ Sum of series $= n^2 + 3(1.5^n - 1)$

C. GEOMETRIC MEAN

If n terms $a_1, a_2, ..., a_n$ are in G.P., then the Geometric Mean G of these n terms is given by

$$G = \sqrt[n]{a_1 \times a_2 \times a_3 \times ... \times a_n}$$

VI. HARMONIC PROGRESSION

The terms of a sequence are said to be in **Harmonic Progression (H.P.)** when their reciprocals are in A.P. In general if a, $a + d$, $a + 2d$, $a + 3d$, ... are successive terms of an arithmetic progression, then $1/a$, $1/(a + d)$, $1/(a + 2d)$ and $1/(a + 3d)$, ... are in harmonic progression.

The nth term (Tn)of a harmonic progression is given by $T_n = \frac{1}{a + (n-1)d}$

Example 15: The third term of a H.P. is 1/3 and the sixth term is 1/9. Find the 31st term of the H.P.

Solution: The third and sixth terms of the H.P. are 1/3 and 1/9 respectively.
Hence, for the corresponding A.P. the third term is 3 and the sixth term is 9.
Hence, $a + 2d = 3$ and $a + 5d = 9$
$\therefore d = 2$ and $a = -1$
Hence the 31st term of this A.P. $= a + (31 - 1)d = (-1) + 30 \times 2 = 59$
Hence for the corresponding H.P., the 31st term is 1/59.

A. HARMONIC MEAN

The **Harmonic Mean** of n numbers is the reciprocal of the arithmetic mean of the reciprocals of these n numbers.
The harmonic mean can be derived using the concept of arithmetic mean.

If a, b and c are in H.P., b is the harmonic mean of a and c. $1/a$, $1/b$ and $1/c$ are in A.P. Thus we have

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{b} - \frac{1}{c}$$

$\therefore \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\therefore b = \frac{2ac}{a+c}$

Hence the harmonic mean for two numbers is given by $b = \frac{2ac}{a+c}$

In general, the harmonic mean of n numbers $a_1, a_2, a_3, a_4, ..., a_n$ is

$$H = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \frac{1}{a_4} + \cdots + \frac{1}{a_n}}$$

B. RELATION BETWEEN ARITHMETIC, GEOMETRIC AND HARMONIC MEANS

Let A, G and H represent the Arithmetic, Geometric and Harmonic means of two positive or two negative quantities a and b. Then,

$A = (a + b)/2$
$G = \sqrt{ab}$
$H = 2ab/(a + b)$
$\therefore A \times H = ab = G^2$
$\therefore A \times H = G^2$

Thus G is the geometric mean of A and H.
Also, we know that $A > G$ for two unequal quantities.
But as G is the geometric mean of A and H, G lies between A and H.
∴ Arithmetic Mean > Geometric Mean > Harmonic Mean

VII. SERIES OF NATURAL NUMBERS

Sum of **first n natural numbers** is given by $S_n = \frac{n(n+1)}{2}$

Sum of the **squares of the first n natural numbers** is given by $S_n = \frac{n(n+1)(2n+1)}{6}$

Sum of the **cubes of the first n natural numbers** is given by $S_n = \left[\frac{n(n+1)}{2}\right]^2$

Example 16: Find the sum of $25^2 + 26^2 + 27^2 + \dots\dots\dots + 60^2$

Solution: $25^2 + 26^2 + 27^2 + \dots\dots\dots + 60^2$

$= (1^2 + 2^2 + 3^2 + \dots\dots\dots + 60^2) - (1^2 + 2^2 + 3^2 + \dots\dots\dots + \mathbf{24^2})$

$= \sum_{r=1}^{60} r^2 - \sum_{r=1}^{25} r^2$

$= \frac{60(60+1)(120+1)}{6} - \frac{24(24+1)(48+1)}{6}$

$= 10 \times 61 \times 121 - 4 \times 25 \times 49 = 73810 - 4900 = 68910$

TEST 1

1. Find the next term of the following series2, 5, 10, 17, 26, ...

   (a) 36 (b) 35 (c) 34 (d) 33 (e) None of these

2. Find the value of x in the sequence 4, 5, 7, 10, 14, x.

   (a) 18 (b) 19 (c) 20 (d) 28 (e) 15

3. The first two terms of an A.P. are 3 and 5 respectively. Find the 5th term of the same sequence.

(a) 28 (b) 15 (c) 23 (d) 25 (e) 11

4. Ten children are standing in a line. Ramesh wants to distribute some chocolates amongst these 10 children such that the first child in the line gets 4 chocolates and every subsequent child gets 3 chocolates more than the previous child. What is the total number of chocolates that Ramesh distributes?

(a) 120 (b) 156 (c) 126 (d) 130 (e) 175

5. Divide the number 124 into 4 parts which are in A.P., such that product of first and fourth term is 128 less than the product of the second and the third term. Find the smallest number amongst these four numbers.

(a) 11 (b) 19 (c) 53 (d) 27 (e) 43

6. The sum of the third and seventh term of an A.P is 8. Find the sum of the first nine terms of this progression?

(a) 24 (b) 32 (c) 36 (d) None of these (e) Cannot be determined

7. The first term of a G.P. is 5 and the common ratio is 2. Find the sixth term of this progression.

(a) 620 (b) 160 (c) 225 (d) 260 (e) 120

8. If 10, *b* and 40 are in G.P., then find the value of *b*.

(a) +400 (b) −20 (c) +20 (d) +25 (e) None of these

9. The ratio of the sum of the first eight terms of a G.P. to the sum of the first four terms of the same G.P. is 97: 81, where the common ratio of the G.P. is a real number. The common ratio is:

(a) 2 (b) 3 (c) $\frac{3}{2}$ (d) $\frac{2}{3}$ (e) $\frac{4}{3}$

10. The geometric mean of two positive numbers is 6 and it exceeds its harmonic mean by 2. Find its arithmetic mean.

(a) 4 (b) 2 (c) 6 (d) 9 (e) 36

TEST 2

11. The first three terms of an H.P. are 1/5, 1/7 and 1/9 respectively. Find the 7th term of the sequence

(a) $\frac{1}{17}$ (b) $\frac{1}{29}$ (c) 17 (d) $\frac{1}{19}$

12. What is the sum of the first 20 terms of the following series
$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + ...$

(a) 5740 (b) 2870 (c) 1435 (d) 4420 (e) None of these

13. Amar bought a car on EMI. He pays Rs 1,000 in the first month, Rs 1,500 in the second month, Rs. 2,000 in the third month and so on, such that in every subsequent month he pays Rs.500 more than the preceding month. He follows this practice for 1 year. If he ends up paying Rs. 20,000 extra under the EMI scheme compared to what he would have paid had he paid the full amount upfront, what is the cost of car (in Rs.)?

(a) 20,000 (b) 25,000 (c) 27,000 (d) 26,000

14. The ratio of the 5^{th} term of an A.P. to the 7^{th} term of the same A.P. is 0.Find the ratio of the 12^{th} term to 13^{th} term of the A.P.?

(a) 3/4 (b) 4/5 (c) 7/8 (d) Cannot be determined.

15. The sides of a quadrilateral are in A.P. The semi perimeter of the quadrilateral is 40. The second largest side of the quadrilateral is three times the smallest side. Find the largest side?

(a) 28 (b) 32 (c) 40 (d) None of these.

16. Sum of all the multiples of 7 in the range 1 - 491 is?

(a) 17402 (b) 14395 (c) 21347 (d) 17395

17. The 1^{st} term of an A.P is 17 and the product of the 2^{nd} and 4^{th} term equals the product of the 5^{th} and 6^{th} term of the A.P. Find the 3^{rd} term of the A.P.?

(a) 7 (b) −14 (c) −35 (d) Cannot be determined.

18. Rajesh plans to save money for gifting his friend. He puts Rs.150 in the piggy bank and then puts Rs.25 more than the previous instance, each time he puts in money. If he regularly puts in money each month, how much money (in Rs.) would be accumulated after 3 years?

(a) 12350 (b) 21150 (c) 41325 (d) None of these.

19. The fifth term of a G.P. is 64 times the second term. If the fourth term is −320, find the sixth term of the G.P.

(a) −5120 (b) 1280 (c) −1280 (d) None of these.

20. The product of 3 terms of an G.P is 1728. If the 3^{rd} term is 4 times the 1^{st} term, find the 3^{rd} term?

(a) 6 (b) 3 (c) −8 (d) Cannot be determined.

21. If you save Rs. 7 on 1^{st} May, Rs.14 on 2^{nd} May, and Rs.28 on 3^{rd} May, how much would you save by 15^{th} May?

(a) 229369 (b) 4163335 (c) 11345 (d) 30945345

22. What is the value of $1^3 + 2^3 + 3^3 + \ldots\ldots + 20^3$?

(a) 44100 (b) 44000 (c) 80000 (d) 40010

23. What is the value of $2^2 + 3^2 + 4^2 + \ldots + 50^2$?

(a) 42900 (b) 42925 (c) 42920 (d) None of these

24. The cost of railway tickets in a particular year is 1.5 times the cost in the previous year. This increase in cost happens each year. The cost of a one-way ticket from Mumbai to Pune was Rs.56 in 2008. What amount (in Rs.) will Mr. Sharma pay for a return journey from Mumbai to Pune in 2013?

(a) 141.75 (b) 283.5 (c) 425.25 (d) None of the above

25. Rohan started practicing Maths from 3rd June. He could solve only 3 problems on the first day. However, he increased his speed such that he solved 6 problems on the second day, 9 on the third, and so on. If he continued this trend throughout June, how many problems did he solve on 25th June of the same year?

(a) 57 (b) 66 (c) 63 (d) None of the above

26. A sum of Rs. 700 has to be used to give seven cash prizes to the students of a school for their overall academic performance. If each prize is Rs. 20 less than its preceding prize, what is the least value of the prize? **[UPSC 2013]**

(a) Rs. 30 (b) Rs. 40 (c) Rs. 60 (d) Rs. 80

TEST 3

27. What is the sum of the first 21 terms of the progression 2, 7, 12, 17, ...?

(a) 998 (b)1168 (c)1042 (d)1092 (e) 972

28. Find the 8^{th} term of the series 384, 192, 96, 48, ...

(a) 3 (b) 6 (c) 8 (d) 12 (e) None of these

29. Find the 19^{th} term of the A.P. with first term 3 and common difference 4.

(a) 81 (b) 69 (c) 79 (d) 75 (e) 86

30. How many terms does the following G.P. 1, 3, 9, 27, ..., 729 have?

(a) 9 (b) 13 (c) 7 (d) 11 (e) 10

31. Find the sum of the 10 terms of a G.P. having first term 5 and common ratio 2.

(a) 4554 (b) 5252 (c) 4232 (d) 4664 (e) 5115

14 Solutions

AVERAGES

TEST 1

1. The average age of A, B, C & D is 26. Thus, the total age of A, B, C and D is 26 × 4 = 104.
   The average age of B & D is 28. Thus, the total age of B & D is 56.
   Thus, the total age of A & C = 104 − 56 = 48.
   Thus, average age of A & C = 48/2 = 24.
   Hence, **option b.**

2. $\frac{A + B + C}{3} = 15$

   $\therefore A + B + C = 45$ ... (i)

   $\frac{(A + 5) + B}{2} = 16$

   $\therefore A + B = 32 - 5 = 27$ ... (ii)

   From these 2 equations,

   $\therefore C = 45 - 27 = 18$

   Hence, **option c.**

3. $\frac{\text{Akash} + \text{Bharat} + \text{Chandra}}{3} = 34$

   ∴ Akash + Bharat + Chandra = 102 ... (i)

   $\frac{\text{Akash} + \text{Bharat} + \text{Chandra} + \text{Dimple}}{4} = 29$

   ∴ Akash + Bharat + Chandra + Dimple = 116

   ∴ Dimple = 116 − 102 = 14

   Hence, **option a.**

4. $\frac{\text{Priya} + \text{Ruchi}}{2} = 32$

   ∴ Priya + Ruchi = 64 ... (i)

   Age of Ruchi = 18

   ∴ 18 of Priya = 64 − 18 = 46

   $\frac{\text{Shiv} + \text{Prem}}{2} = 36$

   Shiv + Prem = 72 ... (ii)

   Age of Prem = 22

   ∴ Age of Shiv = 72 − 22 = 50

   Thus, average age of Shiv & Priya

   $= \frac{\text{Shiv} + \text{Priya}}{2}$

   $= \frac{50 + 46}{2} = 48$

   Hence, **option d.**

5. We have the average score of 4 matches to be

   $\frac{141 + 147 + 162 + 178}{4} = 157$

   Since the score in the 5th match was 7 less than the average, it was 157 − 7 = 150

   Hence, **option e.**

6. Let the 8th number be x.
   Sum of all 15 numbers = 15 × 54 = 810
   Sum of the first 7 numbers + x = 64 × 8 = 512
   x + Sum of the last 7 numbers = 60 × 8 = 480
   Adding equations (i) and (ii),
   Sum of first 7 numbers + 2x + Sum of the last 7 numbers = 992
   ∴ (Sum of first 7 numbers + x + Sum of the last 7 numbers) + x = 992
   The term in the brackets is the sum of all the 15 numbers.
   ∴ 810 + x = 992
   ∴ x = 182
   Hence, **option a.**

7. Let the number of students in the first and the second batch be x and y respectively.
   Hence, the total marks obtained by the students of the first and second batch are 80x and 90y respectively.
   When the two batches are combined, the total marks of the students of the classes are 80x + 90y and the combined number of students is $x + y$.

   $\therefore \frac{(80x + 90y)}{(x + y)} = 84$

   ∴ 80x + 90y = 84x + 84y

   ∴ $x : y = 3 : 2$

   Hence, **option d.**

8. Age of teacher = (Total age of students and teacher taken together) − (Total age of only the students)
   = (41 × 24) − (40 × 23.5)
   = 984 − 940 = 44 years
   Hence, **option c.**

9. The average of the 6 students increases by 2 kg when one student weighing 48 kg is replaced by another.
   ∴ The total increase in weight is 6 × 2 = 12 kg.
   Assume that the weight of each student in the original group was 48 kg.
   Hence, one student weighing 48 kg is replaced by someone who increases the total weight of the group by 12 kg.

Hence, the replacement student has contributed the extra 12 kg.
Hence, the weight of the new student
= 48 + 12 = 60 kg
Hence, **option e.**

Alternatively,
The original group has one student weighing 48 kg.
Let the total weight of the remaining 5 students be n kg and the average weight of all 6 students be a.
$\therefore (n + 48)/6 = a$
$\therefore n + 48 = 6a$ ... (i)
Now, the person weighing 48 kg is replaced with someone who weighs, say x kg.
Hence, the average goes by 2 kg and becomes $a + 2$
$\therefore (n + x)/6 = a + 2$
$\therefore n + x = 6a + 12$
$\therefore n + x = (n + 48) + 12$ ... from (i)
$\therefore x = 60$
Hence, the replacement student weighs 60 kg.
Hence, **option e.**

10. Let the number of people initially present in the group be n.
Hence, the total age of the group is $25n$.
Four new friends with an average age of 21 years join this group.
The total age of the group after the new four friends joined = $25n + (4 \times 21) = 25n + 84$
The average age of the people in the new group is 23.
$\therefore$ Total age of the group after the new four friends joined = $23(n + 4)$
$\therefore 23(n + 4) = 25n + 84$
$\therefore 23n + 92 = 25n + 84$
$\therefore 2n = 8$
$\therefore n = 4$
Thus, originally there were 4 people in the group.
Hence, **option a.**

TEST 2

11. Let the temperatures on Sunday, Monday, Tuesday, Wednesday and Thursday be s, m, t, w and th respectively.
Hence, Sum of the temperatures on Sunday, Monday, Tuesday and Wednesday
$= s + m + t + w$
$= 48 \times 4$
$= 192$ ... (i)
Similarly, sum of the temperatures on Monday, Tuesday, Wednesday and Thursday
$= m + t + w + th$
$= 49 \times 4$
$= 196$ ... (ii)
Subtracting (i) from (ii)
$th - s = 196 - 192 = 4$
$s : th = 12 : 13$
$\therefore 13x - 12x = 4$
$\therefore x = 4$
$\therefore$ Temperature on Sunday = $12x = 48°$
Hence, **option b.**

12. Since, the calculation involves two groups (in this case, departments); the concept of weighted averages is to be used.
Let the number of employees in department B be 'x'.
Using the formula of weighted average,

$$1200 = \frac{(9000 \times 200) + (1400 \times x)}{200 + x}$$

$\therefore 240000 + 1200x = 180000 + 1400x$
$\therefore x = 300$
Hence, **option d.**

13. Since the calculation involves two groups, use the concept of weighted averages.
Let the average age of the group be 'x'.
Using the formula for weighted average,

$$x = \frac{(25 \times 12) + (20 \times 12)}{(15 + 12)}$$

$\therefore x = 22.78$
Hence, **option b.**

14. The average of a student in the first 50 tests is p and his average in the next 10 tests is q.
$\therefore$ His total score is $50p + 10q$ over 60 tests, and his overall average is $p + 2$.

$$\text{So, } \frac{50p + 10q}{60} = p + 2$$

$\therefore 5p + q = 6p + 12$
$\therefore q = p + 12$
Hence, **option d.**

15. Total age of 20 students = $20 \times 9 = 180$ years.
When the teacher is included, there are 21 people in all.
Thus, after including the teacher's age, average age of the 21 people = 9 + 2 = 11 years.
So, total age of the 21 persons = 21×11 = 231.
$\therefore$ Age of the teacher = 231 − 180 = 51 years.
Hence, **option a.**

16. Total age of the three brothers = $10 \times 3 = 30$.
When the father and mother are also considered in this group, the average age increases by 13 i.e. it becomes 10 + 13 = 23
$\therefore$ Total age of the 5 family members = 23×5

= 115

So, total age of father and mother = 115 – 30

= 85

Let the age of the father be x years.

So, age of the mother = x – 5 years.

$\therefore x + x - 5 = 85$

$\therefore x = 45$

Thus, the father is 45 years old.

Hence, **option c.**

17. Five years ago, average age of the couple = 24

So, total age of the couple five years ago

= 24 × 2 = 48

∴ Total age of the couple at present = 48 + 10

= 58

Here, the age of the child is not considered.

But, present average age of the family of three members = 20

∴ Total age of the family at present = 20 × 3

= 60

∴ Age of the child at present = 60 – 58 = 2 years.

Hence, **option a.**

18. Let the average score of the batsman in the first two innings be x runs.

So, his total score in the first two innings = $2x$

and his score in the third innings = $2.5x$

His average across all three innings is 90

So, his total score across all three innings

= 270

$\therefore 2x + 2.5x = 270$

$\therefore x = 60.$

So, the batsman scored $2.5x$ = 150 runs in the third innings.

Hence, **option e.**

19. Average score of 10 players = 26

Total score of the 10 players without the captain

= 26 × 10 = 260

When the captain's score is added, the average increases by 4 i.e. it becomes 30

So, total score of the 11 players including the captain

= 30 × 11 = 330

∴ Runs scored by the captain = 330 – 260 = 70

Hence, **option e.**

20. 10 students had an average score of 80.

So, their total score = 10 × 80 = 800

The remaining 15 students had an average score of 60.

Their total score = 15 × 60 = 900

So, total score of the class = 800 + 900 = 1700

Average score of the class = 1700/25 = 68

Hence, **option b.**

Alternatively,

Since the two groups of students have different group sizes (10 and 15), the weighted average of their scores can be taken to get the average score of the whole class.

∴ Average score of entire class

$$= \frac{(10 \times 80) + (15 \times 60)}{(10 + 15)} = \frac{1700}{25} = 68$$

Hence, **option b.**

TEST 3

21. The average age of all 5 members of the family is 25 years while the average age of 2 members of this family is 13 years.

∴ Total age of the 5 members = 25 × 5 = 125 years.

and, total age of the 2 members = 13 × 2 = 26 years

∴ Total age of the remaining 3 members of the family

= 125 – 26 = 99 years

∴ Average age of the 3 members = 99/3 = 33 years

Hence, **option a.**

22. Since the price of the two groups of suits differs, the average cost of the 5 suits is the weighted average of the two groups.

$$\therefore \text{Average cost} = \frac{(2 \times 179) + (3 \times 189)}{(2 + 3)}$$

$$= \frac{925}{5}$$

= Rs. 185

Hence, **option a.**

23. The average score of a class of p students is 70 while that of a class of n students is 92.

So, total marks of the students of the first class = $70p$

and, total marks of the students of the second class = $92n$

Average marks obtained by students of these two classes

$$= \frac{70p + 92n}{p + n}$$

It is given that this average is equal to 86.

$$\therefore \frac{70p + 92n}{p + n} = 86$$

$\therefore 70p + 92n = 86p + 86n$

$\therefore 16p = 6n$

$\therefore p/n = 6/16 = 3/8$

Hence, **option c.**

24. Let the total runs scored by the bottom six batsmen = x

∴ Total runs scored by the top five batsmen
$= x + 30$
$\therefore x + x + 30 = 210$
$\therefore x = 90$ and $x + 30 = 120$
So, average runs scored by the top five batsmen
$= 120/5 = 24.$
and, average runs scored by the bottom six batsmen
$= 90/6 = 15.$
So, the required difference $= 24 - 15 = 9.$
Hence, **option d.**

25. The average age of the 11 players is the weighted average of the three groups given.
Average age of 11 players

$$= \frac{4 \times 32 + 3 \times 24 + 4 \times 20}{4 + 3 + 4}$$

$$= \frac{280}{11} = 25.45 \cong 25 \text{ years.}$$

Hence, **option d.**

26. Total price of the four items $= 2200 \times 4$
= Rs. 8,800
Let the prices of these items (in Rs.) be x, $5x$, $3x$, $2x$.
$\therefore x + 5x + 3x + 2x = 8800$
$\therefore x = 8800/11 =$ Rs. 800
Hence, price of the costliest item $= 5x$
= Rs. 4,000
Hence, **option d.**

27. There are 7 prime numbers between 50 and 80 i.e. 53, 59, 61, 67, 71, 73, 79.

$$\text{Average} = \frac{53 + 59 + 61 + 67 + 71 + 73 + 79}{7}$$

$$= \frac{463}{7}$$

$= 66.14$
Hence, **option b.**

28. Let the numbers be $x, x + 2, x + 4, x + 6, x + 8$

$$\frac{x + (x + 2) + (x + 4) + (x + 6) + (x + 8)}{5}$$

$= 32$

$$\therefore \frac{5x + 20}{5} = 32$$

$\therefore x + 4 = 32$
$\therefore x = 28$
So, the 5 consecutive even numbers are 28, 30, 32, 34 and 36.
Hence, the smallest of these numbers is 28.
Hence, **option a.**

Note: In such a case, taking the numbers as $x - 4, x - 2, x, x + 2$ and $x + 4$ leads to easier calculations. With these values, we would have directly got $(5x)/5 = 32$ and $x = 32$. However, keep in mind that the smallest number would have been $x - 4$ i.e. $32 - 4 = 28$

29.
$$\frac{x + (x + 1) + (x + 2) + (x + 3) + (x + 4) + (x + 5) + (x + 6)}{7}$$

$= 14$

$$\therefore \frac{7x + 21}{7} = 14$$

$\therefore x = 11$
So, the numbers are 11, 12, 13, 14, 15, 16, 17.
Hence, the mean of the last 4 numbers is given by,

$$\frac{14 + 15 + 16 + 17}{4} = 15.5$$

Hence, the mean is 15.5.
Hence, **option d.**

30. Sum of the 10 numbers $= 10p$
Sum of the 5 numbers having average $m = 5m$
Sum of the 5 numbers having average $n = 5n$
$\therefore 10p = 5m + 5n = 5(m + n)$

$$\therefore p = \frac{1}{2}(m + n)$$

Hence, **option c.**

31. Let p, q, r be the first, second and third number respectively.

$$\therefore \frac{p + r}{2} - \frac{q + r}{2} = 17$$

$$\therefore \frac{p + r - q - r}{2} = 17$$

$\therefore p - q = 34$
Hence, **option d.**

32. Sum of 52 numbers $= 52 \times 30 = 1560$
Since each number is decreased by 2, decrease in the sum $= 52 \times 2 = 104$
∴ New sum $= 1560 - 104 = 1456$

$$\therefore \text{New Arithmetic mean} = \frac{1456}{52} = 28$$

Hence, **option c.**

Note: This is an application of a direct property. Since all the numbers decrease by the same amount, the average also decreases by the same amount. Conversely, if all the numbers had increased by a number m, the average would also have increased by m.

33. Let there be n students in the class.
Let the total marks of the other students be x.

Original average of marks $= \frac{x+60}{n}$

Jay's marks are wrongly entered as 75 instead of 60. So,

New average of marks $= \frac{x+75}{n}$

This increases the average of the class by 0.5

$\therefore \frac{x+75}{n} = \frac{x+60}{n} + \frac{1}{2}$

$\therefore \frac{x+75-x-60}{n} = \frac{1}{2}$

$\therefore \frac{15}{n} = \frac{1}{2}$

$\therefore n = 30$

Hence, the number of students in this class is 30.

Hence, **option d.**

34. Since the number of students in the smaller groups is given in percentage terms, let the total number of students be 100.

Let the mean score of the 45% students be x.

$\therefore (30 \times 68) + (25 \times 60) + (45 \times x) = 65 \times 100$

$\therefore 2040 + 1500 + 45x = 6500$

$\therefore 45x = 2960$

$\therefore x = 65.77$

Hence, **option d.**

35. The average marks of a student is the ratio of the total marks of all the students to the number of students

$= (x^2 + 4x + 3)/(x + 1)$

$= [(x + 1)(x + 3)]/(x + 1)$

$= x + 3$

$\therefore x + 3 = 49$

$\therefore x = 46$

$\therefore$ The number of students in the class $= x + 1$

$= 47$

Hence, **option b.**

PERCENTAGES

TEST 1

1. Matches won by India in the first two weeks

= 50% of 6 = 3

If x is the number of matches they played after the first two weeks then:

Total number of matches played by India

$= x + 6$

The Indian team won all their matches played after the first two weeks.

Hence, total number of matches won by the Indian team $= x + 3$

$\therefore \frac{(x+3)}{(x+6)} = \frac{75}{100} = \frac{3}{4}$

$\therefore 4x + 12 = 3x + 18$

$\therefore x = 6$

Hence, **option a.**

2. Let the initial price of sugar and Sneha's consumption be Rs. x per unit and y units respectively.

Hence, total amount spent by Sneha on sugar = Rs. xy

New price of sugar = Rs. $1.2x$ per unit

Since Sneha wants to spend only 8% than her initial spend on sugar, the amount that she will spend on sugar = Rs. $1.08xy$

$\therefore$ New quantity of sugar that Sneha will buy

$= 1.08xy/1.2x = 0.9y$

Hence, her consumption should reduce by 10%.

Hence, **option b.**

3. Total votes cast = 7500, out of which 20% were invalid.

$\therefore$ Number of valid votes = 80% of 7500

= 6000.

One candidate got 55% of the valid votes

$\therefore$ Valid votes polled by other candidate

= 45% of 6000 = (45/100) × 6000 = 2700

Hence, **option a.**

4. If the price of a commodity decreases by a%, then the percentage increase in the consumption, so that the expenditure remains the same is:

$\frac{a}{100-a} \times 100$

$\therefore$ Increase in consumption

$= \frac{30}{100-30} \times 100$

= 42.86%

Hence, **option b.**

5. Let the t-shirt's price be Rs. 100

$\therefore$ The trouser's price is Rs. 160

i.e. The t-shirt's price is less than the trouser's price by Rs. 60

Since the t-shirt's price is being compared to the trouser's price, the t-shirt's price becomes the final value and the trouser's price becomes the base (or initial) value.

$\therefore$Percentage by which the t-shirt's price is less than

The trouser's price $= \frac{60}{160} \times 100 = 37.5\%$

Hence, **option d.**

6. Let the original price of sugar be Rs. x per kg.

Hence, reduced price = Rs. (80/100) × x

= Rs. (4/5)x.

Also, the total amount is the same in each case i.e. Rs. 80

Since, reduction in price enabled Rohan to purchase 10 kg more for Rs. 80

$\therefore \frac{80}{\frac{4x}{5}} - \frac{80}{x} = 10$

$\therefore 20/x = 10$

$\therefore x =$ Rs. 2

Hence, **option d.**

7. Let the number of chocolates that Seema initially had be x.

$\therefore$ Number of chocolates distributed among the students of the first standard

= 25% of $x = 0.25x$

At this stage, the remaining chocolates

$= x - 0.25x = 0.75x$

$\therefore$ Number of chocolates distributed among the students of the second standard

= 20% of $0.75x$

$= 0.2 \times 0.75x = 0.15x$

Thus, Seema has $x - 0.25x - 0.15x$

$= 0.6x$ chocolates with her.

The actual number of chocolates that she still has is 240

$\therefore 0.6x = 240$

$\therefore x = 400.$

Thus, Seema initially had 400 chocolates.

Hence, **option c.**

8. Ajay got 30/50 = 60% marks in the first test and 60/70 = 85.72% in the second test.

Hence, the percentage increase in his performance was [(85.72 - 60) × 100]/60

= 25.72 × 5/3 = 42.86%

Hence, **option c.**

9. Since there are 3 successive discounts, the formula for successive discounts should be used.

Total % discount

$$= \left(1 - \frac{100 - 12.5}{100} \times \frac{100 - 10}{100} \times \frac{100 - 7}{100}\right) \times 100$$

= 26.76%

Hence, **option d.**

Alternatively,

Let the original price of the product be Rs. 100.

A discount of 12.5% implies that the price decreases by Rs. 12.5

Hence, price after the first discount = Rs. 87.5

The second discount is of 10% on Rs. 87.5 i.e an actual discount of Rs. 8.75

Hence, price after the second discount

= 87.5 - 8.75 = Rs. 78.75

The third discount is of 7% on Rs. 78.75

i.e. an actual discount of 78.75 × 0.07

= Rs. 5.5125 l Rs. 5.51

Hence, price after the third discount

= 78.75 - 5.51 = Rs. 73.24

Thus, the shopkeeper sells the product at a price of Rs. 73.24

Hence, had he given a single discount, the discount amount would have been

100 - 73.24 = Rs. 26.76

Hence, on an original price of Rs. 100, the discount would have been 26.76%

Hence, **option d.**

10. Let there be 100 animals in the mini-zoo.

So, 60 of these are monkeys and 40 are other animals.

Now, 50% of the monkeys have a tail. This implies that 0.5 × 60 i.e. 30 monkeys have a tail.

Now, it is given that 70% of the animals have a tail. Thus, 70 animals have a tail.

Since 30 of these are monkeys, there are 40 other animals who have a tail.

This implies that all 40 "other animals" have a tail i.e. 100% of the "other animals" have a tail.

Hence, **option b.**

TEST 2

11. The daily wages are calculated as hourly rate × work hours.

The hourly wages have increased by 15% and the work hours have reduced by 12.5%.

Hence, the percentage change in the total earnings can be found as shown below:

$$\text{Percentage change} = 15 - 12.5 - \frac{155 - 12.}{100}$$

$= 0.625$

Thus, his daily wages have increased by 0.625%.

Hence, New daily wages = 80 + (80 × 0.00625)

= 80 + 0.5 = Rs. 80.5

Hence, **option c.**

12. A four and six correspond to 4 and 6 runs respectively.

$\therefore$ Runs scored in fours and sixes

= (3 × 4) + (8 × 6) = 12 + 48 = 60

So, runs scored by running between the wickets = 110 - 60 = 50

$\therefore$ Percentage of runs scored by running between the wickets = (50/110) × 100

= 45.45%

Hence, **option b.**

13. A = x% of y = $(x/100) \times y = xy/100$
B = y% of x = $(y/100) \times x = xy/100$
So, A = B
This relationship is not given in any of the answer options.
Hence, **option e.**

14. Between the numbers 1 and 10, there are 2 numbers that have 1 or 9 in the units place i.e. the numbers 1 and 9.
Since, 70 = 7 × 10
Each group of 10 numbers between 1 and 70 will have 2 numbers that satisfy this condition.
∴ Total number of numbers that have units digit 1 or 9 in this range = 2 × 7 = 14
So, the required percentage = (14/70) × 100 = 20%
Hence, **option c.**

15. Let the number be x.
The original value should have been $(5x/3)$ but it became $(3x/5)$

So, error in the number = $\frac{5x}{3} - \frac{3x}{5} = \frac{16x}{15}$

So, % Error = $\frac{\frac{16x}{15}}{\frac{5x}{3}} \times x0 = 64\%$

Hence, **option d.**

16. Ganesh spends 15, 20 and 40% respectively of his salary on fuel, house rent and other expenditure.
So, Ganesh spends (100 – 15 – 20 – 40) = 25% of his salary on his children's education.
So, 25% of his salary is equal to Rs. 5,000
But, amount spent by him on his fuel = 15% of his salary.
So, amount spent on fuel = (15/25) × 5000 = Rs. 3,000
Hence, **option a.**

17. Let us assume that B earns Rs. 100
So, A's income = 20% more than A = 1.2 × 100 = Rs. 120
But B's income is 20% less than that of C.
So, B's income is (100 – 20) i.e 80% of C's income.
So, C's income = (100/80) × B's income
∴ C's income = (100/80) × 100 = Rs.125
Hence, C's income is the highest among the three.
Hence, **option c.**

18. Let the length and breadth of the rectangle be l and b respectively.
So, original area of the rectangle = lb
Now, the length is increased by 50% and the breadth is increased by 20%.
So, the new length and breadth will be $1.5l$ and $1.2b$ respectively.
∴ New area of the rectangle = $1.8lb$
So, increase in area = $1.8lb - lb = 0.8lb$
∴ % increase in area = $[(0.8lb)/(lb)] \times 100$ = 80%
Hence, **option c.**

19. Let Ravi's initial salary be Rs. 100.
So, after an increase of 50%, his salary becomes 1.5 × 100 = Rs. 150.
Now, after a decrease of 50%, his salary becomes 0.5 × 150 = Rs. 75
So, reduction in salary = 100 – 75 = Rs. 25
∴ Percentage decrease = (25/100) × 100 = 25%
Hence, **option a.**

20. Let the initial price be p and initial sales be s.
So, initial revenue = ps
Now, the price reduces by 40% and sales increase by 60%.
So, new price = $0.6p$ and new sales = $1.6s$
∴ New revenue = $0.6p \times 1.6s = 0.96ps$.
So, decrease in revenue = (1 – 0.96) = $0.04ps$
∴ s0.96) = 0.04n revenue = (1 aps/ps) × 100 = 4%
Hence, **option b.**

TEST 3

21. The population now is 100000.
So, after the first year, it becomes (110/100) × 100000 = 110000
After the second year, it becomes (110/100) × 110000 = 121000
Finally, after the third year, it becomes (90/100) × 121000 = 108900
Hence, **option d.**

Alternatively,
The percentage change in population can be applied simultaneously.
Since the population increases by 10%, then increases by 10% and finally decreases by 10%, the population at the end of 3 years = 100000 × 1.1 × 1.1 × 0.9 = 108900
Hence, **option d.**

22. Raju initially got 12 out of 16 questions correct.
He then answered 25% of the remaining questions correctly.
Let the remaining number of questions be $4x$ and let Raju get x out of them correctly.
So, total questions in the exam = 16 + $4x$

$\therefore\ y = \frac{3}{5}x = \frac{3}{5} \times 5 = 3$

Hence, the numbers are 5 and 3.

Hence, **option d.**

31. Let machine A's daily production be x.

$\therefore$ 25% of x = 45% of 2000

$\therefore\ \frac{25}{100} \times x = \frac{45}{100} \times 2000$

$\therefore\ x = 3600$

Hence, the daily production of machine A is 3600.

Hence, **option c.**

32. The population has increased at 2%, 2.5% and 5% in the last 3 years.

Present population

$= 320000 \times \left(1 + \frac{2}{100}\right) \times \left(1 + \frac{\frac{5}{2}}{100}\right)$

$\times \left(1 + \frac{5}{100}\right)$

$\therefore$ Present population

$= 320000 \times \frac{51}{50} \times \frac{41}{40} \times \frac{21}{20}$

$\therefore$ Present population = 8 × 51 × 41 × 21

= 351288

Hence, **option b.**

33. Total marks = 140 + 60 + 100 = 300

To qualify, Amit needs 40% of the total marks i.e. 0.4 × 300 = 120

Let his marks in the third paper be x.

$\therefore\ 70 + 25 + x = 120$

$\therefore\ x = 25$

So, Amit has to score 25 marks in the third paper to just qualify.

Hence, **option c.**

34. Let the required time be n years.

$\therefore\ 1968300 \times \left(1 - \frac{20}{100}\right)^n$

$= 51200 \times \left(1 + \frac{20}{100}\right)^n$

$\therefore\ 1968300 \times \left(\frac{4}{5}\right)^n = 51200 \times \left(\frac{6}{5}\right)^n$

$\therefore\ \left(\frac{6}{5}\right)^n \times \left(\frac{5}{4}\right)^n = \frac{1968300}{51200}$

$\therefore\ \left(\frac{3}{2}\right)^n = \frac{19683}{512}$

$\therefore\ \left(\frac{3}{2}\right)^n = \left(\frac{3}{2}\right)^9$

$\therefore\ n = 9$

Thus, the value of the land and house will be same after 9 years.

Hence, **option c.**

35. Let the original C.P. be Rs. 100 per apple and Reena's initial consumption be 100 apples.

$\therefore$ Initial expenditure = Rs. 10,000

Since the price of apples decreases by 15%, the new C.P. = Rs. 85

Since she increases her consumption by 20%, Reena's new consumption = 120 apples

$\therefore$ New expenditure = 85 × 120 = Rs. 10, 200

Hence, percentage change in expenditure

$= \frac{10200 - 10000}{10000} \times 100 = 2\%$

Hence, **option d.**

INTEREST AND GROWTH RATES

TEST 1

1. Let rate = r% and time = r years.

$\therefore\ 432 = \frac{1200 \times r \times r}{100}$

$\therefore\ r^2 = 36$

$\therefore\ r = 6\%$

Hence, **option b.**

2. S. I. = $\frac{P \times n \times r}{100}$

Here, both P and r are unknown.

Now, we know that the new value of r is 2 greater than the old value. However, in the absence of information on the Principal, the simple interest cannot be found.

Hence, the given data is inadequate.

Hence, **option e.**

3. Let the sum borrowed by Sakshi be Rs. x.

$\therefore\ 12600 = \left(\frac{x \times 2 \times 2}{100}\right) + \left(\frac{x \times 4 \times 3}{100}\right) + \left(\frac{x \times 5 \times 4}{100}\right)$

$\therefore\ 12600 = \frac{x}{25} + \frac{3x}{25} + \frac{5x}{25}$

$\therefore\ x = \frac{315000}{9}$

$\therefore\ x$ = Rs. 35,000

Hence, **option b.**

4. Using the formula for the amount obtained on compound interest we get:

$A = 8000\left(1 + \frac{5}{100}\right)^2$

$\therefore\ A = 8000 \times \frac{21}{20} \times \frac{21}{20}$

$\therefore\ A$ = Rs. 8,820

Hence, **option c.**

5. Amount at the end of the period
= Rs. (30000 + 4347)
= Rs. 34,347.
Let the time be n years.

$$\text{Then}, 34347 = 30000\left(1+\frac{7}{100}\right)^n$$

$$\therefore \frac{11449}{10000} = \left(\frac{107}{100}\right)^n$$

Since, $10000 = 100^2$, check the value of 107^2.
It is indeed 11449.
$\therefore n = 2$ years.
Hence, **option a.**

6. Chintamani borrowed money at simple interest.
Hence, the interest to be paid
$= (P \times N \times R)/100$
$= 50000 \times 3 \times 0.1 =$ Rs. 15,000
He invested the money at compound interest.
Hence, the amount obtained after investment
$= P(1 + R/100)^3 = 50000 \times (1.1)^3 =$ Rs. 66,550
$\therefore$ The compound interest obtained
= 66550 − 50000 = Rs. 16,550
The difference between the simple interest paid and the compound interest earned is his gain/loss
$\therefore$ Chintamani gained = 16550 − 15000
= Rs. 1,550
Hence, **option c.**

7. Let the rate of interest be x for Manoj.
Using the formula for simple interest,
$I = (P \times N \times R)/100$,
Manoj's Interest $= (4000 \times 2 \times x)/100 = 80x$
Aditi's Interest $= [5000 \times 2 \times (x + 0.5)]/100$
$= 100(x + 0.5) = 100x + 50$
The bank receives a total of Rs. 860 as interest from both of them.
Hence,
$80x + 100x + 50 = 860$
$\therefore x = 4.5\%$
$\therefore$ Manoj borrowed the amount at 4.5% per annum and Aditi borrowed the amount at 5% per annum.
Hence, **option d.**

8. For Compound Interest, the amount can be calculated using the following formula,

$$\text{Amount} = P \times \left(1+\frac{r}{100}\right)^n$$

$$\therefore\ 108000 = 62500 \times \left(1+\frac{12}{100}\right)^n$$

$$\therefore \frac{108000}{62500} = (1.12)^n$$

$$\therefore \frac{216}{125} = (1.12)^n$$

$$\therefore \left(\frac{6}{5}\right)^3 = (1.12)^n$$

i. e. $(1.12)^3 = (1.12)^n$
$n = 3$.
Hence, **option d.**

9. The formula of compound interest is given by,

$$\text{Amount} = P \times \left(1+\frac{r}{100}\right)^n$$

According to the given condition,

$$10000 = 5000 \times \left(1+\frac{r}{100}\right)^6$$

$$\therefore \left(1+\frac{r}{100}\right)^6 = 2$$

Now, amount after 18 years,

$$\therefore \text{Amount} = 5000 \times \left(1+\frac{r}{100}\right)^{18}$$

$$\therefore \text{Amount} = 5000 \times \left[\left(1+\frac{r}{100}\right)^6\right]^3$$

$$= 5000 \times (2)^3 = \text{Rs.}40000$$

Hence, **option e.**

10. The difference between the compound interest and the simple interest for 2 years is given by the formula:
Difference $= P \times (R/100)^2$
$\therefore 850 = P \times (0.10)^2$
$\therefore P =$ Rs. 85,000
Hence, **option c.**

Note: The difference between the compound interest and the simple interest for 2 years is actually the interest on the first year's interest. The first year's interest is given by $(R/100) \times P$
Hence, the interest on this will be $R/100 \times (R/100 \times P) = P \times (R/100)^2$

TEST 2

11. Let country A's population become more than country B's population after n years,
(A's population after n years)/ (B's population after n years) $= (1 \times 1.1^n)/(1.5 \times 0.9^n) > 1$
i.e. $2 \times (11)^n > 3 \times 9^n$
Substituting values for n in the above expression,
If $n = 2$, L.H.S. = 242 and R.H.S. = 243
If $n = 3$, L.H.S. = 2662 and R.H.S. = 2187
$\therefore n > 2$; hence, country A's population will become more than country B's population in the middle of the year 2010 (but not on 1st Jan, 2010)
$\therefore$ Country A's population will become more than that of country B's on the 1st Jan of 2011.
Hence, **option d.**

12. Population of the richer countries after 5 years is:

$$1.2 \times \left(1 + \frac{25}{100}\right)^5 = 3.662 \text{ billion}$$

Population of the poorer countries after 5 years is:

$$5.4 \times \left(1 + \frac{15}{100}\right)^5 = 10.861 \text{ billion}$$

So, total world population after 5 years
= 3.662 + 10.861 = 14.523 billion
Hence, **option e.**

13. Since there is a 60% increase in the amount put at simple interest, the simple interest is 60% of the principal.
Let P = Rs. 100. Then, S.I. = Rs. 60 and N = 6 years.

$$\therefore 60 = 100 \times 6 \times \frac{R}{100}$$

$\therefore R = 10\%$
Now, P = Rs. 12,000. N = 3 years and R = 10% p.a.

∴ Compound Interest

$$= 12000\left[\left(1 + \frac{10}{100}\right)^3 - 1\right]$$

$$= 1200 \times \frac{331}{100} = 3972$$

So, the compound interest is Rs. 3,972
Hence, **option c.**

14. Let the sum of money be Rs. P

$$\therefore 50 = \frac{P \times 2 \times 5}{100}$$

∴ P = Rs. 500
Now, using the formula for compound interest,

$$A = 500\left(1 + \frac{5}{100}\right)^2 = 551.25$$

So, the compound interest = $A - P$ = Rs. 51.25
Hence, **option a.**

15. The value of the car depreciates at 20% every year.
So, its value after 1 year = 300000 × (1 – 0.2)
= 0.8 × 300000
Its value after 2 years
= 0.8 × 300000 × (1 – 0.2)
= 0.8^2 × 300000
Similarly its value after 4 years
= 0.8^4 × 300000
= Rs. 1,22,880
Hence, **option e.**

16. The count of bacteria increases by 10% every hour.
If the count is 25000 now, the count after one hour = 1.1 × 25000.
Similarly, the count after two hours
= 1.1^2 × 25000.
And, the count of bacteria after 3 hours
= 1.1^3 × 25000 = 33275.
Hence, **option e.**

17. While calculating interest, we need to consider the period from 6th March to 4th May, with both days also being included.

$$\text{Time} = (26 + 30 + 4) = 60 \text{ days} = \frac{60}{366} \text{ year}$$

$$= \frac{10}{61} \text{ year}$$

P = Rs. 42,700 and R = 8% p.a.

$$\therefore \text{S.I} = \text{Rs.}\, 42{,}700 \times 8 \times \frac{10}{61} \times \frac{1}{100} = 7 \times 80$$

= Rs. 560
Hence, **option c.**

18. For 1st health policy,

$$\text{C.I.} = 5000 \times \left(1 + \frac{4}{100}\right)^2 - 5000$$

$$\therefore \text{C.I.} = 5000 \times \frac{104}{100} \times \frac{104}{100} - 5000$$

∴ C.I. = 5408 – 5000
∴ C.I. = Rs. 408
For 2nd health policy,

$$\text{C.I.} = 8000 \times \left(1 + \frac{6}{100}\right)^2 - 8000$$

$$\therefore 88000 \times \frac{106}{100} \times \frac{106}{100} - 8000$$

∴ C.I. = 8988.8 – 8000
∴ C.I. = Rs. 988.8
∴ Total C.I. = 408 + 988.8 = Rs. 1396.8
Hence, **option a.**

19. P = Rs. 40000, R = 10% p.a.

$$\text{Time} = 2 \text{ years } 6 \text{ months} = 2 + \frac{6}{12}$$

$$= 2\frac{1}{2} \text{ years.}$$

When the compounding is annual and the period is not an integral value, the fractional part of the period (1/2 in this case) can be directly multiplied with the rate. The amount obtained in this case is an approximate value (and not the actual value). However, this can be used only if the answer options are considerably spread out.

$$\text{Amount} = \begin{bmatrix} 40000 \times \left(1 + \frac{10}{100}\right)^2 \\ \times \left(1 + \frac{\frac{1}{2} \times +0}{100}\right) \end{bmatrix}$$

$$\therefore \text{ Amount} = 40000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{21}{20} \times \frac{21}{20}$$

= 121 × 441 = Rs.53,361

∴ C.I. = 53361 − 40000 = Rs.13,361

Hence, **option c.**

Note: The property explained above comes from the binomial theorem. For an expression of the form $(a + b)^n$, when the power is a fraction, the expansion is approximated as $a + nb$

20. Let the sum of money be Rs. 100.
Since it becomes (6/5) of itself, amount = (6/5) × 10 = Rs. 120
∴ Simple Interest = 120 − 100 = Rs. 20
Let the rate of interest be r% p.a.
∴ 20 = (100 × 4 × r)/100
∴ r = 20/4 = 5%
Hence, **option a.**

TEST 3

21. Let the original amount be Rs. x.

∴ $S.I.$ = 2x

$$\therefore 2x = \frac{x \times 15 \times R}{100}$$

$$\therefore R = \frac{40}{3}\% = 13\frac{1}{3}\%$$

Hence, **option e.**

22. Let the rate of interest be R p.c.p.a

$S.I.$ = 4126 − 3468 = Rs. 658

$$\therefore 658 = \frac{3468 \times 2 \times R}{100}$$

∴ R = 9.48% ≅ 9.5%

Hence, **option a.**

23. $$S.I. = \frac{P \times R \times N}{100}$$

$$\therefore 1264 = \frac{P \times 5 \times 2}{100}$$

∴ P = Rs. 12,640

Hence, **option b.**

24. $$\text{Amount} = P \times \left(1 + \frac{R}{100}\right)^n$$

$$= 25000 \times \left(1 + \frac{4}{100}\right)^2$$

$$= 25000 \times \frac{26}{25} \times \frac{26}{25}$$

∴ Amount = Rs. 27,040

Hence, **option b.**

25. A = Rs. 1,00,000, P = Rs. 50,000 and n = 10

$$\therefore 1000 = 500000\left(1 + \frac{R}{100}\right)^{10}$$

$$\therefore \left(1 + \frac{R}{100}\right)^{10} = 2$$

Amount after 20 years will be,

$$50000 \times \left(1 + \frac{R}{100}\right)^{20}$$

$$= 50000 \times \left[\left(1 + \frac{R}{100}\right)^{10}\right]^2$$

= 50000 × 2^2

∴ Value after 20 years = Rs. 2,00,000

Hence, **option d.**

26. $$S.I. = \frac{P \times R \times N}{100}$$

$$S.I. = \frac{5454 \times 4 \times 5}{100}$$

∴ S.I. = Rs. 1090.8 ≈ Rs. 1,091

Hence, **option e.**

27. Current value

$$= \text{Original value} \times \left(1 - \frac{R}{100}\right)^n$$

where R is the rate of depreciation and n is the duration between current price and original price.

$$\therefore 72000 = \text{Original value} \times \left(1 - \frac{2}{100}\right)^2$$

$$\therefore 72000 = \text{Original value} \times \frac{49}{50} \times \frac{49}{50}$$

∴ Original value = Rs. 74,968.76 ≅ Rs. 74,969

Hence, **option b.**

28. P = 5000, R = 20% and n = 2 years

Let P' be the population after 2 years.

$$P' = P \times \left(1 + \frac{R}{100}\right)^n$$

$$\therefore P' = 5000 \times \left(1 + \frac{20}{100}\right)^2$$

∴ P' = 5000 × 1.2^2 = 7200

Hence, **option a.**

29. $$\text{Present worth} = \frac{253}{\left(1 + \frac{6}{100}\right)^2}$$

$$\therefore \text{ Present worth} = 253 \times \frac{100}{106} \times \frac{100}{106}$$

= Rs. 225.169

≅ Rs. 225.17

Hence, **option d.**

30. Let Rs. P be the amount.

$C.I. = A - P$

$$\therefore\ C.I. = \left[P \times \left(1 + \frac{20}{100}\right)^2\right] - P = \frac{11}{25}P$$

$$S.I. = \frac{P \times R \times N}{100} = \frac{P \times 20 \times 2}{100} = \frac{2}{5}P$$

Now, for two years, $C.I. - S.I. = 100$

$$\therefore \frac{11}{25}P - \frac{2}{5}P = 100$$

$$\therefore \frac{11P - 10P}{25} = 100$$

$\therefore P =$ Rs. 2,500

Hence, **option d.**

PROFIT, LOSS AND DISCOUNT

TEST 1

1. In such a case where no actual values are given, the easiest approach is to assume the CP as Rs. 100

There is a mark-up of 20%

∴ MP = 1.2 × 100 = Rs. 120

Now, there is a discount of 20%.

∴ SP = 0.8 × 120 = Rs. 96

∴ Loss = 96 – 100 = Rs. 4

Thus, there is a loss of Rs. 4 on a cost price of Rs. 100.

$$\therefore \text{Percentage loss} = \frac{4}{100} \times 100 = 4\%$$

Hence, **option c.**

2. Let the CP of the article be x.

∵ The shopkeeper sold the article at a loss of 8%

∴ The SP of the article = $0.92x$

But if the shopkeeper had sold the article for Rs. 540 more, he would have made a profit of 10%.

In that case, SP = $1.1x$

∴ $1.1x = 0.92x + 540$

∴ $0.18x = 540$

∴ $x = 3000$

∴ The CP of the article is Rs. 3,000.

Hence, **option d.**

3. Let the original marked price be Rs. 100.

∴ The reduced price should have been Rs. 85.

However, due to the error, the marked price became Rs. 115.

In this case, the customer would have had to pay = 115 – 85 = Rs. 30 extra.

Hence, if the customer pays Rs. 30 more on an item with marked price Rs. 100,

then he pays Rs. 540 more on item with marked price= (540 × 100)/30 = Rs. 1,800

∴ The customer actually paid 15% extra i.e. 1.15 × 1800 = Rs. 2,070

Hence, **option b.**

4. The trader gives a 10% discount on the MP.

∴ SP = 0.9 × MP

Also, the trader makes a 20% profit on the CP.

∴ SP = 1.2 × CP

∴ 0.9 × MP = 1.2 × CP

∴ MP = 1.33 × CP

∴ MP = CP + 0.33CP

∴ The trader marked the item 33.33% over his cost price.

Hence, **option d.**

Alternatively,

Let the cost price be Rs. 100

Profit = 20%

∴ SP = 100 × 1.2 = Rs. 120

Since there is a discount of 10%,

SP = 0.9 × MP

∴ MP = 120/0.9 = Rs. 133.33

So, the required percentage is 33.33%

Hence, **option d.**

5. SP of Rs. 2,500 results in a 20% discount off the marked price,

∴ SP = 2500 = 80% of MP

∴ MP = 2500/0.8

∴ The SP that would result in a 40% discount off the marked price is:

New SP = 60% of MP = 0.6 × 2500/0.8 = 1875

∴ The selling price would be Rs. 1875

Hence, **option a.**

6. Since the two types of rice are mixed in the ratio 3 : 2, assume that the actual quantities of the two types of rice are 3 kg and 2 kg respectively.

Hence, total CP of 5 kg mixture of the two kinds of rice = (30 × 3) + (42 × 2) = Rs. 174

SP of 5 kg mixture = 38 × 5 = 190

$$\therefore \text{Profit\%} = \frac{(190 - 174)}{174} \times 100$$

$$= \frac{16}{174} \times 100 \approx 9.19\%$$

Hence, **option b.**

7. By selling 150 bags, the shopkeeper gains the cost of 250 bags.

Thus, the money earned by selling the remaining 100 bags is his profit.

$$\text{Percentage Gain} = \frac{\text{Remaining Goods}}{\text{Sold Goods}} \times 100$$

$$= \frac{100}{150} \times 100 = 66.67\%$$

Hence, **option e.**

Alternatively,
Selling Price of 150 bags is Rs. 7500.
∴ Selling Price of each bag = y = Rs. 50
Let the Cost Price of each bag be Rs. x
∴ Cost Price of 250 bags = Rs. $250x$
Since sale of 150 bags was enough to recover the cost price of 250 bags.
∴ 7500 = $250x$
∴ $x = 30$
∴ Cost of 150 bags = 150 × 30 = Rs. 4,500

∴ Percentage gain $= \frac{7500 - 4500}{4500} \times 100$

$= \frac{3000}{4500} \times 100 = 66.67\%$

Hence, **option e.**

8. Let the cost price be Rs. x.
When the selling price is Rs. 18,700, the owner loses 15%.
∴ $0.85x = 18700$
∴ x = Rs. 22,000
If the owner wants to gain 15 % then,
Selling price = $1.15x$ = 1.15 × 22000
= Rs. 25,300
Hence, **option c.**

9. Offer 1:
Price of the laptop = Rs. 35,000
After the discount, the price becomes
0.7 × 35000
= Rs. 24,500
Offer 2:
Price of the laptop = Rs. 35,000
After the successive discounts, the price becomes

$35000 \times \left(1 - \frac{20}{100}\right) \times \left(1 - \frac{10}{100}\right)$

i. e. Rs. 25,200

Thus, Offer 1 is better.
Hence, **option a.**

10. Let a be the CP of the first plot.
Let b be the CP of the second plot.
∵ The first plot was sold at a gain of 12%.
∴ The SP of the first plot = 1.12 a
∴ 1.12 a = 1 crore
∴ a = Rs. 89,28,571.43
Similarly,
∵ The second plot was sold at a loss of 12%.
∴ The SP of the second plot = 0.88 b
∴ 0.88 b = 1 crore
∴ b = Rs. 1,13,63,636.37
∴ Total CP = $a + b$
= Rs. 2,02,92,207.79
But total SP = Rs. 2 crore
∴ Loss = Total CP – Total SP = 2,92,207.79
∴ Loss percentage
= [2,92,207.79 / 2,02,92,207.79] × 100
= 1.44%
Hence, **option c.**

Alternatively,
Both the houses were sold at the same price.
Also, the percentage profit in the first case (+a%) is equal to the percentage loss in the second case (−a%).
In such a case, the overall transaction leads to a loss and the percentage loss is (a^2/100)%
In this problem, a = 12.
Hence, percentage loss
= $(12)^2/100 = 144/100 = 1.44\%$.
Hence, **option c.**

TEST 2

11. Since, the shopkeeper allows successive discounts of 10%, 5% and 4%,
∴ Total discount

$= \left[1 - \left(\frac{100 - 10}{100} \times \frac{100 - 5}{100} \times \frac{100 - 4}{100}\right)\right] \times 100$

= 17.92%
∴ Discount = (17.92/100) × 2750 = Rs. 492.8
∴ Selling Price = Marked Price – Discount
= Rs. 2257.2
Hence, **option d.**

12. 25% gain implies SP = 1.25CP
Now, CP is 10% less i.e. 0.9CP and SP is Rs. 2 less i.e (SP – 2).
Since the profit is still 25%
(SP – 2) = 1.25(0.9CP) where SP = 1.25CP
∴ 1.25CP – 2 = 1.125CP
∴ 0.125CP = 2
∴ 0.125Rs. 16.
Hence, **option b.**

13. Let C.P. of each article be Re. 1
∴ C.P. of x articles = Rs. x
C.P. of 20 articles = Rs. 20
C.P. of 20 articles = S.P. of x articles.
∴ S.P. of x articles = Rs. 20.
∴ Profit = Rs. $(20 - x)$
Also, profit = 25%

$\therefore \frac{20 - x}{x} \times 100 = 25$

∴ $x = 16$.
Hence, **option b.**

14. Total rice purchased = 26 + 30 = 56 kg
∴ C.P. of 56 kg rice = (26 × 20 + 30 × 36)
= 520 + 1080 = Rs. 1,600.
He sells the mixture at Rs. 30 per kg.

∴ S.P. of 56 kg rice = (56 × 30) = Rs. 1,680.
Thus, he gains Rs. 80 in the transaction.

$$\therefore \text{Gain} = \frac{80}{1600} \times 100 = 5\%$$

Hence, **option b.**

15. Since the oranges are sold in dozens, convert the original quantity bought into dozens.
Number of oranges bought in dozens
= 100/12.
So, Selling price = (100/12) × 48 = Rs. 400
∴ Profit = 400 – 350 = Rs. 50
% Profit = (50/350) × 100 = 100/7
Hence, **option a.**

16. Let the C.P. of the product be Rs. x
Profit percent when the product is sold at Rs. 1,920 is the same as the loss percent when the product is sold at Rs. 1,280.

$$\therefore \frac{1920 - x}{x} \times 100 = \frac{x - 1280}{x} \times 100$$

∴ 1920 – x = x – 1280
∴ x = 1600
For 25% profit, selling price = 1.25x
= 1.25 × 1600 = Rs. 2,000
Hence, **option a.**

17. Let the marked price of the oven be Rs. x
Since the person gets two successive discounts of 10% and 5%, the final price paid to purchase the oven
= x × 0.9 × 0.96 = 0.855x
Also, through these two discounts, the person saves Rs. 290
∴ Final amount paid to purchase the oven
= x – 290
∴ 0.855x = x – 290
∴ 0.145x = 290
∴ x = 2000
So, the marked price of the oven is Rs. 2,000.
Hence, **option e.**

18. Ramesh offers consecutive discounts of 20% and 5% respectively on Rs. 20,000.
∴ Ramesh offers the TV at 20000 × 0.8 × 0.95
= Rs. 15,200
∴ Ramesh offers discount = 20000 – 15200
= Rs. 4,800
Suresh offers consecutive discounts of 15% and 10% respectively.
∴ Suresh offers the TV at 20000 × 0.85 × 0.9
= Rs. 15,300
∴ Suresh offers discount = 20000 – 15300
= Rs. 4,700
∴ Difference in their discount = 4800 – 4700
= Rs. 100
Hence, **option c.**

19. Let the selling price = Rs. x.
Since the profit is calculated on the selling price, 50% profit corresponds to 0.5x
∴ cost price = selling price – profit = x – 0.5x
= 0.5x

$$\therefore \text{Actual profit \%} = \frac{\text{Profit}}{\text{Cost Price}} \times 100$$

$$= \frac{0.5x}{0.5x} \times 100 = 100$$

Hence, **option a.**

20. Let the marked price of the product be Rs. x.
If Raju gives two successive discounts of 10% each, the selling price of the product = x × 0.9 × 0.9 i.e. 0.81x
∴ Discount offered = x – 0.81x = 0.19x
However, it is given that the discount offered is Rs. 190
∴ 0.19x = 190
∴ x = 1000
∴ selling price = 0.81x = Rs. 810
This corresponds to a profit percent of 8%
Let the cost price be Rs. c
∴ 1.08c = 810
∴ c = 810/1.08 = 750
Hence, **option c.**

21. Let A's initial cost price be Rs. x.
Since A sells to B for Rs. 1,100 at a 10% profit
1.1x = 1100
∴ x = 1000
Now, B's cost price = Rs. 1,100
Since B sells back to A at a 10% loss, selling price of
B = 1100 × 0.9 = Rs. 990
Now, A gets an article costing Rs. 1000 back at Rs. 990.
∴ Gain to A = Rs. 10. Also, he has already gained **Rs. 100 in** the earlier transaction with B.
∴ Total gain by **A = Rs. 10 + Rs.** 100 = Rs. 110
% Gain = (110/1000) × 100 = 11%
Hence, **option c.**

22. Let the selling price be Rs. x
Since the margin on the selling price is 20%, profit = 0.2x.
And cost price = selling price – profit
= x – 0.2x = 0.8x
Cost price is given as Rs. 1,000
∴ 0.8x = 1000
∴ x = 1250
Hence, **option b.**

23. Cost price = Rs. 10
Original selling price = 10 × 1.2 = Rs. 12
New cost price = Rs. 11

New selling price = Rs. 12

$\% \text{ Profit} = \frac{12-11}{11} \times 100 = 9.09\ \%$

Hence, **option d.**

24. Let the printed price of the book be Rs. x.
So, after the first discount, it becomes Rs. $0.8x$
Now an additional 10% discount on $0.8x$ makes the price $(90/100) \times 0.8x = 0.72x$
But this amount gives 8% profit to the shopkeeper.
So, if the cost price is Rs. y, selling price = Rs. $1.08y$
And, $1.08y = 0.72x$
So, $x/y = 1.5$
Hence, the printed amount is 1.5 times the cost price i.e. 50% more than the cost price.
Hence, **option d.**

25. Since the shopkeeper uses a weight of 950 grams per kg, error = 1000 – 950 = 50 gms.

$\text{Gain}\% = \left(\frac{\text{Error}}{\text{True value} - \text{Error}} \times 100\right)\%$

$= \left(\frac{50}{1000-50} \times 100\right)\%$

$= \left(\frac{50}{950} \times 500\right)\%$

$= 5\frac{5}{19}\%$

Hence, **option d.**

TEST 3

26. The shopkeeper gives a discount of 10% of Rs. 2,400.

$\therefore \text{S.P.} = 90\% \text{ of } 2400 = \frac{90}{100} \times 2400$

= Rs. 2,160
This corresponds to a 10% profit.
Let the cost price of the article be Rs. x.
S.P. – C.P. = Profit
$\therefore 2160 - x = 0.1x$

$\therefore x = \frac{2160}{1.1} = 1963.63$

Thus, cost price of the article ≈ Rs. 1,964
Hence, **option a.**

27. S.P. = Rs. 75,00,000 and percentage profit = 20%
∴ S.P. = 120% of C.P.

$\therefore \text{S.P.} = \frac{100}{120} \times 7500000 = \text{Rs. } 62{,}50{,}000$

Now, for a selling price of Rs. 65,00,000, the profit is 6500000 – 6250000 = Rs. 2,50,000

$\therefore \% \text{ profit} = \left(\frac{250000}{6250000} \times 100\right)\% = 4\%$

Hence, **option b.**

28. S.P. = Rs. 1,150 and M.P. = Rs. 1,450
∴ Actual discount = 1450 – 1150 = Rs. 300

$\text{Discount } \% = \frac{\text{Discount}}{\text{Marked Price}} \times 100$

$= \left(\frac{300}{1450} \times 100\right)\%$

= 20.68%
≅ 21%
Hence, **option a.**

29. $\text{C.P. of 1 kg potatoes} = \frac{528}{30} = \text{Rs. } 17.6$

S.P. of 1 kg potato = Rs. 20

$\text{Gain } \% = \left[\frac{(20-17.6)}{17.6} \times 100\right]\%$

= 13.63%
≅ 14%
Hence, **option b.**

30. C.P. of laptop = C.P. of mobile phone = Rs. 40,000
∴ Total C.P. = 40000 + 40000 = Rs. 80,000
Raghav makes a profit of 10% on the laptop
∴ S.P. of laptop = 1.1C.P. = 1.1 × 40000 = Rs. 44,000
He also makes a loss of 6% on the mobile phone.
∴ S.P. of mobile phone = 0.94C.P. = 0.94 × 40000 = Rs. 37,600
∴ Total S.P. = 44000 + 37600 = Rs. 81,600
Since S.P. > C.P., Raghav makes a profit.
Profit = Total S.P. – Total C.P. = 81600 – 80000 = Rs. 1,600

$\therefore \text{Percentage profit} = \frac{1600}{80000} \times 100 = 2\%$

Therefore, Raghav makes a/* profit of 2% on the whole transaction.
Hence, **option d.**

RATIO AND PROPORTION

TEST 1

1. Let x be the number to be added.
$\therefore (19 + x) : (43 + x) = 2 : 3$
$\therefore 57 + 3x = 86 + 2x$
$\therefore x = 29$
∴ 29 must be added to each term in the ratio 19 : 43 so that it becomes equal to 2 : 3
Hence, **option b.**

2. Let the company's investment in the road construction be x.
$\therefore 4 : 5 = x : 6$ or $4/5 = x/6$
$\therefore x = (6 \times 4)/5 = 4.8$
$\therefore$ The company invested Rs. 4.8 million in road construction.
Hence, **option c.**

3. Let the incomes of A and B be $3x$ and $4x$ respectively.
Let their expenditures be $2y$ and $3y$ respectively.
Savings = Income - Expenditure
$\therefore$ A's savings/B's savings = $(3x - 2y)/(4x - 3y)$
The values of x and y are not known.
Hence, the ratio of savings cannot be determined.
Hence, **option d.**

4. Since, the amount collected by B wing is common to both the ratios, it is to be used to compare the collections of all 3 wings.
Hence, find the LCM of 5 and 2.
LCM of 5 and 2 = 10
$\therefore$ The ratio of the amounts contributed by the people of all the three wings = 16 : 10 : 15
$\therefore$ The amount contributed by each wing is $16x$, $10x$ and $15x$ respectively.
$\therefore 16x + 10x + 15x = 20500$
$\therefore x = 500$
i.e. $10x = 5000$
Hence, the amount collected by B wing is Rs. 5,000.
Hence, **option a.**

5. Let one of the parts be x.
$\therefore$ The other part is $(78 - x)$
$\because$ The ratio between the two parts is 7 : 6

$$\therefore \frac{x}{(78 - x)} = \frac{7}{6}$$

$\therefore 6x = 546 - 7x$
$\therefore 13x = 546$
$\therefore x = 42$ and $(78 - x) = 36$
Product of 42 and 36 = 1512
Hence, **option c.**

6. Originally, let the number of seats for Mathematics, Physics and Biology be $5x$, $7x$ and $8x$ respectively.
Number of increased seats are (140% of $5x$), (150% of $7x$) and (175% of $8x$)
i.e. $1.4 \times 5x$, $1.5 \times 7x$ and $1.75 \times 8x$
i.e. $7x$, $10.5x$ and $14x$ i.e. $14x : 21x : 28x$ or 2 : 3 : 4
Hence, **option a.**

7. Let the initial number of members with Mr. Shah be $6k$ and the number of members with Mr. Raheja be $5k$.
24 members went over from Mr.Shah's side to Mr. Raheja's side.
Hence, the number of members now supporting
Mr. Shah is $6k - 24$ while the number of members with Mr. Raheja is $5k + 24$.
This ratio is now 2 : 3
$\therefore (6k - 24) : (5k + 24) = 2 : 3$
$\therefore 18k - 72 = 10k + 48$
$\therefore 8k = 120$
$\therefore k = 15$
$\therefore$ The number of members with Mr. Shah initially = $6k = 90$
Hence, **option a.**

8. Let the numbers be A, B and C
$\therefore A + B + C = 98$
A : B = 2 : 3 and B : C = 5 : 8
Since B is the common term being compared, equalise B in both ratios.
Take the LCM of 3 and 5 i.e. 15
So, A needs to get multiplied by 5 and C by 3
$\therefore$ A : B : C = 10 : 15 : 24

$$\therefore B = \frac{15}{49} \times 98 = 30$$

Hence, **option c.**

9. The ratio of the number of coins is 1 : 2 : 3 for the 50 paise, 25 paise and Rs. 1.50 coins respectively.
$\therefore$ In terms of monetary value, the ratio becomes
$(1 \times 0.5) : (2 \times 0.25) : (3 \times 1.5)$ which equals 0.5 : 0.5 : 4.5, i.e. 1 : 1 : 9.
$\therefore (1/11)^{th}$ of the total value comes from 25 paise coins, i.e. $(1/11) \times 6600$ = Rs. 600 is in the form of 25 paise coins
$\therefore$ The total number of 25 paise coins is $600/0.25 = 2400$
Hence, **option c.**

10. Let the annual income of Mr. X and Mr. Y be Rs. $9x$ and $8x$ respectively.
Also, let their expenditures be Rs. $5y$ and Rs. $4y$ respectively.
Both individually save Rs. 5,000
$\because$ Income - Expenditure = Savings
For Mr. X, $9x - 5y = 5000$ ... (i)
For Mr. Y, $8x - 4y = 5000$ ... (ii)
Solving equations (i) and (ii),
$x = 1250$ and $y = 1250$
$\therefore$ Mr. Y's expenditure = $4y$ = Rs. 5,000
Hence, **option b.**

TEST 2

11. $5x - 13y = 3x - 8y$
$\therefore 2x = 5y$
$\therefore x : y = 5 : 2$
$x^2 : y^2 = 25 : 4$
$2x^2 : 3y^2 = 50 : 12$
Using Componendo and Dividendo law,
$(2x^2 + 3y^2) : (2x^2 - 3y^2) = 62 : 38$
$= 31 : 19$
Hence, **option d.**

12. Let Vinod's share be x.
$\therefore 6x = 10 \times$ (Vinay's share)
$\therefore$ Vinay's share = $3x/5$
Similarly, Vinit's share = $6x/5$
$\therefore x + (3x/5) + (6x/5) = 14x/5 = 798$
$\therefore x = (798/14) \times 5 = 57 \times 5 = 285$
Hence, **option e.**

Alternatively,
Let the share of Vinod, Vinay and Vinit be a, b and c respectively.
Hence, $6a = 10b$ and $6a = 5c$
Hence, $6a = 10b = 5c$.
Hence, $a : b : c = (10 \times 5) : (6 \times 5) : (6 \times 10)$
$= 50 : 30 : 60$
$= 5 : 3 : 6$
Hence, Vinod's share = $(5/14) \times 798 = 285$
Hence, Vinod's share was Rs. 285.
Hence, **option e.**

13. Let x be the fourth proportional.
$\therefore 3/5 = 27/x$
$\therefore x = (27 \times 5)/3 = 45$
Hence, **option a.**

14. From the given equation,

$$k = \frac{(x^2 - y^2)^2}{x - y}$$

Now, $(x^2 - y^2)$ can be written as $(x + y)(x - y)$.

So, we have $k = \dfrac{(x + y)^2 \times (x - y)^2}{x - y}$

$= (x + y)^2 \times (x - y)$
$= (x + y)(x + y)(x - y)$
$= (x + y)(x^2 - y^2)$
Hence, **option c.**

15. Let the weights of the two pieces be $4x$ and $5x$. Therefore, the weight of the original stone was $9x$.
Value of the stone is directly proportional to the square of its weight.
$\therefore$ The values of the original stone and the two pieces are proportional to $81x^2$, $16x^2$ and $25x^2$.
Hence, the required ratio is $81 : (16 + 25)$
$= 81 : 41$.
Hence, **option b.**

16. A : B : C = (20,000 x 24) : (15,000 x 24) : (20,000 x 18) = 4 : 3 : 3.

$\therefore$ B'share $= \dfrac{3}{10} \times 25000 =$ Rs. 7,500

Hence, **option a.**

17. A was a partner for all 12 months. Assume that B joined for x months.

$$\therefore \frac{85000 \times 12}{42500 \times x} = \frac{3}{1}$$

$$\therefore x = \frac{82500 \times 12}{42500 \times 3} = 8$$

Thus, B joined for 8 months.
Hence, **option d.**

18. Let the share of A, B, C and D be Rs. $5x$, Rs. $2x$, Rs. $4x$ and Rs. $3x$ respectively.
C gets Rs. 1,000 more than D.
$\therefore 1x - 3x = 1000$
$\therefore x = 1000$.
$\therefore$Rs. $2x$ = Rs. 2,000.
Hence, **option c.**

19. Let the three numbers be a, b, c
$a : b = 2 : 3$ and $b : c = 5 : 8$
Since b is the common term being compared in both ratios, we equalize b in both ratios.
$\therefore$ Take the LCM of 3 and 5 i.e. 15.
So, multiply a by 5 and c by 3 to get a consolidated ratio.
$\therefore a : b : c = 10 : 15 : 24$
Let $a = 10x$, $b = 15x$ and $c = 24x$
$a + b + c = 98$
$\therefore 10x + 15x + 24x = 98$
$\therefore 49x = 98$
$\therefore x = 2$
So, the second number is $15x = 15 \times 2 = 30$.
Hence, **option b.**

20. It is given that:

$$\frac{x}{y} = \frac{3}{5}$$

$$\therefore \frac{3x}{y} = \frac{9}{5}$$

Let us apply componendo

$$\therefore \frac{3x + y}{y} = \frac{9 + 5}{5} = \frac{14}{5} \quad \text{... (i)}$$

$$\text{Now, } \frac{5x}{y} = \frac{3}{1}$$

$$\therefore \frac{5x - y}{y} = \frac{3 - 1}{1} = 2 \quad \text{... (ii)}$$

Dividing equation (i) by equation (ii) we get:

$$\frac{3x+y}{5x-y}=\frac{14}{5\times 2}=\frac{7}{5}$$

Hence, **option b.**

Alternatively,
Let $x = 3k$ and $y = 5k$
$\therefore 3x + y = 3(3k) + 5k = 14k$
$5x - y = 5(3k) - 5k = 10k$
$\therefore (3x + y) : (5x - y) = 14k : 10k = 7 : 5$
Hence, **option b.**

TEST 3

21. The profit of A and B are in the same ratio as their investment i.e. 3 : 2
Thus, A's share in the profit is 3 : 5.
A's share of the profit = Rs. 855
$\therefore$ (3/5) × x = 855, where x is the total profit after donating to charity.
$\therefore x$ = 855 × (5/3) = Rs. 1,425.
This is 95% of the actual profit.
$\therefore$ Actual profit = 1425 × (100/95) = Rs.1,500
Hence, **option b.**

22. For managing the business, A received 5% of the total profit i.e 5% of Rs. 7400 = Rs. 370.
$\therefore$ Balance profit = 7400 – 370 = Rs. 7,030.
This is divided among A, B and C in the ratio of their investments.
Since they invested money for different periods, the time periods need to be multiplied with the investment values to get their total investments.
$\therefore$ Ratio of their investments
= (6500 × 6) : (8400 × 5) : (10000 × 3)
= 39000 : 42000 : 30000
= 13 : 14 : 10
$\therefore$ B's share in the profit = 7030 × (14/37)
= Rs. 2,660
Hence, **option b.**

23. $a : b = 3 : 4$ and $b : c = 5 : 8$
Since b is the term common to both ratios, equate it by taking the LCM of 4 and 5 i.e. 20
So, multiply a by 5 and c by 4
So, $a : b = 15 : 20$ and $b : c = 20 : 32$
$\therefore a : b : c = 15 : 20 : 32$
Hence, **option a.**

24. We know that:

$$\frac{a}{d}=\frac{a}{b}\times\frac{b}{c}\times\frac{c}{d}=\frac{3}{8}\times\frac{5}{3}\times\frac{4}{5}=\frac{1}{2}$$

$$\therefore\ \frac{d}{a}=\frac{1}{\frac{a}{d}}=2.$$

Hence, **option c.**

25. Let the contribution of C = x.
Then, contribution of B = x + 5000 and contribution of A = x + 5000 + 4000
= x + 9000.
So, $x + x + 5000 + x + 9000 = 50000$
$\therefore 3x = 36000$
$\therefore x = 12000$
Hence, A : B : C = 21000 : 17000 : 12000
= 21 : 17 : 12.
A's share will be in the same ratio as the investment.
$\therefore$ A's share = Rs. [35000 × (21/50)]
= Rs. 14,700
Hence, **option d.**

26. Let the fourth proportional be x.

$$\therefore \frac{5}{8}=\frac{15}{x}$$

$$\therefore x = 15\times\frac{8}{5}=24.$$

Hence, **option e.**

27. Let the original numbers be $3x$ and $5x$.
As per the given data:

$$\frac{3x-9}{5x-9}=\frac{12}{23}$$

$\therefore 23(3x - 9) = 12(5x - 9)$
$\therefore 9x = 99$
$\therefore x = 11$
So, the smaller number is $3x = 3 \times 11 = 33$.
Hence, **option b.**

28. Let us assume:

$$\frac{a}{b}=\frac{11}{76}\text{ and }\frac{c}{d}=\frac{9}{62}$$

We know that $\frac{a}{b}>\frac{c}{d}$ if $ad > bc$ and vice versa

So, $ad = 11 \times 62 = 682$
And $bc = 76 \times 9 = 684$
As $bc > ad$
We get $(c/d) > (a/b)$
Hence, Shyam has selected the larger fraction.
Hence, **option b.**

29. Ramesh ate 2 out of 6 pieces from the first pizza and 5 out of the 9 from the second one.

Ramesh's total share $=\frac{2}{6}+\frac{5}{9}=\frac{16}{18}=\frac{8}{9}$

Similarly Suresh's total share $=\frac{3}{6}+\frac{3}{9}=\frac{15}{18}$
$=\frac{5}{6}$

$\therefore$ Ratio of what Ramesh and Suresh ate $=\frac{\frac{8}{9}}{\frac{5}{6}}$

$= \frac{16}{15}$

Hence, **option a.**

30. Let us assume that:

$\frac{a}{b} = \frac{4}{5}$

Now we know that if $a/b < 1$, then

$\frac{a+1}{b+1} > \frac{a}{b}$

$\therefore \frac{5}{6} > \frac{4}{5}$

Extending the same logic we get:

$\frac{[(a+1)+1]}{[(b+1)+1]} > \frac{a+1}{b+1}$

$\therefore \frac{a+2}{b+2} > \frac{a+1}{b+1}$

$\therefore \frac{6}{7} > \frac{5}{6} > \frac{4}{5}$

Hence, **option d.**

Alternatively,
For these values, the value of the fraction could have been directly calculated.
$4/5 = 0.8$
$5/6 = 0.833$
$6/7 = 0.857$
$0.857 > 0.833 > 0.8$
$\therefore (6/7) > (5/6) > (4/5)$
Hence, **option d.**

Note that this approach may be time consuming for larger fractions.

31. The ratio of the age of A and B is 11 : 8 and the sum of their ages is 38
$\therefore 11x + 8x = 38$
$\therefore x = 2$
$\therefore$ A = 22 and B = 16
Thus, A is 22 years old and B is 16 years old.
So, after 8 years, A will be 30 years old and B will be 24 years old.
$\therefore$ Ratio of ages of A and B = 30 : 24 = 5 : 4.
Hence, **option c.**

32. Let Suresh's present age be x. So, his age, 8 years ago, was $(x - 8)$ years.
So, Ramesh's age 8 years ago was $2(x - 8)$.
$\therefore$ Ramesh's current age $= 2(x - 8) + 8 = 2x - 8$
The current ratio of their ages is 3 : 2.

$\therefore \frac{2x - 8}{x} = \frac{3}{2}$

$\therefore 4x - 16 = 3x$
$\therefore x = 16$
So, Suresh's present age is 16 years.
Hence, **option c.**

33. $A = \frac{1}{3}, B = \frac{1}{4}$ and $C = \frac{1}{5}$

Here, if the denominators are equalised, the required ratio is equal to the ratio of the numerators. To equalise the denominators, take the LCM of 3, 4 and 5 i.e. 60

$\therefore A : B : C = \frac{1}{3} : \frac{1}{4} : \frac{1}{5} = \frac{20}{60} : \frac{15}{60} : \frac{12}{60}$

$= 20 : 15 : 12$

Hence, **option b.**

Alternatively,
When A : B : C is of the form $(1/m) : (1/n) : (1/p)$, the value of A : B : C can be directly found as:
A : B : C = $(n \times p) : (p \times m) : (m \times n)$
Here, $m = 3, n = 4$ and $p = 5$
$\therefore$ A : B : C = (4 (4 A : B : C can be directly found as:
Hence, **option b.**

34. In a partnership, the profits are distributed in the ratio of investments.
Since Sonal is the working partner, 25% of the profit first goes to her before the remaining profit can be shared between the two partners.
Sonal's share as working partner = 25% of 6000 = Rs.1,500
Now, Sonal's total investment = 12000×5 = Rs. 60,000 and Jignesh's total investment = 10000×3 = Rs. 30,000
$\therefore$ 30,000of their investments = 60000 : 30000 = 2 : 1
Amount of profit to be distributed in this ratio = 6000 − 1500 = Rs. 4,500

$\therefore$ Jignesh's share $= 45000\frac{1}{3} = 1500$

Thus, Jignesh gets Rs. 1,500.
Hence, **option d.**

35. Let the total amount with Amarjeet be Rs. x.

Wife's share $= \frac{x}{2}$

$\therefore$ Remaining part $= x - \frac{x}{2} = \frac{x}{2}$

Share of Karan $= \frac{1}{5}$ of $\frac{x}{2} = \frac{x}{10}$

$\therefore$ Remaining part $= \frac{x}{2} - \frac{x}{10} = \frac{2x}{5}$

Manjeet and Manpreet's share $= \frac{1}{2} \times \frac{2x}{5} = \frac{x}{5}$

Manjeet receives Rs. 2,50,000/-.

$\therefore \frac{x}{5} = 250000$

$\therefore 2x$ = Rs. 12,50,000

Thus, Karan's share = $x/10$ = Rs. 1,25,000

Hence, **option d.**

36. Let the number of gold coins be x. So, the number of non-gold coins is $3x$.

Now, as per the given conditions:

$$\frac{x+10}{3x} = \frac{1}{2}$$

$\therefore 2x + 20 = 3x$

$\therefore x = 20$

So, the total number of coins

$= x + 10 + 3x = 20 + 10 + 60 = 90$ coins.

Hence, **option a.**

MIXTURES AND ALLIGATIONS

TEST 1

1. In this case, the rate per kg of each type of pulse is the attribute while the quantity of pulses used is the weight assigned. The cost of the resultant mixture is nothing but the weighted average of the two costs.

Hence,

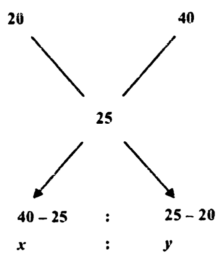
Cost of resultant mixture

$$= \frac{(10 \times 1) + (20 \times 4)}{(1 + 4)}$$

$$= \frac{90}{5} = \text{Rs. 18 per kg}$$

Hence, **option e.**

2. Since the concentration of water in each solution as well as in the final mixture is known, use the alligation cross to find the proportion.

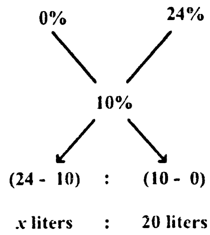


$$\frac{x}{y} = \frac{(40-25)}{(25-20)} = \frac{15}{5} = \frac{3}{1}$$

$\therefore$ The 20% water solution and the 40% water solution should be mixed in the ratio 3 : 1 to get a solution which has 25% water.

Hence, **option a.**

3. Here, one solution has 24% wine and the resultant solution has 10% wine. The second solution being mixed is water (which has 0% wine). Thus, the alligation cross can be made as shown below:



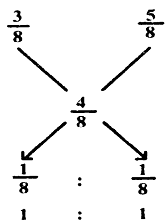
$$\therefore \frac{x}{20} = \frac{14}{10}$$

$$\therefore x = \frac{14 \times 20}{10} = 28$$

$\therefore$ 28 litres of water should be added.

Hence, **option d.**

4.



The first alloy has zinc and tin in the ratio 3 : 5.

$\therefore$ The proportion of zinc in the first alloy is 3/8.

Similarly, the ratio of zinc and tin in the second alloy is 5 : 3.

$\therefore$ The proportion of zinc in the second alloy is 5/8.

Since the ratio of zinc and tin in the mixture is 1 : 1, the proportion of zinc in the mixture of these two alloys is 1/2 (or 4/8).

$\therefore$ The ratio in which the two alloys should be mixed, to get a resultant mixture of zinc and tin in the ratio 1 : 1 can be found using the above diagram.

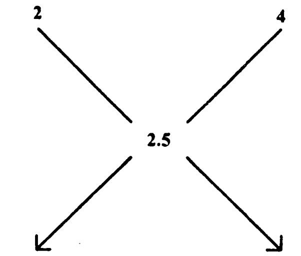
$\therefore$ The required ratio is 1 : 1.

Hence, **option a.**

5. S.P of the mixture = Rs. 3.75 per kg and profit = 50%.

$\therefore$ C.P = 3.75/1.5 = Rs. 2.5 per kg

Hence, the ratio in which the two types of coffee powder should be mixed is:



4 - 2.5 : 2.5 - 2

1.5 : 0.5

3 : 1

Hence, the two types of coffee powder in the mixture should be mixed in the ratio 1.5 : 0.5 i.e. 3 : 1

Hence, **option c.**

6. Capacity of the vessel = Quantity of milk = 72 litres.

Quantity of milk replaced = Quantity of water added = y litres

After first replacement:

$$\frac{\text{Quantity of milk remaining}}{\text{Quantity of total mixture}} = \left(\frac{x-y}{x}\right)$$

After second replacement:

$$\frac{\text{Quantity of milk remaining}}{\text{Quantity of total mixture}}$$

$$= \left(\frac{x-y}{x}\right) \times \left(\frac{x-y}{x}\right)$$

$$= \left(\frac{x-y}{x}\right)^2$$

Thus, after n replacements:

$$\frac{\text{Quantity of milk remaining}}{\text{Quantity of total mixture}} = \left(\frac{x-y}{x}\right)^n$$

Here, the replacement is done twice ie. $n = 2$

Also, the final ratio of milk to water is 25 : 11.

So, the final ratio of milk to solution is 25 : (25 + 11)

$$\therefore \left(\frac{25}{25+11}\right) = \left(\frac{72-y}{72}\right)^2$$

$$\therefore 2\sqrt{\frac{25}{36}} = \left(\frac{72-y}{72}\right)$$

$$\therefore 2\frac{5}{6} = \left(\frac{72-y}{72}\right)$$

$\therefore$ 360 = 432 - 6y

$\therefore y$ = 12 litres

Hence, **option c.**

Note: There is also a negative value of the square root, which comes out to be invalid, as shown below.

$$\frac{-5}{6} = \frac{72-y}{72}$$

$\therefore$ -360 = 432 - 6y

$\therefore y$ = 132 litres

However, the capacity of the vessel is only 72 litres. Thus, 132 litres of milk cannot be removed out of it.

Hence, this value is invalid.

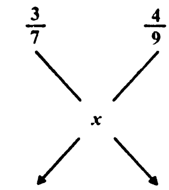
7. The ratio of the students who passed to those who failed in class 1 is 3 : 4.

Hence, 3/7th of the total students in class 1 have passed.

Similarly, 4/9th of the total students in class 2 have passed.

Also, the number of students in each class is the same.

Hence,



$\frac{4}{9} - x$: $x - \frac{3}{7}$

1 : 1

$$\frac{\frac{4}{9}-x}{x-\frac{3}{7}} = \frac{1}{1}$$

$$\therefore \frac{4}{9} - x = x - \frac{3}{7}$$

$\therefore x$ = 55/126

$\therefore$ 55/126 of the total students in both the classes put together passed.

$\therefore$ The passing percentage of all the students taken together = 55/126 passed.n pass

Hence, **option b.**

Alternatively,

$\because$ The number of students in both classes is the same, the ratio of the total number of students who have passed to the total number of students is just the arithmetic mean of the corresponding ratios in the two classes.

$\therefore$ The fraction of the total number of students who have passed

$$= \frac{1}{2}\left(\frac{3}{7}+\frac{4}{9}\right) = \frac{55}{126}$$

Hence, the percentage corresponding to this fraction is 5500/126 ≈ 44%

Hence, **option b.**

8. The shopkeeper sells the mixture at Rs. 54 per litre thereby making a 20% profit.

$\therefore$ Actual cost of the mixture = 54/1.2 = Rs. 45 per litre.

∴ The amounts in which the two mixtures are to be used is given by the alligation rule shown below:

42 50

45

5 : 3

Hence, the Rs. 42 and the Rs. 50 variants should be mixed in the ratio 5 : 3.

∴ The juice costing Rs. 50 forms 3/8th of the total mixture.

∴ 40 litres of Orange-La has (3/8) × 40 = 15 litres of the Rs. 50 variant of juice.

Hence, **option a.**

9. Ratio of copper to the other metals in the resultant alloy

$$= \frac{\frac{1}{4}\times 2 + \frac{1}{5}\times 5}{7}$$

= (1 + 1/2)/7 = 3 : 14

Hence, **option c.**

10. Let a parts of the Rs. 7.2 per kg mixture be mixed with b parts of the Rs. 5.7 per kg mixture.

∴ $a : b$ = (6.3 – 5.7) : (7.2 – 6.3) = 0.6 : 0.9

= 2 : 3

Hence, **option b.**

TEST 2

11. Let the price of the mixture be Rs. x.

So, as per the given ratio,

$(4 - x)/(x - 2.4) = 1/3$

∴ $12 - 3x = x - 2.4$

∴ $4x = 14.4$

∴ $x = 3.6$

So, the cost price of the mixture is Rs. 3.6

To make a 25% profit, it should be sold at 3.6 × 1.25 = Rs. 4.5

Hence, **option b.**

12. Total salary of department A = 30000 × 20

= Rs. 6,00,000

Total salary of department B = 20000 × 50

= Rs. 10,00,000

Total salary of department C = 25000 × 40

= Rs. 10,00,000

Total salary of department D = 15000 × 30

= Rs. 4,50,000

∴ Total salary of all the employees across the four departments

= 600000 + 1000000 + 1000000 + 450000

= Rs. 30,50,000

$$\therefore 50{,}00040 + 3000000\frac{3050000}{20 + 50 + 40 + 30}$$

$$= \frac{3050000}{140}$$

= 21785.71 ≅ Rs. 21785

Hence, the average salary across all four departments is Rs. 21,785.

Hence, **option c.**

13.

0.1% 10%

2%

8 : 1.9

Using the alligation rule shown above, the ratio of strychnine from the two mixtures is 8 : 1.9.

Final amount of the heart stimulant = 10 ml

∴ Amount of 0.1% solution in the final stimulant = (8/9.9) × 10 ≈ 8.1 ml

Hence, **option d.**

14. Let the quantity of the wine in the cask originally be x litres.

8 litres of water is replaced with 8 litres of water. This process is repeated 4 times.

∴ Quantity of wine left in cask after 4 operations

$$\therefore \frac{x\left(1 - \frac{8}{x}\right)^4}{x} = \frac{16}{81}$$

∴ $x = 24$

Thus, there was initially 24 litres of wine in the cask.

Hence, **option b.**

15. Suppose the vessel initially contains 8 litres of liquid. So, it has 3 litres of water and 5 litres of syrup.

Let x litres of this liquid be replaced with water.

So, $3x/8$ litres of water and $5x/8$ litres of syrup get reduced from the mixture while x litres of water gets added.

So, quantity of water in the new mixture

$$= 3 - \frac{3x}{8} + x$$

and, quantity of syrup in the new mixture

$$= 5 - \frac{5x}{8}$$

The mixture comprises equal parts water and equal parts syrup.

$\therefore 3 - \frac{3x}{8} + x = 5 - \frac{5x}{8}$

$\therefore x = \frac{8}{5}$

So, part of the mixture replaced $= \frac{1}{8} \times \frac{8}{5} = \frac{1}{5}$

Hence, **option c.**

16. 4 litres of milk is replaced with 4 litres of water.
Amount of milk left after 3 operations

$= 40\left(1 - \frac{4}{40}\right)^3$ litres

$= 40 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} = 29.16$ litres

Hence, **option d.**

17. Let the original concentration of milk be x%.
The concentration of milk after the replacement is 30%.
Since this replacement is done only once,

$\frac{x}{100} \times \frac{(42-6)}{42} = \frac{30}{100}$

$\therefore \frac{36x}{42} = 30$

$\therefore x = 35\%$

Hence, **option b.**

18. Suppose the can initially contains 7x litres and 5x litres of the liquids A and B respectively.

Quantity of A in mixture left $= 7x - \frac{7}{12} \times 9$

$= 7x - \frac{21}{4}$

Quantity of B in mixture left $= 5x - \frac{5}{12} \times 9$

$= 5x - \frac{15}{4}$

$\therefore 5\frac{7x - \frac{21}{4}}{5x - \frac{15}{4} + 9} = \frac{7}{9}$

$\therefore x = 3$

So, $7x$ = 21 litres of liquid A was initially present in the can.
Hence, **option c.**

19. 100 employees at Grade I in an organization have an average salary of Rs. 42 per month while 150 employees at Grade II in the same organization have an average salary of Rs. 36 per month.
So, average salary
= [(100 × 42) + (150 × 36)]/(100+ 150)
= (4200 + 5400)/250 = 9600/250 = Rs. 38.4
Hence, **option c.**

20. A scientist mixes 80% sulphuric acid with water to get 60% sulphuric acid.
Since water contains 0% sulphuric acid, the ratio in which the two solutions are mixed is:
(60 – 0) : (80 – 60) = 60 : 20 = 3 : 1
Since 9 litres of 80% sulphuric acid was used, quantity of water used was (1/3) × 9
= 3 litres.
Hence, **option b.**

TEST 3

21. The price (per litre) of the resultant mixture is the weighted average of the individual prices.

$\therefore$ Price $= \frac{(24 \times 3) + (21 \times 5)}{(3+5)}$

$= \frac{72 + 105}{8}$

$= \frac{177}{8}$

= Rs. 22.125 per litre
≈ Rs. 22.13 per litre
Hence, **option d.**

22 Original amount of milk $= \frac{4}{5} \times 3000$

$= 2400$ ml

∴ Original amount of water = 3000 – 2400
= 600 ml
Let the amount of water to be added be w ml.

$\therefore \frac{2400}{600 + w} = \frac{3}{2}$

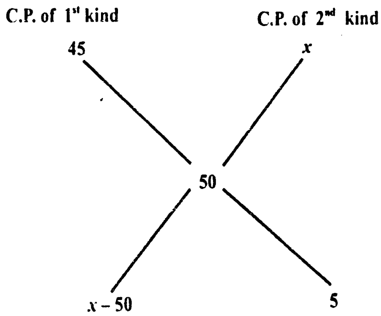
∴ 4800 = 1800 + 3w
∴ w = 1000 ml
Hence, **option a.**

23. The cost of the resultant mixture is the weighted average of the individual costs.
Let x kg of the sugar costing Rs. 44/kg be mixed with y kg of the sugar costing Rs. 52/kg.

$\therefore 49.50 = \frac{(44x) + (52y)}{(x+y)}$

∴ 49.5x + 49.5y = 44x + 52y
∴ 5.5x = 2.5y
Thus, x:y = 2.5:5.5 = 5:11
Hence, **option d.**

24. Let the price (per kg) of the second type of rice be Rs. x
The mixture can be represented as shown below:



$\therefore \frac{x-50}{5} = \frac{5}{4}$

$\therefore 4x - 200 = 25$

$\therefore x = \text{Rs. } 56.25$

Thus, the latter type of rice costs Rs. 56.25 per kg.

Hence, **option c.**

25. Total quantity of Mixture A = 3 + 1 = 4 kg

Total quantity of Mixture B = 5 + 3 = 8 kg

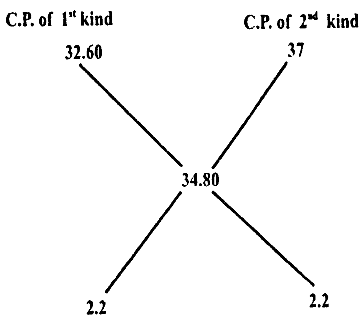
∴ Total quantity of resultant mixture = 8 + 4 = 12 kg

Total quantity of silver in resultant mixture = 1 + 3 = 4 kg

$\therefore$ % silver in mixture $= \frac{4}{12} \times 100 = 33.33\%$

Hence, **option e.**

26. By the rule of alligation,



Required ratio = (2.2) : (2.2) = 1 : 1.

Hence, **option a.**

27. Let x kgs of ordinary rice be mixed.

Selling price of Rs. 45 per kg corresponds to 25% profit.

$\therefore \text{C. P.} = \frac{45}{1.25} = \text{Rs. } 36 \text{ per kg}$

This C.P. is the weighted average of the individual prices.

$\therefore 36 = \frac{(31 \times x) + (47 \times 20)}{(x + 20)}$

$\therefore 31x + 940 = 36x + 720$

$\therefore 5x = 220$

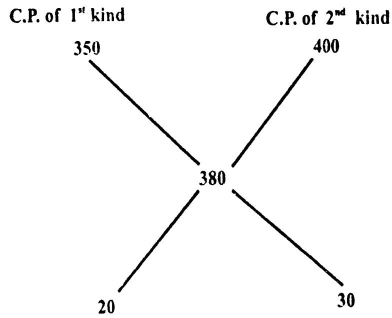
$\therefore x = 44$ kgs

Hence, **option c.**

28. Selling price of Rs. 410.4 per kg corresponds to 8% profit.

$\therefore \text{C. P.} = \frac{410.4}{1.08} = \text{Rs. } 380 \text{ per kg}$

Now, by the alligation rule,



∴ Required ratio = 20:30 = 2:3

Hence, **option e.**

29. Let the per unit cost price of pastry flour be Rs. x.

The per unit cost of cake batter is the weighted average of the per unit price of cake flour and pastry flour.

$\therefore 350.625 = \frac{(5 \times 360) + 3x}{5 + 3}$

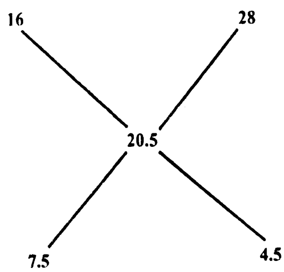
$\therefore (350.625 \times 8) = 1800 + 3x$

$\therefore 2805 = 1800 + 3x$

$\therefore x = \text{Rs. } 335$

Hence, **option b.**

30. The resultant mixture is shown as per the alligation rule as under:



∴ Required ratio = (7.5) : (4.5) = 3 : 2

Hence, **option e.**

VARIATION

TEST 1

1. It is given that:

$y \propto \frac{x}{z}$

$\therefore y = \frac{kx}{z}$

$\therefore k = \frac{yz}{x}$

And hence $\frac{y_1 z_1}{x_1} = \frac{y_2 z_2}{x_2}$

Substituting the values we get:

$\frac{5 \times 4}{2} = \frac{y \times 6}{3}$

$\therefore y = 5$

Hence, **option b.**

2. The following relations can be established:

$P = k_1 \times Q$

$Z = \frac{k_2}{Q}$

$A = k_3 \times \frac{P}{Z}$

$= k_3 \times \frac{(k_1 \times Q)}{\left(\frac{k_2}{Q}\right)} = k' \times Q^2$

$\therefore$ From the given relations, A is directly proportional to Q^2.

Substituting the values from the question,

$27 = k_1 \times 9$

$\therefore k_1 = 3$ and $3 = \frac{k_2}{9}$

$\therefore k_2 = 27$ and $90 = k' \times 81$

$\therefore k' = \frac{90}{81}$

$\therefore$ When $P = 81$, then $Q = \frac{81}{3} = 27$

$A = \frac{90}{81} \times 27^2 = 810$

Hence, **option d.**

3. If the time duration is constant, the number of men and the work done are directly proportional to each other.

$\because$ The number of men becomes 1.5 times (200 × 1.5 = 300)

$\therefore$ The work done i.e. the length of the road will also increase to 1.5 times the original length i.e. 1.5 km

Hence, **option b.**

Alternatively,

If the number of men required is n and the length of the road is l, then

$n_1/n_2 = l_1/l_2$

Hence, $(200/300) = 1/l_2$

$\therefore l_2 = 1.5$ km

Hence, **option b.**

4. Volume of a sphere is directly proportional to the cube of the radius.

$V = (4/3)\pi r^3$

$\because$ The ratio of radii is 1 : 2,

$\therefore$ The ratio of volume would be $1^3 : 2^3$,

i.e. 1 : 8.

Hence, **option c.**

5. Force is inversely proportional to the square of the distance between the charges.

$\therefore F \propto \frac{1}{d^2}$

where F and d are the force and the distance between the two charges respectively.

Hence, $F_1 \times d_1^2 = F_2 \times d_2^2$

Hence, $20 \times (200)^2 = F_2 \times (2000)^2$

$\therefore F_2 = 20 \times (1/10)^2 = 0.2$ N

Hence, **option a.**

6. The volume of a cone varies directly with the product of the square of the radius and the height i.e. $V \propto r^2h$.

$\because$ The radius and height are doubled, the new volume will be proportional to $(2r)^2 \times (2h)$

i.e. $8r^2h = 8V$, i.e. 8 times the old volume

$\therefore$ Percentage change in the volume

$= (8 - 1)/1 \times 100 = 700\%$

Hence, **option e.**

7. Let the bases of the trapezoid be b_1 and b_2, and the height be h.

The equation for the area of the trapezoid can be written as:

$S = k \times h \times (b_1 + b_2)$

where k is the constant of proportionality.

Substituting the given values,

$285 = k \times 19 \times (11 + 19)$

$\therefore k = 0.5$

$\therefore$ The area of the second trapezoid,

$S = 0.5 \times 10 \times (10 + 15)$

$\therefore S = 125\ m^2$

Hence, **option c.**

8. The equation for the number of cranes c can be written as: $c = k \times y/w$

Substituting the values from the question,

$4 = k \times \frac{8}{6}$

$\therefore k = 3$

In the second case, the equation can be written as:

$c = 3 \times y/w$

$5 = 3 \times 20/w$

$\therefore w = 12$

Hence, **option d.**

9. Let the area be a, radius be r and the perimeter be p of the circle.

Then,

$a = k/r^3$ ... (i)

$p = k/a$ ... (ii)

∵ The constant of proportionality, k, is equal for both the cases.

From equation (ii), $a = k/p$,

From equation (i), $a = k/r^3$

∴ $k/p = k/r^3$

∴ $p = r^3$

If $r = 8$, $p = 8^3 = 512$ units

Hence, **option d.**

10. Let the marks of Mahesh, Ramesh and Durgesh be m, r, and d respectively.

Hence, $m = k \times r^2 \times d^4$

Let m' denote Mahesh's new marks.

$m' = k \times (2r)^2 \times (d/2)^4$

∴ $m' = k \times 4 \times r^2 \times d^4/16 = (k \times r^2 \times d^4)/4$

∴ $m' = m/4$

∴ The marks of Mahesh will reduce by 75%.

Hence, **option b.**

TEST 2

11. Work Done = Number of Students × Hours worked per day × Number of days worked.

Here, the work to be done is "solving n number of problems". Since the number of problems to be solved in the second case is five times the number of problems to be solved in the first case, the work done in the second case is five times the work done in the first case.

Let the number of students required in the second case be x.

$$\therefore \frac{W}{5W} = \frac{8 \times 5 \times 9}{x \times 4 \times 15}$$

$$\therefore x = \frac{8 \times 5 \times 9 \times 5}{4 \times 15} = 30$$

Thus, 30 students will be required.

Hence, **option b.**

12. The book has 300 pages at the rate of 28 lines per page.

Now, if the number of pages has to become only 280. For this to happen, the number of lines per page has to increase.

So, the number of pages and the number of lines per page are inversely proportional to each other.

∴ the number of × Number of lines per page $= k$

Let the number of lines per page in the second case be p

∴ $28 \times 300 = 280p$

∴ $p = (28 \times 300)/280 = 30$

Thus, if the book is to have 280 pages, it should have 30 lines per page.

Hence, **option b.**

13. 24 workers can finish the work in 15 days. Since the work is to be done in 12 days now, the number of workers required has to increase.

Thus, the number of days required is inversely proportional to the number of workers.

∴ Number of workers × Number of days = k

Let the number of workers needed in the second case be x.

∴ $24 \times 15 = x \times 12$

∴ $x = (24 \times 15)/12 = 30$

Thus, the number of additional workers required is $30 - 24 = 6$

Hence, **option a.**

14. Current (I) is inversely proportional to resistance (R)

∴ $I_1R_1 = I_2R_2 = I_3R_3$

Here, $I_1 = 2$ amperes, $R_1 = 3$ ohms,

$I_2 = 5$ amperes and $R_3 = 5$ ohms

∴ $R_2 = (2 \times 3)/5 = 1.2$ ohms

And $I_3 = (2 \times 3)/5 = 1.2$ ohms

Hence, **option e.**

15. Let the weight of the diamond be w decigrams and the price be Rs. p.

∴ $p \; \alpha \; w^2$

∴ $p = kw^2$

Now when $w = 20$ dg, $p = 3600$

∴ $3600 = k \times 20^2$

∴ $k = 9$

Now, the diamond is broken in three pieces in the ratio 2 : 3 : 5.

So, the weights will be 4 dg, 6 dg and 10 dg respectively.

∴ Total price = $k(2^2 + 3^2 + 5^2) = 38k = 9 \times 38$ = Rs. 342

∴ Loss = 3600 − 342 = Rs. 3258

Hence, **option e.**

16. The greater the height of the pole, the longer is its shadow.

So, the height of the pole and the length of the shadow are directly proportional to each other.

When the height of the pole is 3 m, the length of the shadow is 3.6 m.

So, when the length of the shadow is 54 m, the height of the pole is $(3 \times 54)/3.6 = 45$ m

Hence, **option a.**

17. Let us denote electric field strength by E, charge by q and distance by r.

It is given that:

$$E \; \alpha \; \frac{q}{r^2}$$

$\therefore E = \frac{kq}{r^2}$

Putting the given values in the above equation we get:

$9 \times 10^9 = k \times \frac{1}{1^2}$

$\therefore k = 9 \times 10^9$

Now, we need to find E when q = 2C and r = 2m

$\therefore E = 9 \times 10^9 \times \frac{2}{2^2} = 4.5 \times 10^9 N/C$

Hence, **option d.**

18. Let us denote gravitational force by F, masses by m_1, m_2 and distance between them be r.
It is given that:

$F \alpha \frac{m_1 m_2}{r^2}$

$\therefore F = \frac{k m_1 m_2}{r^2}$

Now, substituting the given values we get:

$\frac{5}{3} = k \times 1 \times \frac{1}{1^2}$

$\therefore k = \frac{5}{3}$

Now, F = 5/3 N, $m_1 = m_2$ = 2kg
So, we can find r.

$\frac{5}{3} = \frac{\left(\frac{5}{3}\right) \times 2 \times 2}{r^2}$

$\therefore r = \sqrt{2 \times 2} = 2$ m

Hence, **option a.**

19. Let us denote the weight supported by w, diameter by d and height by h.
It is given that:

$w \alpha \frac{d^4}{h^2}$

$\therefore w = \frac{kd^4}{h^2}$

$\therefore k = \frac{wh^2}{d^4}$

$\therefore \frac{w_1 h_1^2}{d_1^4} = \frac{w_2 h_2^2}{d_2^4}$

Putting the values we get:

$\frac{64 \times 9^2}{2^4} = w_2 \times \frac{9^2}{3^4}$

$\therefore w_2 = 324$ metric ton

Hence, **option e.**

20. The water bill can be given as: $w = x + ky$ where x corresponds to the fixed charge (in Rs.) and y is the cost per litre (in Rs.).

So, the water bill for February and March is as given below:

$x + 4000y = 8500$ ... (i)

$x + 6000y = 11000$ ... (ii)

Subtracting (i) from (ii), we get

$y = 1.25$

Putting y = 1.25 in (i), we get

$x = 3500$

So, the cost per litre is Rs. 1.25 and the fixed charges are Rs. 3,500.

Thus, when the water consumption is 6800 litres per month, the water bill is:

3500 + (6800 × 1.25) = 3500 + 8500
= Rs. 12,000

Hence, **option a.**

TEST 3

21. Let w gms be the weight and l cm be the length of the iron bar respectively.

$\therefore w \propto l$

$\therefore w = kl$ ...k is the constant of proportionality.

$\therefore 287.5 = k \times 25$

$\therefore k = 11.5$

For l = 36 cm

$w = 11.5 \times 36 = 414$ gms

Hence, **option b.**

22. Let the weight on moon and earth be w_1 and w_2 kgs respectively.
w_1 and w_2 are directly proportional.

$w_1 \propto w_2$

$\therefore w_1 = kw_2$

$\therefore 6 = k \times 80$

$\therefore k = 3/40$

Now, w_2 = 98 kgs

$\therefore w_1 = 3/40 \times 98$

$\therefore w_1 = 7.35$ kg

Thus, Akash's new weight on the moon is 7.35 kgs.

Hence, **option a.**

23. Let f_1, f_2 and l_1, l_2 be the frequencies and wavelengths of two sound waves respectively such that $l_1:l_2 = 3:2$.
The frequency and wavelength of the two waves are given to be inversely proportional to each other.

$\therefore hf \propto \frac{1}{l}$

$\therefore \frac{f_1}{f_2} = \frac{l_2}{l_1}$

$\therefore \frac{f_1}{f_2} = \frac{2}{3}$

Thus, ratio of the frequencies is 2:3

Hence, **option e.**

24. $q \propto \frac{1}{p}$

$\therefore pq = k$

$\therefore k = 4 \times 54 = 216$

Now, q = 8

$\therefore p = 216/8 = 27$

Hence, **option c.**

25. Current and resistance are inversely proportional to each other for constant voltage.
Let *I* amps and *R* ohms be the current and resistance of a circuit respectively.

$\therefore I \propto \frac{1}{R}$

$\therefore IR = k$

$\therefore k = 65 \times 44 = 2860$

Now, $R = 52$ ohms

$\therefore I = 2860/52 = 55$ amps

Hence, **option a.**

26. $x \propto \frac{1}{y^3}$

$\therefore xy^3 = k$

$\therefore k = 2 \times 5^3 = 2 \times 125 = 250$

For $y = 2$

$$x = \frac{250}{2^3} = \frac{250}{8} = 31.25$$

Hence, **option d.**

27. Let *R* ohms and *L* m be the electrical resistance and length of a wire respectively.

$R \propto L$

$\therefore R = kL$

When $R = 12.5$ ohms and $L = 5$ m

$k = 12.5/5 = 2.5$

Now, when $R = 35$ ohms

$\therefore L = R/k = 35/2.5 = 14$ m

Hence, **option d.**

28. Distance = Speed × Time
Since the distance to be covered is constant, the speed and time are inversely proportional to each other.

$\therefore s \propto \frac{1}{t}$

$\therefore k = st$

A time of 4 hours and 12 minutes corresponds to 4.2 hours.

$\therefore k = 60 \times 4.2 = 252$

When $s = 80$ kmph,

$t = k/s = 252/80 = 3.15$ hours

3.15 hours corresponds to 3 hours and 9 minutes.

$\therefore$ Time taken by Rohan to cover the same distance at 80 kmph is 3 hours and 9 minutes.

Hence, **option d.**

29. Work done is directly proportional to the number of men and number of days required to complete the work i.e. construct the wall.
Let *w*, *d* and *m* be the work done, number of days and number of men respectively

$\therefore w \propto d \times m$

$\therefore w = k \times d \times m$

$\therefore 3 = k \times 2 \times 5$

$\therefore k = 0.3$

When $d = 3$ and $m = 8$

$w = 0.3 \times 3 \times 8 = 7.2$ m

So, 8 men can construct a wall 7.2 m long in 3 days.

Hence, **option e.**

30. Both the points satisfy the equation $xy = k$.
Let $(x_1, y_1) = (4, 15)$ and $(x_2, y_2) = (3, y)$

$\therefore x_1 \times y_1 = x_2 \times y_2$

$\therefore 4 \times 15 = 3 \times y$

$\therefore y = 20$

Hence, **option e.**

TIME AND WORK

TEST 1

1. A alone completes the work in 4 days.

$\therefore$ Work completed by A in 1 day = 1/4

B alone completes the work in 5 days.

$\therefore$ Work completed by B in 1 day = 1/5

$\therefore$ Total work done in one day = 1/4 + 1/5 = 9/20

$\therefore$ Number of days to complete the work = 20/9

Hence, **option b.**

2. Let B alone take *b* hours to complete the work.

$\therefore$ Work completed by B in one hours = $1/b$

A takes 25 hours to complete the work alone.

$\therefore$ Work completed by A in one hour = 1/25

Work done by A and B together in 1 hour

$$= \frac{1}{25} + \frac{1}{b}$$

A and B can complete the job together in 10 hours.

$$\therefore \frac{1}{25} + \frac{1}{b} = \frac{1}{10}$$

$$\therefore \frac{1}{b} = \frac{1}{10} - \frac{1}{25}$$

$$\therefore \frac{1}{b} = \frac{3}{50}$$

$\therefore b = \frac{50}{3} = 16.66$ hours

Hence, **option c**.

Alternatively,

Using the concept of assumed total work.

Let the total work to be done be the L.C.M of 25 and 10 i.e. 50 units.

Hence, A can do 50/25 = 2 units of work per hour.

Similarly, A and B together can do 50/10 = 5 units of work per hour

Hence, B alone can do 5 – 2 = 3 units of work per hour.

Hence, B can complete 50 units of work in 50/3 = 16.66 hours.

Hence, **option c**.

3. Rate at which the tank empties = (Capacity of the tank)/(Time taken to empty the tank)

$= \frac{1300}{7.2} = 180.55$ litres/min

Hence, **option c**.

4. The man is half as efficient as the woman.

The woman completes 2/3rd of a task in 1 day.

∴ The man completes half of it i.e.1/3rd of a task in 1 day.

∴ The man completes the task in 3 days.

Hence, **option d**.

5. Let the total work be 1 unit

In half a day, $\left(\frac{1}{2}\right)\left(\frac{7}{8}\right)$ of work is done

Work left $= 1 - \frac{7}{16} = \frac{9}{16}$

Hence, **option a.**

6. In such a case, Total work = Number of workers

× Working hours per day × Number of days

Also, since a room of specific dimensions is being built, the work can be equated in terms of the volume of the room.

Hence, Total work

= Length × Breadth × Height

Hence,

Let the number of men required in the second case be x.

$\therefore \frac{10 \times 7 \times 16}{x \times 7 \times 8} = \frac{60 \times 5 \times 14}{50 \times 6 \times 28}$

$\therefore x = 40$

Hence, **option c.**

7. Ajay and Vijay can individually complete the project in 8 and 16 days respectively.

Hence, in one day Ajay and Vijay will individually complete (1/8)th and (1/16)th part of the project respectively.

Since, they work alternately, the portion of the project completed by them in 2 days will be equal to

$\left(\frac{1}{8} + \frac{1}{16}\right) = \frac{3}{16}$

∴ Over 5 such sessions (i.e.10 days), they can complete

$5 \times \frac{3}{16} = \frac{15}{16}$ parts of the project.

Now, 1 - 15/16 = 1/16th of the project is remaining.

On the 11th day, it is Ajay's turn. Ajay's work rate is 1/8 and the work to be done is 1/16

$\therefore$ Time required by Ajay $= \frac{\text{Work to be done}}{\text{Work rate}}$

$= \left(\frac{\frac{1}{16}}{\frac{1}{8}}\right) = 0.5$

Hence, an additional half a day is required.

Hence, both of them together require 10.5 days, if they work alternately

(starting with Ajay).

Hence, **option d**.

8. Pipe A alone fills the tank in 4 hours.

∴ Portion of the tank filled by A alone in 1 hour = 1/4

Pipe B alone fills the tank in 12 hours.

∴ Portion of the tank filled by B alone in 1 hour = 1/12

Pipe C alone empties the tank in 3 hours.

∴ Portion of the tank emptied by C alone in 1 hour = 1/3

∴ When all three pipes are opened simultaneously, then in 1 hour:

Portion of tank filled = Portion of tank filled by

A + Portion of tank filled by B – Portion of tank emptied by C

Tank filled in 1 hour $= \frac{1}{4} + \frac{1}{12} - \frac{1}{3} = 0$

Hence, in one hour, there is no net inflow or outflow in the tank.

Hence, at the end of five hours as well, the tank neither spills nor gets empty.

Thus, it remains full.

Hence, **option b**.

9. Pipe A alone empties the tank in 3 hours.

∴ Tank emptied by A in 1 hour = 1/3

Pipe B alone fills the tank in 9 hours.

∴ Tank filled by B in 1 hour = 1/9

Pipe C alone fills the tank in 12 hours.

∴ Tank filled by C in 1 hour = 1/12

When all the three pipes are opened simultaneously, then in 1 hour:

Tank emptied = Tank emptied by A − Tank filled by B − Tank filled by C

$$= \frac{1}{3} - \frac{1}{9} - \frac{1}{12} = \frac{5}{36}$$

∴ Portion of the tank emptied in 1 hour = 5/36

Thus, the tank gets empty in 36/5 or 7.2 hours.

Hence, **option a**.

10. The number of days taken by the man, woman and child (working together) to complete the work = 6

Work done in 1 day when all 3 work together = 1/6

Let the man take m days to finish the task.

Number of days taken by a woman = $3m$ and number of days taken by a child = $6m$

Work done by them in one day can be given as,

$$\frac{1}{m} + \frac{1}{3m} + \frac{1}{6m} = \frac{1}{6}$$

$$\therefore \frac{9}{6m} = \frac{1}{6}$$

∴ $6m = 54$

∴ Time taken by the child alone to complete the job is 54 days.

Hence, **option d.**

Note: We did not cancel 6 in the equation above as the time taken by a child ($6m$) was needed. Had we cancelled, we would have got m and then we would have had to multiply it again by 6.

TEST 2

11. Let the total task be equivalent to the LCM of 14, 8 and 7 i.e. 56 units.

∵ A and B together complete the task in 14 days

∴ They can together complete 56/14 = 4 units/day

∴ $A + B = 4$

Similarly,

$B + C = 7$ and

$C + A = 8$

Solving the 3 equations simultaneously,

$A = 2.5$ units/day, $B = 1.5$ units/day and $C = 5.5$ units/day

∴ C does the maximum work in a single day, and is the most efficient.

Hence, **option c.**

Alternatively,

∵ A and B together take more time than C and B together to complete the same task, the additional time required must be because of A.

∴ A is slower than C.

∴ C is more efficient than A.

Similarly, A and B together take more time than A and C together to complete the same task. Hence, in this case, the additional time taken must be because of B.

∴ B is slower than C.

∴ C is more efficient than B.

∴ C is more efficient than A as well as B.

∴ C is the most efficient.

Hence, **option c.**

12. The least efficient person in the group is B (refer to the previous question).

Let A take a days, B take b days and C take c days to complete the task.

Task completed by A in 1 day $= \frac{1}{a}$

Task completed by B in 1 day $= \frac{1}{b}$

Task completed by C in 1 day $= \frac{1}{c}$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{14} \quad \text{...(i)}$$

$$\frac{1}{b} + \frac{1}{c} = \frac{1}{8} \quad \text{...(ii)}$$

$$\frac{1}{a} + \frac{1}{c} = \frac{1}{7} \quad \text{...(iii)}$$

(iii) − (ii) gives us,

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{56} \quad \text{...(iv)}$$

(i) − (iv) gives us,

$$\frac{2}{b} = \frac{3}{56} \text{ or } \frac{1}{b} = \frac{3}{112}$$

Putting the value of $1/b$ in equation (i), $1/a = 5/112$.

Putting the value of $1/b$ in equation (ii), $1/c = 11/112$.

∴ A, B and C take 112/5, 112/3 and 112/11 days respectively to complete the task alone.

When multiple fractions have the same numerator, the largest fraction is the one that has the smallest denominator.

∴B takes the longest time to complete the task alone.

∴ B, the least efficient amongst the three takes 112/3 days to complete the task alone.

Hence, **option b.**

13. Amount of work **done** by Arjun in one day = 1/7
Amount of work done by Karan in one day = 1/11
Amount of work done by Arjun and Karan in one day = 1/7 + 1/11 = 18/77
∴ Number of days required to complete the work = 77/18 = 4.28 days
Hence, **option c.**

14. Pipe A can fill the tank in 4 minutes.
Part of tank filled by pipe A in one minute = 1/4
Pipes A and B can together fill the tank in 3 minutes.
Part of tank together filled by pipes A & B in one minute = 1/3
In one minute, pipe B can fill
$\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$th of the tank
Thus, B alone can fill the tank in 12 minutes.
Hence, **option b.**

15. In one day X can finish $1/15^{th}$ of the work.
In one day Y can finish $1/10^{th}$ of the work.
Let us say that in one day Z can complete, $1/Z^{th}$ of the work.
When all the three work together, in one day the can complete,
$= \frac{1}{15} + \frac{1}{10} + \frac{1}{Z}$th of the work
According to the question,
$\frac{1}{15} + \frac{1}{10} + \frac{1}{Z} = \frac{1}{5}$
Or, Z = 30 days
So, ratio of their efficiencies $= \frac{1}{15} : \frac{1}{10} : \frac{1}{30}$
= 2 : 3 : 1
So, out of Rs. 720, share of Z = 1/6 × 720 = Rs. 120
Hence, **option a.**

16. A and B can individually complete the project in 20 and 30 days respectively.
Let the total work pertaining to the project be the LCM of 20 and 30 i.e. 60 units.
So, A can do 60/20 = 3 units of work per day and B can do 60/30 = 2 units of work per day.
Thus, when A and B work together, they do 3 + 2 = 5 units of work per day.
A quits 10 days before the project is completed.
So, for the last 10 days, B works alone.
In these 10 days, B completes 10 × 2 = 20 units of work
Thus, A and B completed 60 – 20 = 40 units of work when they worked together.
So, A and B worked together for 40/5 = 8 days
So, total time for project to be completed = 8 + 10 = 18 days
Hence, **option e.**

17. Let the total amount of work be the LCM of 12, 15 and 20 i.e 60 units.
Let a, b and c be the number of of units of work respectively done by A, B and C alone.
∴ $a + b$ = 60/12 = 5 units/day
$b + c$ = 60/15 = 4 units/day
$c + a$ = 60/20 = 3 units/day
On solving these equations, we get a = 2 units/day.
Thus, A working alone can do 2 units of work per day.
So, A working alone can finish the work in 60/2 = 30 days
Hence, **option b.**

18. Since A is twice as good a worksman as B, A takes half the time as B to complete the work if they are working alone.
Let A working alone finish the work in a days.
So, A does $(1/a)$ of the work in 1 day.
So, B working alone finishes the work in $2a$ days. So, B does $(1/2a)$ of the work in 1 day.
Thus, in 1 day, A and be together do $(1/a)$ + $(1/2a)$ of the work i.e. $(3/2a)$ of the work.
A and B can together complete the work in 18 days. So, they can do (1/18) of the work in 1 day.
∴ $\frac{3}{2a} = \frac{1}{18}$
∴ a = 27
Hence, A alone can finish the work in 27 days.
Hence, **option c.**

19. Assume that 1 man can do 1 unit of work per day.
So, 3 men can do 3 units of work per day.
Since 3 men take 6 days to complete the work, total work = 3 × 6 = 18 units
The 3 men work for 2 days. So, work done in 2 days = 3 × 2 = 6 units
Amount of work left = 18 – 6 = 12 units
Now, there are 3 + 3 = 6 men working on this piece of work.
∴ Time taken by 6 men to complete the remaining work = 12/6 = 2 days
Hence, **option b.**

20. A and B can together dig the trench in 8 hours.
So, work done by A and B in one hour = 1/8
A alone can dig the trench in 12 hours.
So, work done by A alone in one hour = 1/12
So, work done by B alone in one hour
= (1/8) – (1/12) = 1/24
So, B can dig the trench alone in 24 hours.
Hence, **option e.**

TEST 3

21. Let the total work be the LCM of 18 and 15 i.e. 90 units.
So, B alone can do 90/15 = 6 units/day
A alone can do 90/18 = 5 units/day
In the first 10 days, B does 10 × 6 = 60 units of work.
So, time taken by A to do the remaining 30 units of work = 30/5 = 6 days
Hence, **option d.**

22. B, working alone, takes 23 days to complete the work.
A is 30% more efficient than B.
So, A is 1.3 times as efficient as B.
So, time taken by A working alone
= Time taken by B working alone/1.3
= 23/1.3 = 230/13 days
Let the total work be a common multiple of 23 and 230/13, say 230 units.
So, B does 230/23 = 10 units of work per day and A does 230/(230/13) = 13 units of work per day.
So, A and B working together do 10 + 13 = 23 units of work per day.
So, A and B together complete the work in 230/23 = 10 days.
Hence, **option c.**

23. Let the total work be 40 units
So, A and B can respectively do 2 units/day and 1 unit/day, if working alone.
Together, they can do 3 units of work per day.
If both work at 80% efficiency, they can do 0.8 × 3 = 2.4 units per day.
So, time taken = 40/2.4 = 50/3 days
Hence, **option d.**

24. A, working alone, finishes 80% of the work in 20 days.
So, if A is working alone, he can finish the entire work in 25 days.
Let the total work be a multiple of 25, say 150 units.
So, A does 6 units of work per day.
Thus, in 20 days, A completes 120 units of work.
Now, A and B together finish the remaining 30 units of work in 3 days i.e. they do 10 units of work per day.
Since A does 6 out of those 10 units, B does the remaining 4 units per day.
So, time taken by B to do 150 units of work
= 150/4 = 37.5 days
Hence, **option c.**

25. Suppose pipe A alone takes x hours to fill the tank.
Then, pipes B and C will take $x/2$ and $x/4$ hours to fill the tank.

$$\therefore \frac{1}{x}+\frac{2}{x}+\frac{4}{x}=\frac{1}{5}$$

$$\frac{7}{x}=\frac{1}{5}$$

$\therefore x = 35$ hours
Hence, **option c.**

26. Let the capacity of the tank be 60 units.
So, pipe A and pipe B can individually fill 4 units/min and 3 units/min respectively.
Together, they can fill 7 units per minute.
They work together for 4 minutes and fill 28 units in this time.
So, 32 units are still to be filled.
B can fill this in 32/3 minutes.
So, total time = 4 + (32/3) = 44/3 minutes i.e. 14 minutes and 40 seconds
Hence, **option d.**

27. Let the slower pipe alone fill the tank in x minutes.
Then, the faster pipe will fill it in $x/3$ minutes.
Together they fill the tank in 36 minutes.

$$\therefore \frac{1}{x}+\frac{3}{x}=\frac{1}{36}$$

$\therefore x = 144$ minutes
Hence, **option c.**

28. Let the capacity of the tank be 84 units.
So, $A + B + C = 14$ units/hour
In 2 hours, they fill 2 × 14 = 28 units
The remaining 56 units are filled by A and B in 7 hours.
So, number of units filled by A and B in 1 hour
= 56/7 = 8 units
$\therefore A + B = 8$
$\therefore C = 14 - 8 = 6$ hours
So, C can fill the tank in 84/6 = 14 hours
Hence, **option c.**

29. Efficiency is defined as the amount of work done in one day.
Working together A and B can do

$\frac{1}{15}+\frac{1}{10}=\frac{5}{30}=\frac{1}{6}$ amount of work

So the efficiency is 1/6.
Hence, **option a.**

30. 10 people can complete the project in 20 hours.
So, total work to be done = 10 × 20 = 200 people-hours
This group of 10 works for 10 hours.
So, work done in 10 hours = 10 × 10 = 100 people-hours
∴ Work remaining after 10 hours = 200 − 100 = 100 people-hours
Now, total number of people in the group = 10 + 6 = 16
∴ Time required = Work left/Total number of people = 100/16 = 6.25 hours
Hence, **option b.**

TEST 4

31. Ramya alone takes 6 days to complete some work and Shreya is twice as efficient as Ramya.
So, Shreya alone takes 3 days to complete the work.
Let the total work correspond to 12 units.
So, Ramya does 12/6 = 2 units of work per day and Shreya does 12/3 = 4 units of work per day.
When both work together, work done by them in a day = 2 + 4 = 6 units
∴ Time taken by both of them together = 12/6 = 2 days
Hence, **option d.**

32. Let the daily work done by one boy be x and that by one girl by y.
Since 4 boys and 6 girls complete the work in 10 days, the work done by them in a day is

$\therefore 4x+6y=\frac{1}{10}$ ... (i)

Similarly,

$\therefore 12x+8y=\frac{1}{4}$ ... (ii)

Multiplying (i) by 3 and solving equation (i) and (ii) simultaneously, we get

$x=\frac{7}{400}, y=\frac{1}{200}$

∴ Work done by 20 boys and 30 girls in 1 day = $20x + 30y = 1/2$
∴ Time taken by 20 boys and 30 girls to finish the work = 2 days
Hence, **option a.**

33. Omkar's 1 day's work $=\frac{1}{8}$ ... (i)

(Kushal + Sanket)'s 1 day's work

$=\frac{1}{4}$ ... (ii)

(Omkar + Sanket)'s 1 day's work

$=\frac{1}{3}$ ... (iii)

Adding equation (i) and (ii), we get
(Omkar + Kushal + Sanket)'s 1 day's work

$=\frac{1}{8}+\frac{1}{4}=\frac{3}{8}$ ... (iv)

Subtracting equation (iii) from (iv), we get

∴ Kushal's 1 day's work $=\frac{3}{8}-\frac{1}{3}=\frac{1}{24}$

Hence, Kushal alone completes the work in 24 days.
Hence, **option d**
Alternatively,
Let the total work done be the LCM of 8, 4 and 3 i.e. 24 units
So, work done by Omkar in one day = 24/8 = 3 units
Similarly, work done by Kushal and Sanket together in one day = 6 units
and, work done by Omkar and Sanket in one day = 8 units
∴ Work done by Sanket in one day = 8 − 3 = 5 units
and, work done by Kushal in one day = 6 − 5 = 1 unit
∴ Time taken by Kushal alone to complete the work
= 24/1 = 24 days
Hence, **option d.**

34. (A + B)'s 1 day's work $=\frac{1}{3}$

(C + D)'s 1 day's work $=\frac{1}{4}$

E work = dy's work $=\frac{1}{4}$

$\therefore ((A+B)+(C+D)+E)'$s 1 day's work

$=\frac{1}{3}+\frac{1}{4}+\frac{1}{4}$

$=\frac{1}{3}+\frac{1}{2}$

$=\frac{5}{6}$

∴ A, B, C, D and E together can finish the work in

$\frac{6}{5}$ i.e. $1\frac{1}{5}$ days.

Hence, **option c.**

35. The pump fills the tank in 2 hours while the pump (with the leak) fills the tank in 2.5 hours.
Let the capacity of the tank be 10 litres.
So, the pump can fill 10/2 = 5 litres/hour
Along with the leak, the pump can fill 10/2.5 = 4 litres/hour
This implies that 5 − 4 = 1 litre flows out of the tank through the leak every hour.
So, if the tank is full and the pump is closed,
Time taken by the leak to empty the tank = Capacity of tank/Rate of leak = 10/1 = 10 hours
Hence, **option b.**

36. The amount paid to the person is directly proportional to the duration that he works for.
Let the money that he earns be Rs. x.

$$\therefore x = \frac{500 \times 14}{10} = \text{Rs.}700$$

Hence, **option a.**

37. Let the capacity of the tank be 80 litres.
So, amount filled by pipe A in 1 minute = 80/16 = 5 litres
Similarly, amount filled by pipe B in 1 minute = 80/20 = 4 litres
and, amount emptied by pipe C in 1 minute = 80/8 = 10 litres
∴ Effective amount filled in tank when it is empty and all three pipes are simultaneously opened = 5 + 4 − 10 = −1 litre
This means that when all three pipes are simultaneously opened, 1 litre of water flows out every hour (and nothing gets filled).
So, the tank can never get filled.
Hence, **option d.**

38. Let the capacity of the tank be 50 litres.
So, amount filled by pipe A in 1 minute = 50/10 = 5 litres
and, amount filled by pipe B in 1 minute = 50/25 = 2 litres
Both pipes are simultaneously opened for 5 minutes.
Amount filled by both pipes in 5 minutes = (5 + 2) × 5 = 35 litres
∴ Amount to be filled by pipe B alone = 50 − 35 = 15 litres
Time taken by pipe B to fill the tank = 15/2 = 7.5 minutes
Hence, **option b.**

39. Amount of water raised ∝ Number of pumps × Number of days × Rate of working (in hours per day)
Let the required number of days be x.

$$\therefore \frac{20}{16} \times \frac{6}{8} \times \frac{4}{x} = \frac{4800}{3840}$$

$$\therefore x = \frac{20 \times 6 \times 4 \times 3840}{16 \times 8 \times 4800}$$

∴ x = 3 days
Hence, **option a.**

40. Number of tables ∝ Number of carpenters × Number of days × Rate of working (in hours/day)
Let the required number of tables be x.

$$\therefore \frac{12}{16} \times \frac{20}{24} \times \frac{6}{10} = \frac{240}{x}$$

$$\therefore x = \frac{240 \times 16 \times 24 \times 10}{12 \times 20 \times 6}$$

∴ x = 640
So, 640 tables can be made.
Hence, **option a.**

41. Here, the number of days the food will suffice is inversely proportional to the number of soldiers.
At the end of 10 days, the food would have sufficed for 20 more days with 1000 soldiers.
Since the number of soldiers has doubled at the end of 10 days; the number of days the food would suffice gets halved.
Hence the food will last for 20/2 i.e 10 more days
Hence, **option d.**

42. Let the consumption of petrol be 1 litre/day.
Hence the total consumption is 10 litres.
Now, Arun consumes 25% more petrol every day.
Hence he consumes 1.25 litres of petrol every day.
Let y be the number of days the petrol will last.
∴ $1.25 \times y = 10$
∴ $y = 8$
Hence, **option d.**

TIME AND DISTANCE

TEST 1

1. Time = Distance/Speed
∴ Time taken for the bus to travel from City A to City B = 220/55 = 4 hours
∴ Time taken for the bus to travel from City B to City C = 220/110 = 2 hours
∴ Total time taken for travelling from City A to City B and then to City C = 4 + 2 = 6 hours
Hence, **option c.**

2. Since all three contestants have completed the race, they have travelled the same distance.
∵ The distance is constant, their speed is inversely
proportional to the time taken to complete the race.
∴ The ratio of speeds = 1/8 : 1/3 : 1/6
= (3/24) : (8/24) : (4/24)
= 3 : 8 : 4
Hence, **option b**.

3. Let the total time that Meher takes to complete the entire journey be t hours.
Meher spent 20%, 50% and the remaining 30% of the time walking, in the bus and in the cab respectively.
Thus, time spent walking, in the bus and in the cab is 02t, 0.5t and 0.3t respectively.

$$\therefore \text{Average speed} = \frac{\text{Total Distance covered}}{\text{Total Time taken}}$$

$$= \frac{10(0.2t) + 40(0.5t) + 50(0.3t)}{t}$$

= 37 km/hr
Hence, **option a**.

4. Let the time that Arun usually takes to reach his office on time be t minutes.
∵ The distance from his house to his office is constant and speed is inversely proportional to time,

$$\frac{40}{60} = \frac{t-10}{t+15}$$

Note that in the above equation, there is no need to convert the speed to m/min, as the ratio is being taken. As such, the converting factor cancels out.
∴ 2(t + 15) = 3(t – 10)
∴ 2t + 30 = 3t – 30
∴ t = 60 min
Thus, he generally takes 60 minutes to reach his office exactly on time.
Hence, **option d**.

5. Since both the trains start simultaneously and then meet each other, they have travelled for the same amount of time.
Hence, the distance travelled by the trains is directly proportional to their speed.
If d is the distance travelled by the slower train, then (d + 320) is the distance travelled by the faster train.
Hence,

$$\frac{d+320}{d} = \frac{100}{80}$$

∴ d = 1280 km
∴ The distance between the two stations
= d + (d + 320) = 2d + 320 = 2560 + 320
= 2880 km
Hence, **option c**.

6. Usha beats Parvati by 10 m in a 100 m race and Parvati beats Anuja by 5 m in the same race.
Hence, when Usha covers 100 m, Parvati covers 90 m in the same race in the same time.
Similarly, when Parvati covers 100 m, Anuja covers
95 m in the same race in the same time.
∴ When Parvati covers 90 m, Anuja covers
(90 × 95)/100 = 85.5 m
Thus, when Usha covers 100 m, Anuja covers 85.5 m.
∴ Usha beats Anuja by 100 – 85.5 = 14.5 m
Hence, **option c**.

7. Since the length of the train is required in metres, convert the speed of the train from km/hr to m/s.
The train runs at 108 km/hr = 108 × 5/18
= 30 m/s
Compared to the train, the length of the pole is considered to be negligible.
∵ The length of the pole will be negligible, and the train crosses it in 13 seconds;
∴ The distance covered by the train in 13 seconds is
equal to the length of the train.
∴ Length of train = Speed (in m/s) × Time (in seconds)
= 30 × 13 = 390 metres
∴ The train is 390 metres long.
Hence, **option a**.

8. From Mumbai to Delhi, the airplane goes from west to east while the air current flows from east to west.
Hence, in this case, the airplane and air current go in opposite directions.
Hence, the airplane will travel at 1100 – 100 = 1000 km/hr while going from Mumbai to Delhi.
From Delhi to Mumbai, both, the airplane and the air current go in the same direction i.e. from east to west.
Hence, the airplane will travel at 1100 + 100 = 1200 km/hr while Coming from Delhi to Mumbai.
∴ The total time taken fot the whole journey

$$= \frac{4800}{1000} + \frac{4800}{1200} = 4.8 + 4 = 8.8 \text{ hours}$$

Hence, **option d**.

9. Let the speed of the ship upstream be 'u' miles/hr and speed downstream be 'd' miles/hr.
Hence,

$$\frac{40}{u} + \frac{90}{d} = 10$$
$$\frac{60}{u} + \frac{60}{d} = 10$$

Solving for u and d,
d = 15 miles/hr and u = 10 miles/hr
∴ The speed of the ship in still water

$$= \frac{d+u}{2} = \frac{15+10}{2} = 12.5 \text{ miles/hr}$$

Hence, **option b.**

10. Total distance to be covered = Sum of the lengths of the trains = 210 + 120 = 330 m
Since the answer options give the time in seconds, convert the speed of the 2 trains from km/hr to m/s.
Relative speed = Sum of the speeds
= 50 + 70 = 120 km/hr

$$= 120 \times \frac{5}{18} = 33.33 \text{ m/s}$$

$$\therefore \text{Time taken} = \frac{\text{Distance to be covered}}{\text{Relative speed}}$$

$$= \frac{330}{33.33} = 9.9 \text{ seconds} \approx 10 \text{ seconds}$$

Hence, **option c.**

TEST 2

11. Let Deepika's speed be s_d km/hr.
Let Gurmeet's speed be s_g km/hr.
Deepika walks in the same direction as Gurmeet.
∴ The relative speed of Deepika with respect to Gurmeet is ($s_d - s_g$) km/hr.
∵ Gurmeet's speed is 4 km/hr, in 45 minutes Gurmeet would have travelled (45/60) × 4 = 3 kms.
Now, Deepika covers up these 3 kms in 36 minutes at a relative speed of ($s_d - s_g$).
∵ Speed = Distance/Time
∴ ($s_d - s_g$) = 3 km/36 minutes = (3 × 60)/36
= 5 km/hr
However, s_g = 4 km/hr
∴ $s_d - 4 = 5$
∴ s_d = 9 km/hr
Hence, **option e.**

12. Both the speeds given here are multiple of 18, hence convert them to m/s in the beginning itself.
Hence,
43.2 km/hr = 43.2 × 5/18 = 12 m/s
54 km/hr = 54 × 5/18 = 15 m/s
∵ The two cyclists start in opposite directions, the relative speed would be the sum of the two speeds
= 12 + 15 = 27 m/s
The faster cyclist will meet the slower one for the first time after 1080/27 = 40 sec
∴ The faster cyclist meets the slower one after every
40 sec.
The faster cyclist completes one round every 1080/15 = 72 sec
The slower cyclist completes one round every 1080/12 = 90 sec
∴ The two cyclists will be meet for the first time at the starting point after the L.C.M. of the time taken by each to complete a round;
L.C.M. (72, 90) = 360 sec.
∴ When they meet at the starting point for the first time they would have meet each other 360/40 = 9 times (which includes their meeting at the starting point for the first time)
Hence, **option d.**

13. When distance is constant, speed and time are inversely proportional to each other.
The distance is covered in 5 hours at a speed of 240 kmph.
For the same distance to be covered in 5/3 hours, speed should be (5 × 240)/(5/3)
= 720 km/hr
Hence, **option e.**

14. Let Abhay's speed be x km/hr and the time taken by Sameer be y hours.

$$\therefore \frac{30}{x} - \frac{30}{2x} = (y+2) - (y-1) = 3$$

$$\therefore \frac{30}{2x} = 3$$

∴ x = 5 km/h
Hence, **option a.**

15. Let the distance travelled by x km. Robert saves 2 hours by increasing his speed by 5 kmph.

$$\therefore \frac{x}{10} - \frac{x}{15} = 2$$

∴ x = 60 km
So, time taken to travel 60 km at 10 km/hr
= 60/10 = 6 hours
So, Robert started 6 hours before 2 P.M. i.e., at 8 A.M.
Now, Robert wants to reach by 1 P.M. i.e. in 5 hours.

Required speed = 60/5 = 12 km/h

Hence, **option c.**

16. Let the speed of the train be x km/hr and that of the car be y km/hr.

$$\therefore \frac{120}{x} + \frac{480}{y} = 8$$

$$\therefore \frac{1}{x} + \frac{4}{y} = \frac{1}{15}$$

$$\text{Also, } \frac{200}{x} + \frac{400}{y} = \frac{25}{3},$$

because this mode of transport requires 8 hours and 20 minutes,

$$\therefore \frac{1}{x} + \frac{2}{y} = \frac{1}{24}$$

On solving these we get $x = 60$ and $y = 80$

So, ratio of speed of train to speed of car = 3 : 4

Hence, **option b.**

17. $\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}}$

$$= \frac{(50 \times 3) + (60 \times 2)}{5} = \frac{270}{5} = 54 \text{ km/hr}$$

Hence, **option b.**

18. Since the length of the train, length of the platform and the time (in the answer options) are given in terms of metres and seconds, convert the speed of the train to m/s.

Speed of the train = 132 kmph = 132 × (5/18) = (110/3) m/s

Distance covered in passing the platform

= Length of the train + Length of the Platform

= (110 + 165) = 275 m.

$$\text{So, Required time } = \frac{275}{\frac{110}{3}} = 7.5 \text{ sec.}$$

Hence, **option c.**

19. Let the distance be d.

Time required to go from Andheri to Bandra will be (d/10) hours.

Now, time required while coming back = (d/15) hours.

So, using the formula for average speed:

$$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

$$\text{Average Speed} = \frac{2d}{\frac{d}{10} + \frac{d}{15}} = \frac{2 \times 10 \times 15}{10 + 15}$$

= 12km/h

Hence, **option a.**

20. Let the length of the train be x m.

Speed of the train = 54 km/hr = 54 × (5/18) = 15 m/sec.

Time taken to cross the man = 20 s.

∴ Length of the train = 15 ×20 = 300 m.

Let the length of the platform be y m.

Time taken to cross the platform = 36 s.

∴ Total distance covered

= Length of the train + length of the platform

= 300 + y

∴ 300 + y = 15 × 36 = 540

300 + y = 540

∴ y = 240 m

∴ Length of the platform = 240 m.

Hence, **option b.**

TEST 3

21. The car travels at 25 miles per hour for 40 miles and then at s miles per hour for another 120 miles. So, the total distance covered is 160 miles and the average speed over this distance is 40 miles per hour.

So, we get

$$40 = \frac{160}{\frac{40}{25} + \frac{120}{s}}$$

$$\therefore \frac{120}{s} = 4 - \frac{40}{25} = \frac{60}{25}$$

$$\therefore s = 25 \times \frac{120}{60} = 50 \text{ miles per hour.}$$

Hence, **option c.**

22. Length of the train = x m.

Speed of the train = 72 km hr = 72 × (5/18) = 20 m/sec

Time taken = 26 sec.

Length of the Platform = 250 m.

∴ Total Length = (x + 250) m.

∴ (x + 250) = 20 × 26 = 520

∴ x + 250 = 520

∴ x = 270

∴ Length of the train = 270 m.

Hence, **option e.**

23. The distance travelled by the boat while travelling in the direction of the stream is 120 km.

Let the distance to be covered by the boat when it is travelling against the stream be x km.

The boat goes down the river at a speed of 20 + 4 = 24 km/hr and up the river at a speed of 20 - 4 = 16 km/hr.

Since the time taken is same

∴ 120/24 = x/16

∴ x = 80 km.

Hence, **option a.**

24. Let the speed of the boat in still water be 'b' km/h.
∴ Downstream speed = $b + 10$ (As the speed of the stream is 10 km/h).
And upstream speed = $b - 10$
As per the given condition:

$$\frac{60}{b+10} + \frac{60}{b-10} = 4.5$$

$b = 30$ km/hr satisfies the above equation
∴ Downstream speed = 30 + 10 = 40 km/h
His onward journey was done at a speed of 40 km/h and the distance covered was 60 kms.
So, the time taken for the onward journey = 1.5 hours.
Hence, **option d.**

25. Before the turn, both cars travel for 8 km each. So, just before they take the turn, the two cars are (8 + 8) = 16 km apart.
Once they take the turn, they travel 6 km each.
So, after the turn, they are 12 km apart horizontally.
So, actual distance between them = $[(12)^2 + (16)^2]^{1/2}$ = 20 km
Hence, **option b.**

26. Let the speed of the boat be b and speed of the current be f.
∴ $b + f = 40$ and $b - f = 14$
Solving these two equations we get:
$f = 13$ km/hr
Hence, **option a.**

27. As both the people move in opposite directions, their relative speed = (5 + 10) = 15 km/h
So, time after which they meet each other = (15/15) = 1 hour.
Hence, **option a.**

28. A will complete one round in 300/5 = 60s.
B will complete one round in 300/10 = 30s.
LCM of 30 and 60 = 60.
Hence, A and B will meet together after 60s.
Hence, **option d.**

29. A beats B by 20m and A beats C by 40 m.
So, when A covers 200 m, B covers 180 m and C covers 160 m.
So, ratio of speeds of B and C = 180 : 160 = 9 : 8
Now, when B covers 200 m, he beats C by 24 m.
So, C covers 176 m
∴ Ratio of speeds of B and C = 200 : 176 = 25 : 23
Hence, the data is not consistent.
Hence, **option e.**

30. A beats B by 20m in 100m race.
So, ratio of speeds of A and B = 100 : 80 = 5 : 4.
Now, B beats C by 25m in the same race.
∴ Ratio of speeds of B and C = 100 : 75 = 4 : 3.
Hence, ratio of speeds of A, B, and C = 5 : 4 : 3
∴ Ratio of speeds of A and C = 5 : 3.
Hence, **option d.**

TEST 4

31. Let the original speed and original time be s km per minutes and t minutes respectively.
Here the distance covered by the car is same.

$$\therefore s \times t = \frac{3}{4} s \times (t + 15)$$

∴ $4t = 3t + 45$
∴ $t = 45$
Hence, the time originally taken by the car is 45 minutes.
Hence, **option b.**

Note: Here we have taken the time in km per minutes as the time in the question as well as the options is given in minutes. Also, the actual speed of the car in either case is not given. Just the ratio of the speeds is given. So, irrespective of the unit of speed, it gets cancelled out.

32. Aniket travels at s kmph for the first 20 minutes and at 3s kmph for the next 10 minutes.

$$\therefore 50 = \frac{20}{60} \times s + \frac{10}{60} \times 3s$$

∴ $300 = 20s + 30s$
∴ $s = 60$ kmph
Hence, **option d.**

33. Average Speed = Total Distance Covered/Total Time Taken

$$\text{Time} = \frac{10}{30} + \frac{10}{50} = \frac{8}{15} \text{ hours.}$$

Total distance = 10 + 10 = 20 km

$$\therefore \text{Speed} = \frac{20}{\frac{8}{15}} = 20 \times \frac{15}{8} = 37.5 \text{ kmph}$$

Hence, **option b.**

34. Let the speed of the boat be u kmph and that of the river be v kmph
∴ $u - v = 6/3 = 2$ ...(i)
and,
$u + v = 20/4 = 5$ ...(ii)
(ii) − (i) gives $v = 1.5$
So, speed of the river = 1.5 kmph.

Hence, **option b.**

35. Let the distance between the source and destination be x km.
The car reaches 20 minutes late with an average speed of 54 kmph.
Difference in the time taken at the two different speeds = 20 minutes = 1/3rd of an hour

$\therefore \frac{x}{54} - \frac{x}{60} = \frac{1}{3}$

$\therefore \frac{x}{18} - \frac{x}{20} = 1$

$\therefore 20x - 18x = 360$

$\therefore x = 180$ km

Hence, **option b.**

36. Let the speed of the boat in still water be u kmph and that of the stream be v kmph
$\therefore u - v = 24/12 = 2$ ... (i)
and, $u + v = 24/4 = 6$ ... (ii)
On solving these equations simultaneously, $u = 4$ and $v = 2$
So, speed of the boat in still water = 4 kmph and speed of the stream = 2 kmph.
Hence, **option d.**

37. Chirag is twice as fast as Piyush who is four times as fast as Bhavesh.
So, Chirag is eight times as fast as Bhavesh.
For a common distance, speed and time are inversely proportional.
So, time taken by Chirag
= (time taken by Bhavesh)/8 = 48/8
= 6 minutes.
Hence, **option b.**

38. Let the two-way distance covered by Varun be x m.
Time taken to walk x m + Time taken to ride x m = 40 minutes
$\therefore$ Time taken to walk $2x$ m + Time taken to ride $2x$ m = 80 minutes
Varun gains 12 minutes if he rides both ways i.e if he rides both ways, he covers the distance in 28 minutes.
So, the time taken to ride both ways i.e. ride $2x$ m = 28 minutes
$\therefore$ Time taken to walk $2x$ m = 80 – 28 = 52 minutes.
Hence, **option a.**

39. Mohan covers half the distance in three-fourth of the total time i.e. he covers 5 kms in 30 minutes (or 0.5 hours)
So, he has to now cover 5 kms in 10 minutes (or 1/6 hours)
So,

Speed = $\frac{5}{\frac{1}{6}} = 5 \times 6 = 30$ kmph.

Hence, **option d.**

40. The car and train are moving in the same direction.
Relative speed = 66 – 30 = 36 kmph

$= 36 \times \frac{5}{18} = 10$ m/sec

Total distance to be covered by train to overtake the car = 320 + 160 = 480 m

$\therefore$ Time = $\frac{\text{Distance}}{\text{Speed}} = \frac{480}{10} = 48$ seconds.

Hence, **option a.**

41. Since the two trains are travelling in the same direction, their speeds are subtracted.
Relative speed = 84 – 52 = 32 kmph

$= 32h\frac{5}{18} = \frac{80}{9}$ m/sec

Since the fast train has to only cross the man, the length of the slow train is not considered. The man gets considered as a moving body with negligible length.

$\therefore$ The fast train has $\frac{80}{9} \times 18 = 160$ meters.

Hence, **option a.**

42. Speed of the car = $\frac{300}{12} = \frac{75}{3}$ kmph

Speed of the bus = $\frac{300}{16} = \frac{75}{4}$ kmph

$\therefore$ Ratio of their speeds = $\frac{75}{3} : \frac{75}{4} = \frac{1}{3} : \frac{1}{4}$

= 4 : 3

Hence, **option d.**

Alternatively,
When distance is constant, speed and time are inversely proportional.
$\therefore$ Speed of car : speed of bus = time taken by bus : time taken by car = 16 : 12 = 4 : 3
Hence, **option d.**

43. Total distance = 120 + 140 = 260 m

Speed of the first train = $\frac{120}{12} = 10$ m/sec

Speed of the second train = $\frac{140}{10} = 14$ m/sec

Since the trains travel in opposite directions, relative speed = 10 + 14 = 24 m/sec

$\therefore \frac{260}{24} = 10.83$ seconds

Hence, the trains will cross each other in 10.83 seconds.
Hence, **option c.**

44. Since the runners run in opposite directions, their speeds get added.
∴ Relative speed = 20 + 30 = 50 m/s
Distance = length (circumference) of track = 800 m

$$\text{Time} = \frac{\text{Length of the track}}{\text{Relative speed}} = \frac{800}{50}$$

= 16 seconds.
Hence, **option c.**

45. Let the speeds of train A and train B be x and y kmph respectively such that $x > y$
Travelling in the same direction, relative speed = $x - y$
∴ $x - y = 150/5 = 30$ ... (i)
Travelling in opposite directions, relative speed = $x + y$
∴ $x + y = 150/3 = 50$ ... (ii)
Solving equation (i) and (ii) simultaneously, we get
$x = 40$ and $y = 10$
So, the speed of the two trains is 40 kmph and 10 kmph
Hence, **option b.**

46. The relative speed between the thief and policeman is (10 – 8) km/hr = 2 km/hr

$$\therefore \text{Time} = \frac{\text{Distance}}{\text{Relative Speed}} = \frac{0.1}{2} = 0.05 \text{ hours}$$

= 3 minutes
Hence, **option b.**

47. Let the original speed be x km/hr.

$$\therefore \frac{63}{x} + \frac{72}{x+6} = 3$$

∴ $63x + 378 + 72x = 3(x^2 + 6x)$
∴ $x^2 + 6x = 45x + 126$
∴ $x^2 - 39x - 126 = 0$
∴ $x = 42$ km/hr
Hence, **option c.**

48. Let the distance covered be D km, amount of diesel used be L litres, the total payment be Rs. T and the time taken be t hours.
∴ $T = 6D + 40L$ ... (i)
In option 1, Mileage = 8 = D/L
∴ $L = D/8$ ... (ii)
Putting equation (ii) in (i), we get,
$11D = T = 2120$
∴ $D = 192.72$ km
∴ Average speed = D/t = 192.72/20 =9.636 km/hr

Similarly solving for the other options we get the values of the average speed (in km/hr) for options 2, 3 and 4 as 7.8, 8.23 and 8.54 respectively.
Thus, car A maintained the maximum speed.
Hence, **option a.**

49. Let the time taken to cover the distance by walking be x km.
Let the time taken to cover the distance by driving be y km.
∴ $x + y = 6$ ... (i)
$2x = 10$ ... (ii)
On solving, $y = 1$
Thus he will need 1 × 2 = 2 hours to drive both ways.
Hence, **option a.**

NUMBER SYSTEMS

TEST 1

1. $1.5 + 1.8 \div 0.9 + (3 - 4) - 1.2 \times 2$
$= 1.5 + 1.8 \div 0.9 - 1 - 1.2 \times 2$
$= 1.5 + 2 - 1 - 2.4 = 0.1$
Hence, **option e.**

2. $[3 + (2 - 4) \times 7 + 3 - (8 \times 2 - 12) \div 4]$
$= [3 + (-2) \times 7 + 3 - (16 - 12) \div 4]$
$= [3 + (-2 \times 7) + 3 - (4 \div 4)]$
$= (3 - 14 + 3 - 1) = -9$
Hence, **option a.**

3. $$\left(5 + \left[\frac{1}{3} + \frac{1}{4}\right] \times 12 - 3\right) - 3 \times \frac{1}{3}$$

$$= 5 + \left(\frac{7}{12} \times 12\right) - 3 - 1$$

$= (5 + 7 - 3) - 1$
$= 9 - 1 = 8$
Hence, **option a.**

4. $$\frac{13}{3} - \left\{\frac{1}{6} \times \left(3 + \frac{11}{5} + 5 - \frac{21}{5}\right)\right\}$$

$$= \frac{13}{3} - \left\{\frac{1}{6} \times \left(8 - \frac{10}{5}\right)\right\}$$

$$= \frac{13}{3} - \left\{\frac{1}{6} \times (6)\right\} = \frac{13}{3} - 1 = \frac{10}{3} = 3\frac{1}{3}$$

Hence, **option b.**

5. $3 + 6 \div 3 \times 2 = 3 + 2 \times 2 = 3 + 4 = 7$
Hence, **option a.**

6. $$\frac{[2^4 + (16 - 3 \times 4)]}{[(6 + 3^2) + (7 - 4)]} = \frac{16 + (16 - 12)}{15 \div 5}$$

$$= \frac{16 + 4}{5}$$

$$= \frac{20}{5} = 4$$

Hence, **option b.**

7. $(7 - \sqrt{9}) \times (4^2 - 3 + 1)$
$= (7 - 3) \times (16 - 3 + 1)$
$= 4 \times 14 = 56$
Hence, **option c.**

8. $(33 - 2 \times 7) + (5 \times 3 - 22)$
$= (33 - 14) + (15 - 22)$
$= 19 - 7 = 12$
Hence, **option e.**

9. $(15 \div 3 + 4) - (3^2 - 7 \times 2)$
$= (5 + 4) - (9 - 14)$
$= 9 - (-5) = 14$
Hence, **option b.**

10. $(3 + 2)^2 - 5 \times 3 + 2^3$
$= 5^2 - 15 + 8$
$= 25 - 15 + 8 = 18$
Hence, **option c.**

TEST 2

11. Let the required number be N.
Since, the remainder is 2 when the number is divided by 13, you can write,
$N = 13Q + 2$
Hence, $N^2 = (13Q + 2)^2$
$\therefore N^2 = (13Q)^2 + 2(13Q)(2) + (2)^2 = 169Q^2 + 52Q + 4$
The first two terms are divisible by 13, but 4 is not divisible by 13.
∴ The remainder is 4.
Hence, **option b.**

12. Let the original number be x.
According to the question,
$$\frac{7}{4} \times x - \frac{4}{7} \times x = 99$$
$$\therefore \frac{(49 - 16)x}{28} = 99$$
$$\therefore \frac{33x}{28} = 99$$
$\therefore x = 84$
Hence, **option d.**

13. The shortest way to solve such a question is to substitute the value of n using the answer options, and then to see which value of $2n + 1$ is not a prime number.
For $n = 3, 2n + 1 = 7$
$n = 5, 2n + 1 = 11$
$n = 4, 2n + 1 = 9$
$n = 6, 2n + 1 = 13$
Thus, among the four numbers given in the options, only $n = 4$ yields a non-prime number.
However, because one of the options is "None of these", we also need to check $n = 1$ and $n = 2$
For $n = 1, 2n + 1 = 3$
$n = 2, 2n + 1 = 5$
Thus, the smallest value of n for which $2n + 1$ is not prime is $n = 4$
Hence, **option c.**

14. The prime numbers between 60 and 75 are 61, 67, 71 and 73.
∴ sum = 61 + 67 + 71 + 73 = 272
Hence, **option d.**

15. Let the quotient in both the case be q.
Using the first statement,
$\therefore 123 = nq + 13$
$\therefore nq = 110$
Let the remainder when 492 is divided by $4n$ be r.
$\therefore 492 = (4n)q + r = 4(nq) + r$
$\therefore 492 = 4(110) + r$
$\therefore 4r = 492 - 440 = 52$
Hence, **option d.**

16. The largest 3-digit number is 999, which is divisible by 3.
The number just less than 999 is 998, which is also divisible by 2.
So, consider the number just less than 998 i.e. 997.
Consider a perfect square immediately greater than 999.
322 = 1024
Hence, we consider the prime numbers less than 32 and see if they divide 997.
The prime numbers less than 32 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 and 31.
None of these divides 997.
Hence, 997 is the largest 3-digit prime number.
Hence, **option d.**

17. The sum of an odd number and even number is always odd.
Since x is odd and y is even, $(x + y)$ will be odd.
Hence, **option a.**

18. Let the smaller number be x.
So, the larger number is $(x + 12238)$
On dividing the larger number by the smaller, we get 76 as the quotient and 13 as the remainder.
$\therefore x + 12238 = 76x + 13$
$\therefore 75x = 12225$
$\therefore x = 163.$
Hence, the smaller number is 163.
Hence, **option c.**

19. Let the required fraction be x.
Since the fraction is less than 1, its reciprocal is greater than 1.

$$\therefore \frac{1-x^2}{x} = \frac{11}{30}$$

$\therefore 30(1 - x^2) = 11x$
$\therefore 30 - 30x^2 = 11x$
$\therefore 30x^2 + 11x - 30 = 0$
$\therefore 30x^2 + 36x - 25x - 30 = 0$
$\therefore 6x(5x + 6) - 5(5x + 6) = 0$
$\therefore (6x - 5)(5x + 6) = 0$

$$\therefore x = \frac{5}{6} \text{ or } x = -\frac{6}{5}$$

Since x is a positive fraction less than 1, $x = 5/6$.
Hence, **option a.**

20. The perfect squares closest to 755 are
$27^2 = 729$ and
$28^2 = 784$
Since the required number is to added to 755, we consider 784 as the relevant perfect square.
$\therefore$ Number to be added = 784 - 755 = 29
Hence, **option d.**

TEST 3

21. $1\frac{2}{3} - \frac{1}{6} + \frac{1}{2} - \frac{1}{3} + x = \frac{8}{3}$

$$\therefore \frac{5}{3} - \frac{1}{6} + \frac{1}{2} - \frac{1}{3} + x = \frac{8}{3}$$

$$\therefore \frac{10 - 1 + 3 - 2}{6} + x = \frac{8}{3}$$

$$\therefore x = \frac{8}{3} - \frac{5}{3}$$

$$\therefore x = \frac{3}{3} = 1$$

Hence, **option a.**

22. $\frac{\sqrt{0.81} \times \sqrt{0.25}}{\sqrt{x}} = \sqrt{2.25}$

$$\therefore \frac{0.9 \times 0.5}{\sqrt{x}} = 1.5$$

$\therefore \sqrt{x} = 0.3$
$\therefore x = 0.09$
Hence, **option b.**

23. $438 \div 73 \times 424 \div 53 = x$
$\therefore x = (438/73) \times (424/53)$
$\therefore x = 6 \times 8$
$\therefore x = 48$
Hence, **option c.**

24. $76 \div 19 \div 3 \times 6 + 104 \div 13 \times 2 = x$

$$\therefore 76 \times \frac{1}{19} \times \frac{1}{3} \times 6 + 104 \times \frac{1}{13} \times 2 = x$$

$\therefore x = 8 + 16 = 24$
Hence, **option c.**

25. An improper fraction has its absolute value greater than 1.

$$31\frac{3}{8} = \frac{31 \times 8 + 3}{8} = \frac{251}{8}$$

Hence, **option a.**

Alternatively,
Option 2, 3, 4 and 5 are proper fractions as their values are less than 1.
Hence, we can directly eliminate these options. Option 1 is the only fraction that is improper in nature.
Hence, **option a.**

26. $\left[1 + \left(\frac{1}{2} + \frac{1}{3}\right) \times 18 - 4\right] \times \frac{1}{2} = x$

$$\therefore x = \left(1 + \frac{5}{6} \times 18 - 4\right) \times \frac{1}{2}$$

$$\therefore x = (1 + 15 - 4) \times \frac{1}{2} = 12 \times \frac{1}{2}$$

$\therefore x = 6$
Hence, **option e.**

27. A proper fraction is a fraction whose absolute value is less than 1.

$$1\frac{2}{5} = \frac{7}{5} = 1.4$$

$$1\frac{1}{2} = \frac{3}{2} = 1.5$$

$$\frac{11}{9} = 1.22$$

$$\frac{15}{17} = 0.88$$

$$\frac{13}{12} = 1.08$$

Thus, the fraction in option 4 is a proper fraction.
Hence, **option d.**

28. $[(16)^2 \div 8 \times 14] \div 2 = 16 \times x$

$$\therefore \left(256 \times \frac{1}{8} \times 14\right) \times \frac{1}{2} = 16x$$

$$x = \frac{32 \times 14}{16 \times 2} = 14$$

Hence, **option d.**

29. $3.3 + 33.03 + 333.003 + 0.33 + 3.03 = x$
$x = 372.693$
Hence, **option c.**

30. Let x and y be the two numbers.

$\therefore x^2 + y^2 = 970$

and $xy = 483$

$(x - y)^2 = x^2 + y^2 - 2xy$

$\therefore (x - y)^2 = 970 - 2(483)$

$\therefore (x - y)^2 = 970 - 966$

$\therefore (x - y)^2 = 4$

$\therefore (x - y) = \pm 2$

Since the absolute difference between the two numbers is required, we take only the positive value

$\therefore (x - y) = 2$

Hence, **option a.**

NUMBER THEORY

TEST 1

1. $12 = 2^2 \times 3$

$24 = 2^3 \times 3$

$16 = 2^4$

$32 = 2^5$

$8 = 2^3$

The factor common to all the numbers is 2 and its highest power available among all the given numbers is 2^2 i.e. 4

Hence, the HCF is 4.

Hence, **option b.**

2. All the signals will become red again for the first time at the LCM of the time taken by each signal respectively to turn red.

$8 = 2^3$

$12 = 2^2 \times 3$

$16 = 2^4$

$20 = 2^2 \times 5$

LCM of 8, 12, 16 and 20 = $2^4 \times 3 \times 5$ = 240s = 4 minutes

Thus, all four signals will turn red again for the first time after 4 minutes.

Hence, **option d.**

3. $32 = 2^5$

$128 = 2^7$

$512 = 2^9$

$1024 = 2^{10}$

Thus, the LCM of the 4 numbers will be the highest power of 2 i.e. $2^{10} = 1024$

Hence, $X = 1024$ and sum of digits of X

$= 1 + 0 + 2 + 4 = 7$

Hence, **option b.**

4. As one group consists of students from only one class, 91, 143 and 208 students should be divided into equal sized groups of the largest possible size.

Thus, you need to find HCF (91, 143, 208).

$91 = 7 \times 13$

$143 = 11 \times 13$

$208 = 2^4 \times 13$

Thus, HCF (91, 143, 208) = 13

Therefore, the largest possible group size is 13.

Hence, **option d.**

5. Let the required number be x.

When 98 is divided by x, we get a remainder 2.

Therefore, when 96 is divided by x, the remainder should be 0.

Therefore, 96 is divisible by x.

Similarly, 144, 264 and 360 are divisible by x.

The largest number that will divide these numbers will be the HCF of these numbers.

Therefore, find the HCF of 96, 144, 264 and 360.

$\therefore x$ = HCF (96, 144, 264, 360)

By factorization,

$96 = 2^5 \times 3$

$144 = 2^4 \times 3^2$

$264 = 2^3 \times 3 \times 11$

$360 = 2^3 \times 3^2 \times 5$

$\therefore x$ = HCF (96, 144, 264, 360) = $2^3 \times 3 = 24$

$\therefore$ The required number is 24.

Hence, **option b.**

6. Let the required number be x.

HCF × LCM of two numbers

= Product of the two numbers

$\therefore 7200 \times 18 = 450 \times x$

$$\therefore x = \frac{7200 \times 18}{450} = 288$$

Hence, **option c.**

7. The highest 4 digit number divisible by 12, 40, 32 and 72 will be a multiple of the L.C.M. of the given four numbers.

$12 = 2^2 \times 3$

$40 = 5 \times 2^3$

$32 = 2^5$

$72 = 2^3 \times 3^2$

$\therefore$ L.C.M. = $2^5 \times 3^2 \times 5 = 1440$

Now, the highest 4 digit number is 9999

It is clear that 9999 is not divisible by 1440.

Therefore, the required number has to be some number smaller than 9999 and divisible by 1440.

If we divide 9999 by 1440, we get a remainder of 1359

Hence, the highest 4 digit number divisible by 1440

= 9999 − 1359 = 8640

$\therefore N = 8640$

Hence, **option c.**

8. Since 2 is the HCF of the two numbers, let the uncommon factors in these two numbers be x and y respectively.
Therefore, the 2 numbers will be $2x$ and $2y$ respectively.
$\therefore 2x \times 2y = 1924$
$\therefore xy = 481$
But 481 can be expressed as 13×37
Thus, the larger of 2 numbers is $a = 37 \times 2 = 74$ and
$b = 13 \times 2 = 26$
Hence, **option a.**

9. Let the equal remainder be r.
So, $1086 - r$, $946 - r$ and $995 - r$ are divisible by the required number.
Thus, $(1086 - r) - (946 - r)$ is also divisible by the required number.
Thus, 140 is divisible by the required number.
Similarly, 49 and 91 are also divisible by the required number.
Thus, the required number has to be the HCF of 49, 91 and 140.
$49 = 7^2$
$91 = 7 \times 13$
$140 = 2^2 \times 5 \times 7$
Thus, the required number i.e. the HCF is 7
Hence, **option d.**

10. Let the other number be x.
For two numbers,
H.C.F × L.C.M = product of two numbers
So, $36 \times 1950 = 234 \times x$
Or, $x = 300$
Hence, **option b.**

TEST 2

11. On observation we see that,
$3 - 1 = 2$, $4 - 2 = 2$ and $5 - 3 = 2$
Since the difference is constant i.e. 2, the required number = LCM (3, 4, 5) – 2
The L.C.M of 3, 4, 5 is 60
$\therefore$ The required number is = 60 – 2 = 58
Hence, **option a.**

12. LCM of 3,4,5 and 8 is 120.
The least 4 digit number, which is a multiple of 120 is 1080.
Hence, the answer should be 1080 which is not given in any of the options.
Hence, **option e.**

13. S_1, S_2 and S_3 respectively come back to their original position in 72, 84 and 96 seconds.
So, they will together come back to their original position at every instance that is a multiple of the LCM of their time i.e. the LCM of 72, 84 and 96.
$72 = 2^3 \times 3^2$
$84 = 2^2 \times 3^1 \times 7^1$
$96 = 2^5 \times 3^1$
$\therefore$ LCM of 72, 84 and 96 = $2^5 \times 3^2 \times 7^1 = 2016$ seconds
Thus, the three springs will come back together every 2016 seconds.
$\therefore$ They will come together for the second time after $2016 \times 2 = 4032$ seconds
$= 67\frac{1}{5}$ mins
Hence, **option b.**

14. Since the bells ring together after 15525 seconds, 15525 must be the LCM of the time intervals at which the three bells ring.
Since one of the intervals is a prime number, find a prime factor of 15525
$15525 = 3^3 \times 5^2 \times 23 = 27 \times 25 \times 23$
Here, 27 is a perfect cube, 25 is a perfect square and 23 is a prime number. Also, all three are co-primes.
$\therefore$ Required sum = 23 + 25 + 27 = 75 seconds
Hence, **option a.**

15. 43, 91 and 183 when divided by a particular number leave the same remainder, say r.
Let the highest number that satisfies this condition be n.
So, $43 - r$ is divisible by n.
Similarly, $91 - r$ and $183 - r$ are also divisible by n.
So, $(183 - r) - (91 - r)$ is also divisible by n i.e. 92 is divisible by n.
Similarly, $(183 - r) - (43 - r)$ i.e. 140 is also divisible by n.
Also, $(91 - r) - (43 - r)$ i.e. 48 is also divisible by n.
So, n is the highest number that divides 48, 92 and 140 i.e. n is the H.C.F. of 48, 92 and 140.
$48 = 2^4 \times 3^1$
$92 = 2^2 \times 23^1$
$140 = 2^2 \times 5 \times 7$
Thus, H.C.F. of 48, 92 and 140 is 2^2 i.e. 4
Hence, **option a.**

16. Let the least multiple of 7 required by $7n$.
This number when divided by 6, 9, 15 and 18 leaves a remainder of 4.
So, $7n - 4$ is divisible by 6, 9, 15 and 18.
So, $7n - 4$ has to be the L.C.M. of 6, 9, 15 and 18.
$6 = 2^1 \times 3^1$
$9 = 3^2$
$15 = 3^1 \times 5^1$

$18 = 2^1 \times 3^2$
So, L.C.M. of 6, 9, 15 and 18 = $2^1 \times 3^2 \times 5^1 = 90$
When 4 is added to it, the number becomes 94
However, 94 is not a multiple of 7.
So, check for further multiples of 90.
We can confirm that 184 and 274 are also not divisible by 7.
However, 364 is divisible by 7.
So, 364 is the smallest such number.
Hence, **option d.**

17. Let the number to be added be r.
So, 2497 + r is divisible by 3, 4, 5 and 6.
This also means that 2497 + r is divisible by the L.C.M. of 3, 4, 5 and 6 i.e. by 60.
So, when 2497 is divided by 60, some remainder will be left
This remainder + r = 60
∴ r = 60 – remainder
On dividing 2497 by 60, the remainder is 37.
∴ Number to be added = (60 - 37) = 23.
Hence, **option c.**

18. The time at which all the three people meet will be the L.C.M. of the time taken by each person individually to complete one round.
$252 = 2^2 \times 7^1 \times 9^1$
$308 = 2^2 \times 7^1 \times 11^1$
$198 = 2^1 \times 9^1 \times 11^1$
∴ L.C.M. of 252, 308 and 198 = $2^2 \times 7^1 \times 9^1 \times 11^1 = 2772$.
So, A, B and C will again meet at the starting point in 2772 seconds i.e. 46 minutes and 12 seconds.
Hence, **option d.**

19. Since the numbers are in the ratio 3 : 4 : 5, let the numbers be $3x$, $4x$ and $5x$ respectively.
Then, their L.C.M. = $60x$.
However, their L.C.M. is given to be 2400.
∴ $60x = 2400$ i.e. $x = 40$.
So, the numbers are (3 × 40), (4 × 40) and (5 × 40) i.e. 120, 160 and 200.
So, their H.C.F. is 40
Hence, **option a.**

20. Since the required number leaves a remainder of 6 and 5 when dividing 1657 and 2037 respectively, it divides (1657 – 6) and (2037 – 5) i.e. 1651 and 2032.
So, the required number is the H.C.F. of 1651 and 2032
Since these are relatively larger numbers, divide 2032 by 1651. The remainder when 2032 is divided by 1651 is 381.
Now, divide 1651 by 381. The remainder of this division is 127.
Now, divide 381 by 127. The remainder of this division is 0.
So, 127 is the H.C.F of 1651 and 2032.
So, 127 is the required number.
Hence, **option b.**

TEST 3

21. Let the numbers be $2x$ and $3x$.
So, their L.C.M. = $6x$.
However, the L.C.M. is given as 48
∴ $6x = 48$
∴ $x = 8$.
So, the numbers are 16 and 24.
Hence, their sum is 40.
Hence, **option c.**

22. Let the numbers be a and b.
∴ $a + b = 55$
H.C.F. × L.C.M. = $a \times b$
∴ $ab = 5 \times 120 = 600$.
∴ 20m of reciprocals $= \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{55}{600}$
$= \frac{11}{120}$
Hence, **option c.**

23. $36 = 2^2 \times 3^2$
$84 = 2^2 \times 3 \times 7$
∴ H.C.F. = $2^2 \times 3 = 12$.
Hence, **option d.**

24. Let the numbers be $13a$ and $13b$.
∴ $13a \times 13b = 2028$
∴ $ab = 12$.
Now, the pairs of co-primes with product 12 are (1, 12) and (3, 4).
So, the required numbers are (13 × 1, 13 × 12) and (13 × 3, 13 × 4).
Clearly, there are 2 such pairs.
Hence, **option b.**

25. Product or two primes = H.C.F. of two numbers × L.C.M. of two numbers
Let the numbers be $37a$ and $37b$.
∴ $37a \times 37b = 4107$
∴ $ab = 3$.
Now, co-primes with product 3 are (1, 3).
So, the required numbers are (37 × 1, 37 × 3) i.e., (37, 111).
∴ The greater number is 111.
Hence, **option c.**

26. The given numbers are 1.08, 0.36 and 0.90.
These numbers can also be written as (108/100), (36/100) and (90/100)
So, the H.C.F. is the H.C.F. of the numerators and the L.C.M. of the denominators.

Since the denominators are the same in each fraction, L.C.M. of the denominators is 100.
H.C.F. of 108, 36 and 90 is 18.
∴ H.C.F. of given numbers = 18/100 = 0.18.
Hence, **option c.**

27. Since n is divisible by 3, 5 and 12, the next number divisible by 3, 5 and 12 will be n + LCM of (3, 5, 12)
Now, the LCM of (3, 5, 12) = 60
So, the next number after n, which is divisible by 3, 5, 12 is $n + 60$.
Hence, **option d.**

28. Mr. Brackett and his sons can take a break every 140, 210 and 280 minutes respectively.
So, the three of them will meet each other at the L.C.M. of 140, 210 and 280 minutes.
But, LCM of (140, 210 and 280) = 840 minutes i.e. 14 hours.
So, they can meet each other every 14 hours.
Hence, **option c.**

29. Let the two numbers be x and $x + 3$.
For any two natural numbers m and n, $m \times n$ = HCF
$(m, n) \times$ LCM (m, n)
$\therefore x(x + 3) = 54$
$\therefore x^2 + 3x = 54$
$\therefore x^2 + 3x - 54 = 0$
$\therefore x^2 + 9x - 6x - 54 = 0$
$\therefore (x + 9)(x - 6) = 0$
$\therefore x = 6$ or $x = -9$
Since the given numbers are natural numbers, $x = 6$
and $x + 3 = 9$
Thus, the numbers are 6 and 9.
Hence, **option a.**

30. The greatest possible length of the piece is equal to the H.C.F. of the length of the 3 strings.
Since the greatest possible length is to be found in cm, convert the length of each string into cm.
So, the length of the strings is 700 cm, 385 cm and 1295 cm respectively.
$700 = 2^2 \times 5^2 \times 7$
$385 = 5 \times 7 \times 11$
$1295 = 5 \times 7 \times 37$
∴ H.C.F = 5 × 7 = 35
Hence, greatest possible length of each piece is 35 cm.
Hence, **option a.**

31. Since the balls are to be distributed equally such that each student gets balls of exactly one colour and no balls are left over, the maximum number of balls is the H.C.F. of 294, 252 and 210.
$294 = 2 \times 3 \times 7^2$
$252 = 2^2 \times 3^2 \times 7$
$210 = 2 \times 3 \times 5 \times 7$
∴ H.C.F = 2 × 3 × 7 = 42
Hence, the maximum number of balls that each student can get is 42.
Hence, **option b.**

32. Let the required number be n.
So, n leaves a remainder of 8 when divided by any of 12, 15, 20 or 54.
So, $n - 8$ is divisible by 12, 15, 20 or 54.
Since the least number is required, $n - 8$ is the LCM of 12, 15, 20 and 54.
∴ The required least number
= LCM (12, 15, 20, 54) + 8
12 = 22 × 3
15 = 3 × 5
20 = 22 × 5
54 = 2 × 33
∴ LCM = 22 × 33 × 5 = 540
∴ Required least number = 540 + 8 = 548
Hence, **option d.**

33. Any number with its H.C.F as 20 must contain 20 as a factor.
Since the number-pairs must be between 35 and 90, consider the multiples of 20 between 35 and 90 i.e. 40, 60 and 80
Now, among these three numbers, find the H.C.F of each pair.
$40 = 2^3 \times 5$
$60 = 2^2 \times 3 \times 5$
$80 = 2^4 \times 5$
H.C.F (40, 60) = $2^2 \times 5 = 20$
H.C.F (60, 80) = $2^2 \times 5 = 20$
H.C.F (40, 80) = $2^3 \times 5 = 40$
Hence, the pairs that satisfy the given condition are (40, 600 and (60, 80) i.e. 2 pairs.
Hence, **option b.**

34. $900 = 2^2 \times 3^2 \times 5^2$
$1800 = 2^3 \times 3^2 \times 5^2$
H.C.F = 100 = $2^2 \times 5^2$
Let x be the third number.
So, x will definitely have $2^2 \times 5^2$ as its factors since H.C.F is the product of the lowest powers of common factors.
Also, LCM = $2^4 \times 3^2 \times 5^3$
Since the highest of 2 and 5 in 900 and 1800 is 2^3 and 5^2, x will definitely have $2^4 \times 5^3$ as its factors since LCM is the product of the highest powers of common prime factors.
x cannot have any power of 3 else a power of 3 would also have been present in the H.C.F.

$\therefore x = 2^4 \times 5^3 = 2000$
Hence **option b**.

TEST 4

35. 123x is divisible by 7, therefore, 123 - 2x should also be divisible by 7.
Now, 123 - 2x is divisible by 7,
when 123 - 2x = 119, 105
These are the only multiples of 7 possible if x is a single digit.
Therefore, 123x is divisible by 7 when x = 2, 9
Hence, options 2 and 3 can be eliminated.
Now, when x = 2, the number becomes 1232.
The sum of the digits of this number
= 1 + 2 + 3 + 2 = 8
∴ This number is not divisible by 3.
Now, when x = 9, the number becomes 1239.
The sum of the digits of this number
= 1 + 2 + 3 + 9 = 15
This number is divisible by 3.
∴ 123x is divisible by 3 as well as 7 when x = 9
Hence, **option d.**

36. It is given that:
$n = 8k + 3$
$\therefore 6n = 6 \times (8k + 3)$
$\therefore 6n = 48k + 18$
Since we want the remainder when $6n$ is divided by 8, express the R.H.S. in terms of a multiple of 8.
$\therefore 6n = 8 \times (6k + 2) + 2$
So, when $6n$ is divided by 8, remainder is 2.
Hence, **option c.**

37. Since 7513a4821b is divisible by 10, the digit at the units place has to be 0.
$\therefore b = 0$
Since 7513a4821b is also divisible by 11,
(digits at odd places) – (digits at even places)
= 0 or 11k (i.e. the difference is either zero or a multiple of 11)
(digits at odd places) – (digits at even places)
$= (7 + 1 + a + 8 + 1) - (5 + 3 + 4 + 2 + b)$
$= (7 + 1 + a + 8 + 1) - (5 + 3 + 4 + 2 + 0)$
$= 17 + a - 14$
$= 3 + a$
If this sum is 0, $a = -3$. However, a cannot be negative.
If this sum is a multiple of 11, a = 8 or 19 or 30 and so on...
However, a is a single digit number.
$\therefore a = 8$
$\therefore a = 8$ and $b = 0$
Hence, **option b.**

38. As the number of tickets is equal to the number of cells, and the number of rows is equal to the number of columns, the number of tickets is a perfect square.
Since the table has 58 rows and columns,
Total number of tickets = total number of cells
= 58 × 58 = 3364
The seller has already sold 3128 tickets.
∴ He still has 3364 - 3128 = 236 tickets.
Hence, **option a.**

39. Arjun arranges the tiles in a row-column grid on the wall such that each row and column has 14 tiles.
∴ He needs 14 × 14 = 196 tiles to cover the whole wall.
Arjun bought 200 tiles.
∴ Tiles still left = 200 – 196 = 4
Hence, **option c.**

40. As 4584193ab is divisible by 5, b can be either 5 or 0.
But 4584193ab is also divisible by 8.
∴ 3ab should be divisible by 8 i.e. it must be an even number.
∴ b must be 0.
∴ 3a0 is completely divisible by 8
This is true for 320 and 360.
$\therefore a = 2$ or $a = 6$
$\therefore a + b = 2 + 0 = 2$ or $a + b = 6 + 0 = 6$
None of these values are given in the options.
Hence, **option d.**

41. The smallest and largest 4-digit numbers are 1000 and 9999.
While 1000 is divisible by 4, 9999 is not. The largest
4-digit number divisible by 4 is 9996
1000 = 4 × 250 and 9996 = 4 × 2499
Thus, 1000 is the 250th multiple of 4 and 9996 is the 2499th multiple of 4.
So, number of multiples of 4 from 1000 to 9996
= 2499 – 250 + 1 = 2250
Since all the multiples of 4 are divisible by 4, there are 2250 4-digit numbers that are completely divisible by 4.
Hence, **option b.**

42. The 3-digit numbers divisible by 3 are 102, 105, 108, ..., 999.
This forms an A.P where a = 102, d = 3 and l = 999.
The n^{th} term (tn) of an A.P. is
$tn = a + (n - 1)d$
$\therefore 999 = 102 + (n - 1)3$
$\therefore 999 - 102 = 3n - 3$
$\therefore 897 + 3 = 3n$
$\therefore 900 = 3n$

$\therefore n = \frac{900}{3} = 300$

When the first and last term of an A.P. are known, sum of the first n terms (Sn) is:

$S_n = \frac{n}{2}(a + l)$

$\therefore S_{300} = \frac{300}{2}(102 + 999)$

$\therefore S_{300} = 150(1101)$

$\therefore S_{300} = 165150.$

Hence, sum of all the 3-digit numbers divisible by 3 is 165150.

Hence, **option c**.

43. To check divisibility by 4, check if the last two digits of that number are divisible by 4.

The last two digits of the given numbers are 76, 22, 17, 84, 52 and 38 respectively.

Of these, only 76, 84 and 52 are divisible by 4.

So, only 9376, 6484 and 9352 are divisible by 4.

Now, we check the divisibility by 8 for only these three numbers.

To check divisibility by 8, check if the last three digits of that number are divisible by 8.

The last three digits of the available numbers are 376, 484 and 352 respectively.

Of these, only 376 and 352 are divisible by 8.

So, only 9376 and 9352 are divisible by 8.

Thus, the only numbers divisible by both 4 and 8 are 9376 and 9352.

Hence, **option c**.

44. Let the number which is divided be denoted by x and the quotient be denoted by q.

$\therefore x = 5q + 3$

On squaring, we get,

$x^2 = (5q + 3)^2$

$\therefore x^2 = 25q^2 + 30q + 9$

Dividend = (Divisor × Quotient) + Remainder

Since we want the remainder when x^2 is divided by 5, express the equation given above in the format of the formula given above.

$\therefore x^2 = 25q^2 + 30q + 5 + 4$

$\therefore x^2 = 5(5q^2 + 6q + 1) + 4$

Hence, the remainder when x^2 is divided by 5 is 4.

Hence, **option d**.

45. $70b$ is divisible by 11.

$\therefore (7 + b) - 0 = 0$ or $(7 + b) - 0$ is a multiple of 11.

If $(7 + b) - 0 = 0$, $b = -7$.

This is not possible as b is the digit of a number.

So, $(7 + b) - 0$ is a multiple of 11.

$\therefore b = 4$ or 15 or 26 and so on

Since b is the digit of a number, $b = 4$

$\therefore 968 - 26a = 704$

$\therefore 26a = 968 - 704 = 264$

$\therefore a = 4$

$\therefore a - b = 4 - 4 = 0$

Hence, **option b**.

46. The day when all five groups meet is the LCM of the interval at which each group meets.

The interval for each group is: Gardening - 2 days, Electronic - 3 days, Chess - 4 days, Yachting - 5 days, Photography - 6 days

LCM of 2, 3, 4, 5 and 6 is 60

Thus, all groups meet once in 60 days.

Hence, in a period of 180 days, they meet 3 times on the same day..

Hence, **option a**.

47. The gardener has 1000 plants. Since he plants them such that number of plants in rows and columns is the same, total number of plants should be a perfect square.

The perfect square nearest to 1000 and greater than 1000 is 1024 i.e. 32^2

Hence, he needs to have at least 24 more plants.

Hence, **option b**.

LINEAR EQUATIONS

TEST 1

1. When the coefficients of an equation are interchanged to form a second equation, it is often easier to form two new equations by adding and subtracting the original equations and then solving the two new equations.

$15m + 17n = 21$ ... (i)

$17m + 15n = 11$ ... (ii)

Adding equations (i) and (ii),

$32m + 32n = 32$

$\therefore m + n = 1$ ... (iii)

Subtracting equation (i) from equation (ii),

$2m - 2n = -10$

$\therefore m - n = -5$ ... (iv)

Solving equation (iii) and equation (iv),

$m = -2$ and $n = 3$

Hence, **option d**.

2. Let the number of hundred rupee notes with Carla be x and the number of fifty rupee notes be y.

$x + y = 32$ ... (i)

$100x + 50y = 2750$ ... (ii)

$\therefore 2x + y = 55$

$\therefore x + (x + y) = 55$

$\therefore x + 32 = 55$ ... (from (i))

$\therefore x = 23$
$\therefore$ The number of hundred rupee notes is 23.
Hence, **option a**.

3. Let the fraction be x/y.
From the first condition,

$$\frac{x+1}{y} = \frac{7}{19} \quad \text{... (i)}$$

From the second condition,

$$\frac{x}{y+1} = \frac{1}{3} \quad \text{... (ii)}$$

Solving equation (i),
$19x + 19 = 7y$
$\therefore 19x - 7y = -19$ ... (iii)
Solving equation (ii),
$3x = y + 1$
$\therefore 3x - y = 1$ ... (iv)
Multiplying equation (iv) by 7, and subtracting from equation (iii),
$x = 13$ and $y = 38$

$\therefore$ The original fraction $= \frac{13}{38}$

Hence, **option d.**

Alternatively,
This problem can be solved by using the options.
Check by adding 1 to the denominator of every option. Whichever option becomes 1/3 after simplification can be a possible answer.
In this case, only the last option turns out to be 1/3 after adding 1 to the denominator.
Hence, **option d.**

4. Let the present ages of Aishwarya and Deepika be x and y respectively.
According to the first condition,
$x + 10 = 2y$
According to the second condition,
$(x - 6) = 5/3 \times (y - 6)$
After simplifying both the equations,
$x - 2y = -10$ ... (i)
$3x - 5y = -12$ ... (ii)
Multiplying (i) by 3, and subtracting from equation (ii),
$x = 26$ and $y = 18$
Hence, **option b.**

5. Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$

$7u + 13v = 27$ ... (i)
$13u + 7v = 33$ ... (ii)
Multiplying equation (i) by 13 and equation (ii) by 7,
$91u + 169v = 351$
$91u + 49v = 231$
Solving, $v = 1$ and $u = 2$

$\therefore x = \frac{1}{2}$ and $y = 1$

$\therefore x + y = \frac{3}{2}$

Hence, **option a.**

6. Let 'x' be the number of bananas that Amar brought to school.
The Physics and Chemistry teacher were given $x/4$ and $x/6$ bananas respectively.
After giving 2 bananas to the Head Master, the chemistry teacher still had 4 bananas left.
$\therefore x/6 - 2 = 4$
$\therefore x = 6 \times 6 = 36$
The Physics teacher was given one-fourth of the total number of bananas,
$\therefore$ The Physics teacher received $x/4 = 9$ bananas.
Hence, **option d.**

7. Population of A (in millions) = 9
Let the number of people in city B (in millions) be b, while those in city C (in millions) be c.
From the given information,
$b = 9 + c$ ... (i)
and $c = 9 + b/2$
i.e. $2c = 18 + b$ ... (ii)
Solving equations (i) and (ii),
$b = 36$ and $c = 27$
$\therefore$ The total number of people in cities A, B and C combined = 9 + 36 + 27 = 72 million.
Hence, **option d.**

8. Let Sam, Harry and Jake have x, y and z number of candies with them respectively.
If Sam gives three candies to Jake, then he is left with
$x - 3$ candies and Jake now has $z + 3$ candies.
$\therefore x - 3 = (z + 3) + 2$ ... (i)
If Harry gives two candies to Jake, he is now left with
$y - 2$ candies whereas Jake has $z + 3 + 2 = z + 5$ candies.
$\therefore y - 2 = (z + 5) + 2$ ... (ii)
Solving equation (i) and equation (ii), we get,
$y - x = 1$
$\because$ Sam and Harry together had 19 candies.
$\therefore x + y = 19$
$\therefore x = 9, y = 10$ and $z = 1$
Jake has now $z + 5 = 6$ candies
Hence, **option d.**

9. Let the number of rows be y and number of students in each row be x.

Thus, the total number of students = xy.
In each case, the number of students per row and number of rows will change, but the total number of students will remain constant.
$xy = (x - 5) \times (y + 6)$
$xy = xy + 6x - 5y - 30$
$6x - 5y = 30$ ... (i)
Also,
$xy = (x + 5) \times (y - 2)$
$\therefore xy = xy - 2x + 5y - 10$
$\therefore 2x - 5y = -10$ ... (ii)
Subtracting equation (ii) from equation (i),
$4x = 40$
$\therefore x = 10$
Substituting the value of x in (ii), $y = 6$.
$\therefore$ Number of students = $10 \times 6 = 60$
Hence, **option e.**

10. In this problem, there are three variables and two equations to solve from.
$\therefore$ You cannot find out the value of all the variables. The method involved here is to see whether 'the price Harry has to pay' can be found out from the set of equations given.
Let the cost of an apple, a mango and an orange be x, y and z respectively.
$\therefore 3x + 7y + z = 120$ ... (i)
and $4x + 5y + z = 164.5$ ... (ii)
The price that Harry has to pay can be written as,
$x + 11y + z$
See if you can get the value of this equation without solving the equations completely.
Multiplying equation (i) by 3 and equation (ii) by 2,
$9x + 21y + 3z = 360$ ... (iii)
and $8x + 10y + 2z = 329$ ...(iv)
Subtracting equation (iv) from equation (iii),
$x + 11y + z = 31$
$\therefore$ The price Harry has to pay is Rs. 31.
Hence, **option b.**

TEST 2

11. Let the two digit number be xy which can be represented as $(10x + y)$.
Given that,
$$\frac{10x + y}{x + y} = 4 \quad \text{... (1)}$$
When the digits are reversed, it becomes $(10y + x)$
According to the question,
$10y + x = 2(10x + y) - 6$ ... (2)
From equation (1) we get,
$10x + y = 4(x + y)$
$10x + y = 4x + 4y$
$6x = 3y$
$2x = y$ ... (3)
Solving equation (2),
$10y + x = 2(10x + y) - 6$
$10y + x = 20x + 2y - 6$
$8y = 19x - 6$
From equation (3) we have $y = 2x$, substituting the same in the above equation
$8(2x) = 19x - 6$
$3x = 6$
$x = 2$
$\therefore y = 2x = 2 \times 2 = 4$
Thus, the number is 24.
Hence, **option a.**

12. Let the wife's salary be Rs. x.
So, the salary of the husband is Rs. $(x + 800)$
According to the question,
$$\frac{x + 800}{4} + \frac{x}{8} = 500$$
$2(x + 800) + x = 500 \times 8$
$2x + 1600 + x = 4000$
$3x + 1600 = 4000$
$3x = 4000 - 1600$
$3x = 2400$
$x = 800$
$\therefore$ 8e's salary = Rs. 800 and husband's salary = Rs. (800 + 800) = Rs. 1,600
So total salary = Rs. (800 +1600) = Rs. 2,400
Saving = Rs. 500
So, expenditure = Rs. (2400 − 500) = Rs. 1,900
Hence, **option d.**

13. $15x + 10y = 90$ ... (i)
$x + y = 7$ ... (ii)
Multiplying (ii) by 10 and subtracting the result from (i) we get,
$5x = 20$
$x = 4$
$\therefore y = 3$
$\therefore$ Now if pens cost Rs 3 and pencils Rs. 4,
Cost = $15 \times 3 + 10 \times 4 = 45 + 40$ = Rs. 85
Hence, **option a.**

14. $7x + 8(2 - x) + 10 = 4x - 4$
$\therefore 7x + 16 - 8x + 10 = 4x - 4$
$\therefore 5x = 16 + 10 + 4 = 30$
$\therefore x = 6$
Hence, **option b.**

15. Let the total money available be Rs. x.
Amount spent on buying a house = $0.5x$
So, amount left = Rs. $0.5x$
Now, he spends the half the remaining amount to buy a car.
Thus, he spends Rs. $0.25x$ to buy the car and is left with Rs. $0.25x$

He now spends 20% of this amount to buy a motorcycle.

So, he spends $0.2 \times 0.25x$ = Rs. $0.05x$ to buy a motorcycle.

So, he is left with $0.2x$.

$\because x = 1000000$

$\therefore$ The amount that he is left with = 0.2 .2 y a mot = 200000 i.e. Rs. 2 lakhs

Hence, **option c.**

16. Let $10a + b$ be the original number.

So, the number when reversed becomes $10b + a$.

Hence, $(10b + a) = 3(10a + b) - 1$

$\therefore 29a = 7b + 1$

$\therefore a = (7b + 1)/29$

a and b have to be single-digit positive integers.

Only $b = 4$ satisfies this condition.

For this value of b, $a = 1$

Hence, $b = 4$ and $a = 1$

Hence, the original number is 14.

Hence, **option d.**

17. $11a + 17b = 73$ ... (I)

$17a + 11b = 67$ ... (II)

When the coefficients of a and b are interchanged, the equations can be solved faster by first adding them and then subtracting them.

Adding I and II, we get,

$28a + 28b = 140$

$\therefore 14a + 14b = 70$

$\therefore a + b = 5$ ... (III)

Now, II – I is,

$6a - 6b = -6$

$\therefore b - a = 1$ ... (IV)

Solving (III) and (IV), we get

$a = 2$ and $b = 3$

Hence, **option c.**

18. Let the number of questions correctly attempted by Suresh be a and the number of questions incorrectly attempted be b.

Hence, we have;

$a + b = 70$ ... (I)

$3a - b = 170$ ... (II)

Add I and II to get,

$4a = 240$

$\therefore a = 60$

Thus, Suresh correctly attempted 60 questions.

Hence, **option a.**

19. Let the original number be $10a + b$. So, the number obtained by reversing the digits of the number is $10b + a$.

As per the given condition,

$(10b + a) - (10a + b) = 72$

$\therefore 9b - 9a = 72$

$\therefore b - a = 8$

Both, a and b have to be single-digit numbers.

For $a = 1$, $b = 9$

For $a = 2$, $b = 10$ (which is not possible)

$\therefore a = 1$ and $b = 9$

So, there is only one such number i.e. 19.

Hence, **option a.**

20. Let Suresh have x coins of Rs. 2 and y coins of Rs. 5.

$\therefore x + y = 100$ ... (I)

and $2x + 5y = 350$ ... (II)

$5 \times$ (I) – II gives

$3x = 150$

$\therefore x = 50$

Thus, Suresh has 50 Rs. 2 coins.

Hence, **option a.**

TEST 3

21. Let $10a + b$ be the original number.

Thus, the reversed number is $10b + a$.

$\therefore 4(10a + b) = (10b + a) + 3$

$\therefore 39a = 6b + 3$

$$\therefore a = \frac{6b + 3}{39}$$

a and b have to be single digit numbers.

a is a single digit number only for $b = 6$

Now, for only $b = 6$, a is an integer.

For $b = 6$, $a = 1$

Hence, the original number is 16.

Hence, **option c.**

22. Let $10a + b$ be the number.

Hence, reverse of the number is $10b + a$

Hence, difference between the two numbers is $(10b + a) - (10a + b) = 9b - 9a = 9(b - a)$

It is given that $b = a + 3$

$\therefore b - a = 3$

$\therefore 9(b - a) = 9 \times 3 = 27$

Hence, **option d.**

23. $3a + 4b = 40$ ... (I)

$7a + 3b = 49$ ... (II)

$4 \times$ (II) – $3 \times$ (I) we get,

$19a = 76$

$\therefore a = 4$ and $b = 7$

Hence, **option c.**

24. Let Ramesh had a 5 rupee coins initially.

Hence, the number of 2 rupee coins with him is $2a$.

Hence, he had Rs. $4a + 5a$ = Rs. $9a$ with him.

Now, if numbers of coins were to be interchanged, i.e. he would have had $2a$ 5 rupee coins. and a 2 rupee coins.
So, the amount of money that he would have had would be $2a + 10a$ = Rs. $12a$.
Hence, the required difference = $12a - 9a = 3a$
This difference is given as Rs. 30
$\therefore 3a = 30$
$\therefore a = 10$
Hence, number of coins = $3a = 3 \times 10 = 30$
Hence, **option e.**

25. $a - 3b + 3c = -4$ ... (I)
$2a + 3b - c = 15$ ... (II)
$4a - 3b - c = 19$ ... (III)
II + III gives,
$6a - 2c = 34$ ... (IV)
Similarly, I + II gives,
$3a + 2c = 11$ ... (V)
IV + V gives,
$9a = 45$
$\therefore a = 5$
Hence, **option e.**

26. $4x + y - 2z = 0$ ... (I)
$3x - 3y + 3z = 9$ ... (II)
$6x - 2y + z = 0$ ... (III)
By I + II - III, we get,
$x = 9$
Hence, **option c.**

27. Let Anuradha's present age = x
$\therefore x + 8 = 2(x - 6)$
$\therefore x = 20$ years
Hence, **option d.**

Alternatively,
This can also be solved quickly by using answer options. Since current age is asked, each option corresponds to present age. So, take each option, add 8 to it and also subtract 6 from it separately. See if the former number is double the latter. The answer option that satisfies this condition is the answer. For instance, the age in option (a) is 14.
$14 + 8 = 22$ and $14 - 6 = 8$
$22/8 \neq 2$
Thus, this is not the present age.
Consider option (d).
$20 + 8 = 28$ and $20 - 6 = 14$
$28/14 = 2$
Hence, the present age is 20 years.
Hence, **option d.**

28. Let the number of tigers be x and the number of human visitors be y.
The number of heads is 84.
$\therefore x + y = 84$ ... (i)
Since each tiger has 4 legs and each human visitor has 2 legs,
$\therefore 4x + 2y = 272$ ... (ii)
Multiplying equation (i) by 2 and solving equation (i) and (ii) simultaneously, we get
$x = 52$ and $y = 32$
So, there are 52 tigers and 32 human visitors.
Hence, **option b.**

29. Since the age of the younger son is given, the age of the other two people can be easily found without any equations.
Since the younger son is 14 years old right now, two years ago his age was 12 years. Two years ago, since the elder son was twice as old as the younger son, the elder son's age was 12×2 i.e. 24 years.
So, the elder son is 26 years old now and will be 28 years old, two years from now.
At that time, the father will be twice as old as the elder son.
So, the father will be $2 \times 28 = 56$ years old two years from now.
Hence, the father is currently 54 years old.
Hence, **option d.**

30. Let the daughter's present age be x years.
So, the mother's present age = $7x$ years.
The daughter's age after two years
= $x + 2$ years.
So, the mother's age after two years
= $7x + 2$ years.
$\therefore 7x + 2 = 5(x + 2)$
$\therefore x = 4$ years
$\therefore 7x = 28$ years.
Hence, **option a.**

TEST 4

31. Let x tickets of Rs.16 and y tickets of Rs.8 be sold.
$\therefore x + y = 14$
And, $16x + 8y = 160$ i.e. $2x + y = 20$
Solving these two simultaneous equations, we get $x = 6$ and $y = 8$.
Hence, **option e.**

32. Let x, y be the number of blue and green bottles respectively.
$\therefore x = \frac{1}{2}y$ ... (i)
$\therefore y = x + \frac{1}{3} \times 12$
$\therefore y = x + 4$ ... (ii)
Putting the value of equation (i) in (ii), we get
$\therefore y = \frac{1}{2}y + 4$

$\therefore y = 8$

$\therefore$ Number of green bottles = 8

$\therefore$ Number of blue bottles $= \frac{1}{2} \times y = \frac{1}{2} \times 8$

$= 4$

Hence, **option a.**

33. Population of village A = 4000

Let x and y be the population of villages B and C respectively.

$\therefore x = 2y$

and

$\therefore y = 4000 + \frac{1}{4}$ of x

$\therefore y = 4000 + \frac{1}{4} \times (2y)$

$\therefore y = 4000 + \frac{y}{2}$

$\therefore y = 8000$

$\therefore x = 2 \times 8000 = 16{,}000$

$\therefore$ Total population of Village B is 16,000.

Hence, **option a.**

34. Let Rs.x and Rs.y be the salary of Riya and Jiya respectively.

$\therefore x = y + 1200$ .... (i)

and

$\frac{1}{3}x + \frac{1}{6}y = 800$

Multiply this equation by 6

$\therefore 2x + y = 4800$ .... (ii)

Solving equation (i) and (ii) simultaneously, we get

$x = 2000$ and $y = 800$

$\therefore$ Jiya's salary = Rs.800

Hence, **option d.**

35. Let Rs.x and Rs.y be the cost of 1 pencil and 1 eraser respectively.

$\therefore 20x + 8y = 116$ ... (i)

and

$x + y = 7$ .... (ii)

Solving equation (i) and (ii) simultaneously, we get

$x = 5$ and $y = 2$

When the cost of 1 pencil and 1 eraser is interchanged, $x = 2$ and $y = 5$

So, the cost of 20 pencils and 8 erasers

$= 20x + 8y = (20 \times 2) + (8 \times 5)$

$= 80$

Hence, the required cost is Rs. 80

Hence, **option a.**

36. Given $\frac{3x}{1 + \frac{1}{1 + \frac{x}{1-x}}} = 1$

$\therefore \frac{3x}{1 + \frac{1}{\frac{(1-x)+x}{1-x}}} = 1$

$\therefore \frac{3x}{1 + \frac{1}{\frac{1}{1-x}}} = 1$

$\therefore \frac{3x}{1 + (1 - x)} = 1$

$\therefore \frac{3x}{(2 - x)} = 1$

$\therefore 3x = 2 - x$

$\therefore 4x = 2$

$\therefore x = \frac{2}{4} = \frac{1}{2}$

Hence, **option a.**

37. Let the four original numbers be A, B, C and D.

$\therefore A + 10 = B - 15 = C \times 2 = D/3 = x$

$\therefore A = x - 10$, $B = x + 15$, $C = x/2$ and $D = 3x$

Also, A + B + C + D = 170

$\therefore (x - 10) + (x + 15) + \left(\frac{x}{2}\right) + (3 \times x) = 170$

$\therefore 2x + 5 + 3x + \frac{x}{2} = 170$

$\therefore 5x + 5 + 0.5x = 170$

$\therefore 5.5x = 165$

$\therefore x = \frac{165}{5.5} = 30$

Hence, $A = x - 10 = 30 - 10 = 20$

$B = x + 15 = 30 + 15 = 45$

$C = x/2 = 30/2 = 15$

$D = 3x = 3 \times 30 = 90$

Hence, sum of the smallest and largest numbers = 15 + 90 = 105.

Hence, **option c.**

38. Let the current age of Roshni be x years.

Then, Roshni's age 5 years ago and multiplied by $2 = 2(x - 5) = 2x - 10$

Also, $2x - 10 - \frac{2}{3}x = x$

$\therefore 2x - \frac{2}{3}x - x = 10$

$\therefore x = 30$

Hence, Roshni's current age = 30 years.

Hence, **option a.**

39. Let the first member's salary be Rs.x.

$\therefore$ Second member's salary = Rs.$(x + 5000)$

and third member's salary

$= (x + 5000 + 5000) =$ Rs.$(x + 10000)$

$\therefore$ Total salary of family = Rs.$(3x + 15000)$

According to the given data,

$\frac{x}{10} + \frac{(x + 5000)}{5} + \frac{(x + 10000)}{4} = 9000$

$$\frac{x}{10}+\frac{(2x+10000)}{10}+\frac{(x+10000)}{4}=9000$$

$$\frac{(3x+10000)}{10}+\frac{(x+10000)}{4}=9000$$

$$\frac{(12x+40000+10x+100000)}{40}=9000$$

$\therefore 22x + 140000 = 360000$

$\therefore x = 220000$

$\therefore x = 10000$

$\therefore$ Total salary of family = (3 × 10000) + 15000 = Rs.45,000

$\therefore$ Monthly expenditure
= Total salary – Savings
= 45000 – 9000 = Rs.36,000

Hence, **option d.**

40. Let Rahul and Rohit originally have Rs.x and Rs.y respectively.
When Rahul gives Rs.50 to Rohit,
$2(x - 50) = y + 50$
$\therefore 2x - 100 = y + 50$
$\therefore 2x - y = 150$ ... (i)
Now, if Rohit gives Rs. 40 to Rahul, then,
$x + 40 = 14(y - 40)$
$\therefore x - 14y = 600$
On multiplying by 2, we get,
$2x - 28y = -1200$ ... (ii)
On solving equations (i) and (ii) simultaneously,
we get, $y = 50$ and $x = 100$
Hence, Rahul and Rohit originally have Rs.100 and Rs.50 respectively.
Hence, **option b.**

41. Let the amount with A, B, C and D be a, b, c and d respectively.
$a = b - 1$
$\therefore a < b$
$c = d + 5$
$\therefore c > d$
$d = 3 + b$
$\therefore d > b$
$\therefore c > d > b > a$
Thus, a gets the smallest amount.
Hence, **option a.**

SEQUENCES, PROGRESSIONS AND SERIES

TEST 1

1. Each term can be written as $n^2 + 1$
Thus, 1st term = $1^2 + 1$, 2nd term = $2^2 + 1$, 3rd term = $3^2 + 1$ and so on
Thus, the next term would be $6^2 + 1 = 37$
Hence, **option e.**

2. The difference between the 1st and 2nd term is 1.
The difference between the 2nd and 3rd term is 2.
The difference between the 3rd and 4th term is 3.
The difference between the 4th and 5th term is 4.
Therefore, the difference between the 5th and 6th term should be 5.
Hence, the 6th term should be 14 + 5 = 19
Hence, **option b.**

3. The first two terms of the A.P. are 3 and 5 respectively.
Hence, $a = 3$ and $d = 2$
The n^{th} term of an A.P. (T_n) is $a + (n - 1)d$,
$\therefore$ The 5th term of the A.P. = $T_5 = 3 + 2(5 - 1)$ = 11
Hence, **option e.**

4. The first child in the line gets 4 chocolates and every subsequent child gets 3 chocolates more than the previous one.
Therefore, the number of chocolates received by each child forms an arithmetic progression with $a = 4$ and
$d = 3$.
$\therefore$ The last child i.e. the 10th child gets
$4 + 3(10 - 1) = 31$ chocolates
Therefore, the total number of chocolates distributed
by Ramesh is: $\frac{n}{2}[a + T_n] = \frac{10}{2}[4 + 31]$
$= 175$
Hence, **option e.**

5. Let the four terms of the A.P be $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$ respectively.
Adding all these terms, we get,
$4a = 124$
$\therefore a = 31$
Also, $(a - 3d)(a + 3d) = (a - d)(a + d) - 128$
$\therefore a^2 - 9d^2 = (a^2 - d^2) - 128$
$\therefore 8d^2 = 128$
$\therefore d = \pm 4$
Consider $d = +4$
Hence, the 4 numbers are 19, 27, 35 and 43.
Consider $d = -4$
Hence, the 4 numbers are 43, 35, 27 and 19.
Therefore, the smallest number in either case is 19.
Hence, **option b.**

6. $T_3 + T_7 = 8$
$\therefore (a + 2d) + (a + 6d) = 8$
$\therefore 2a + 8d = 8$

The sum of the first nine terms of the progression is given by $(n/2) \times [2a + (n - 1)d]$
i.e. $(9/2) \times [2a + 8d]$
i.e. $(9/2) \times 8 = 36$
Hence, **option c.**

7. The n^{th} term of a G.P. is given by $T_n = ar^{n-1}$, where a is the first term of the G.P. and r is the common ratio.
Here, $a = 5$ and $r = 2$
$\therefore$ The 6^{th} term of the G.P. $= 5 \times 2^{(6-1)}$
$= 5 \times 2^5 = 5 \times 32 = 160$
Hence, **option b.**

8. Since 10, b and 40 are in G.P., b is their geometric mean.
$b = \sqrt{10 = th}$
$\therefore b = \pm 20$
Since only 20 is in the options, option (c) is the correct answer.
Hence, **option c.**

9. For a G.P., sum of the first n terms can be expressed as S_n.
$$S_n = \frac{a(1-r^n)}{(1-r)}$$
where a is the initial term and r is the common ratio of the G.P.
$$\therefore \frac{S_8}{S_4} = \frac{\left[\frac{a(1-r^8)}{1-r}\right]}{\left[\frac{a(1-r^4)}{1-r}\right]}$$
$$\frac{97}{81} = \frac{[(1-r^4)(1+r^4)]}{(1-r^4)}$$
$$r^4 = \frac{97}{81} - 1$$
$$r^4 = \frac{16}{81}$$
$$r = \pm\frac{2}{3}$$
Hence, **option d.**

Note: Though it is not known initially whether r is greater than or less than 1, it does not make a difference. This is because in either case, in the R.H.S, the term $(1 - r^n)$ or $(r^n - 1)$ gets cancelled out in the numerator as well as in the denominator leaving behind $1 + r^n$.

10. Let A, G and H represent the Arithmetic, Geometric and Harmonic mean of the two positive numbers.
Now, $G = 6$ and $G = H + 2$
$\therefore H = 4$
The relation between the Arithmetic, Geometric and Harmonic Mean of two numbers is given by
$A \times H = G^2$
$\therefore A \times 4 = 36$
$\therefore A = 9$
Hence, **option d.**

TEST 2

11. The reciprocal of the terms of an H.P. form an A.P.
Hence, the reciprocals of the first three terms of this H.P. are in A.P.
$\therefore$ 5, 7 and 9 are in A.P. such that $a = 5$ and $d = 2$.
The seventh term of this A.P. using the formula
$a + d(n - 1) = 5 + 2(7 - 1) = 17$
$\therefore$ The 7^{th} term of the H.P. is 1/17.
Hence, **option a.**

12. The given series is the series of sum of squares of the first 20 natural numbers.
sum to n terms of square of a series is given by
$$n \times \frac{(n+1) \times (2n+1)}{6}$$
Thus, sum to first 20 terms is given by
$$20 \times \frac{(20+1) \times (40+1)}{6}$$
i.e. 2870
Hence, **option b.**

13. The amount that Amar pays as EMI every month forms an A.P. with $a = 1000$ and $d = 500$.
Since he pays the EMI for one year, $n = 12$
So, total amount paid in instalments
$= (n/2) \times [2a + (n - 1)d]$
$= (12/2) \times [(2 \times 1000) + (11 \times 500)]$
$= 6 \times (2000 + 5500) = 45000$
Since this is Rs. 20,000 more than what he would have paid had he paid the entire amount upfront, the amount that he would have paid upfront = 45000 – 20000 = 25000.
So, cost of the car = Rs. 25,000.
Hence, **option b.**

14. The n^{th} term of an A.P. is $a + (n - 1)d$
So, the 5^{th} term is $a + 4d$ and the 7^{th} term is $a + 6d$
$$\therefore \frac{a+4d}{a+6d} = 0$$
$\therefore a + 4d = 0$
$\therefore a = -4d$
$\therefore$ Ratio of 12^{th} term to 13^{th} term
$$= \frac{a+11d}{a+12d} = \frac{-4d+11d}{-4d+12d} = \frac{7d}{8d}$$

$= \frac{7}{8}$

Hence, **option c.**

15. The sides of a quadrilateral are in A.P.

Let the sides of the quadrilateral be

$a - 3d, a - d, a + d, a + 3d$

$\therefore$ Semi perimeter

$= \frac{a - 3d + a - d + a + d + a + 3d}{2} = 40$

$\therefore 4a = 80$

$\therefore a = 20$

Now, $a + d = 3(a - 3d)$

$20 + d = 3(20 - 3d)$

$\therefore 20 + d = 60 - 9d$

$\therefore d = 4$

$\therefore$ Largest side $= a + 3d = 20 + 3 \times 4 = 32$

Hence, **option b.**

16. There are total of 491 numbers in the range.

Since we are looking for multiples of 7, look for the highest multiple of 7 less than or equal to 491

$491 = (70 \times 7) = 1$

Thus, there are 70 multiples of 7 less than 490

Now, the multiples of 7 form an A.P. such that

$a = d = 7$ and

$n = 70$

So, the sum of these multiples is given by

$S_n = \frac{n}{2} \times [2a + (n - 1)d]$

$\therefore S_{70} = \frac{70}{2} \times [(2 \times 7) + (69 \times 7)]$

$= 35 \times 71 \times 7 = 17395$

Hence, **option d.**

17. Here; $a = 17$.

n^{th} term of an A.P is given by $T_n = a + (n - 1)d$

$T_2 = 17 + d$ $\quad$ $T_4 = 17 + 3d$

$T_5 = 17 + 4d$ $\quad$ $T_6 = 17 + 5d$

$\therefore (17 + d)(17 + 3d) = (17 + 4d)(17 + 5d)$

$\therefore 289 + 51d + 17d + 3d^2 = 289 + 85d + 68d + 20d^2$

$\therefore 68d + 3d^2 = 153d + 20d^2$

$\therefore 17d^2 = -85d$

$\therefore d = -5$

$\therefore 3^{rd}$ term is $T_3 = a + 2d = 17 + 2(-5) = 7$

Hence, **option a.**

18. Since Rajesh puts money in the piggy bank for 3 years, he puts in money 36 times.

The first amount put is Rs. 150, the next is Rs. 175, then Rs. 200 and so on. These amounts form an A.P. with

$a = 150$ and $d = 25$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore S_{36} = \frac{36}{2}[2(150) + (3601)25]$

$= 18\ [300 + 875] = 21150$

Thus, Rs. 21,150 get accumulated.

Hence, **option b.**

19. n^{th} term (T_n) in a G.P. is given as $T_n = ar^{n-1}$

$\therefore T_5 = ar^4$ $\quad$ $T_2 = ar$

$\therefore ar^4 = 64(ar)$

$\therefore r^3 = 64$

$\therefore r = 4$

Also, $T_4 = -320$

$\therefore ar^3 = -320$

$\therefore a4^3 = -320$

$\therefore a = -5$

$\therefore T_6 = ar^5 = (-5)4^5 = -5120$

Hence, **option a.**

20. Let the 3 terms of the G.P be $\frac{a}{r}, a, ar$

$\therefore \frac{a}{r} \times a \times ar = 1728$

$\therefore a^3 = 1728$

$\therefore a = 12$

Also; $ar = 4\left(\frac{a}{r}\right)$

$12r = \frac{4 \times 12}{r}$

$\therefore r^2 = 4$

$\therefore r = \pm 2$

$\therefore$ 1st term $= \frac{12}{2}$ or $\frac{12}{-2}$ i.e. (6 or -6)

So, the exact value of the first term cannot be determined.

Hence, **option d.**

21. You save Rs. 7 on 1st May, Rs.14 on 2nd May, and Rs.28 on 3rd May

Thus, this is a G.P. $a = 7$ & $r = 2$.

$n = 15$, since the period is upto 15th May.

$\therefore S_{15} = \frac{7(2^{15} - 1)}{(2 - 1)} = \frac{7[2^{10}.2^5 - 1]}{1}$

$= \frac{7[1024532 - 1]}{1} =$ Rs. 2,29,369

Hence, **option a.**

22. $(1^3 + 2^3 + 3^3 + .. + n^3) = \frac{1}{4}n^2(n + 1)^2$

$\therefore (1^3 + 2^3 + 3^3 + .. + 20^3)$

$= \frac{1}{4} \times 20^2(20 + 1)^2$

$= \frac{1}{4} \times 0 + 1.(21)^2$

$= 100 \times 21 \times 21 = 44100$

Hence, **option a.**

Note: The sum of cubes of the first n natural numbers can also be found by the following relationship:
Sum of cubes of first n natural numbers
= (sum of first n natural numbers)^2
In this case, n = 20.
So, sum of first 20 natural numbers
= (20 × 21)/2 = 210
So, sum of cubes of first 20 natural numbers
= 2102 = 44100

23. $2^2 + 3^2 + 4^2 + ... + 50^2 = (1^2 + 2^2 + 3^2 + ...+ 50^2) - 1^2$
$1^2 + 2^2 + 3^2 + ...+ n^2 = [n \times (n + 1) + (2n + 1)]/6$
$\therefore (1^2 + 2^2 + 3^2 + \cdots + 50^2)$
$= \frac{1}{6} \times 50 \times (50 + 1) \times ((2 \times 50) + 1)$
$= \frac{1}{6} \times 50 \times 51 \times 101$
= 25 × 17 × 101 = 42925
∴ 22 + 32 + 42 + ... + 502 = 42925 − 12
= 42925 − 1 = 42924
Hence, **option d.**

24. Since the cost of a railway ticket increases by a factor of 1.5 each year, the cost forms a G.P. where a = 56 and r = 1.5
Since the first term (i.e. cost of ticket) corresponds to the year 2008, the cost of tickets in 2013 will correspond to the 6th term of the G.P.
The n^{th} term of a G.P. is given as: $t_n = a \times r^{(n-1)}$
$\therefore t_6 = 56 \times 1.5^{(6-1)}$
$\therefore t_6 = 56 \times 1.5^5$
$\therefore t_6 = 425.25$
This is the one-way fare between Mumbai and Pune in 2013.
So, amount paid by Mr. Sharma for the return journey = 425.25 × 2 = Rs. 850.5
Hence, **option d.**

25. Since the difference between number of problems solved is constant, this is an A.P where a = 3 and d = 3.
Also, number of days = n = (25 − 3) + 1 = 23
Here, the number of problems solved on 25th June corresponds to the 23rd term of this A.P.
n^{th} term of an A.P. is given as $t_n = a + (n - 1)d$
$\therefore t_{23} = 3 + (23 - 1)3 = 3 + (22 \times 3) = 69$
So, Rohan will solve 69 problems on 25th June.
Hence, **option d.**

26. The amount with the 7 students is in A.P. with S_n = 700, n = 7 and d = 20
$S_n = \frac{n}{2}(2a + (n-1)d)$
$\therefore 700 = \frac{7}{2}(2a + 6(20))$
$\therefore a = 40$
Hence, **option 2.**

TEST 3

27. The given progression is an A.P. with first term = a = 2 & common difference = d = 5
For the first 21 terms, n = 21
$S_n = \frac{n}{2}(2a + (n-1)d)$
$S_{21} = \frac{21}{2}(2 \times 2 + (21 - 1) \times 5)$
$\therefore S_{21} = \frac{21}{2} \times (4 + 100) = 21 \times 52$
$\therefore S_{21} = 1092$
Hence, **option d.**

28. The given series is a G.P. with a = 384 & r = 0.5
The n^{th} term of a G.P. $(T_n) = ar^{n-1}$
$\therefore T_8 = 384 \times 0.5^{(8-1)}$
$= 384 \times 0.5^7 = 3$
Hence, **option a.**

29. $T_n = a + (n - 1)d$
$\therefore T_{19} = 3 + (19 - 1) \times 4$
$\therefore T_{19} = 3 + 72$
$\therefore T_{19} = 75$
Hence, **option d.**

30. In the given G.P., a = 1 and r = 3
n^{th} term of a G.P. $(T_n) = ar^{(n-1)}$
For a G.P. with n terms, T_n is also the last term.
$\therefore 729 = 1 \times 3^{(n-1)}$
$\therefore 3^6 = 3^{(n-1)}$
$\therefore n - 1 = 6$
$\therefore n = 7$
Thus, there are 7 terms in the given G.P.
Hence, **option c.**

31. $S_n = \frac{a(r^n - 1)}{r - 1} \quad r > 1$
a = 5, r = 2
$\therefore S_{10} = \frac{5(2^{10} - 1)}{2 - 1}$
$\therefore S_{10} = 5 \times 1023$
$\therefore S_{10} = 5115$
Hence, **option e.**