

DATA INTERPRETATION

Concept Builder and Tests

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Data Interpretation and Analysis

I. INTRODUCTION

Data Interpretation deals with reading and understanding a large amount of information that may be provided in a certain format. Information can be understood in different ways based on the manner in which it is represented. Data Interpretation involves a study of various ways of representing data and then analysing it as per requirements.

Data Analysis, on the other hand deals with the use of mathematical and analytical techniques to correctly analyse data which may be tabulated or worded or may be in the form of charts and graphs. In any case, the data consists of numbers, figures and statistics on which certain numerical operations have to be performed to answer the questions.

Let us first take a look at some basic arithmetic terms and their application in Data Interpretation.

II. BASIC TERMS AND CALCULATIONS

This section is an introduction to the basic components of Data Interpretation – the building blocks of Data Analysis which are used multiple times to solve any DI question.

The two major concepts required here are Average and Percentage.

A. AVERAGE

The mathematical definition of the Average (or Arithmetic Mean) of a series of terms (or numbers) is as follows.

$$\text{Average} = \frac{\text{Sum of all terms}}{\text{Total number of terms}}$$

Example 1: The weights of 3 people are 65 kg, 75 kg and 85 kg. What is their average weight?

Solution: The total weight of 3 people = 65 + 75 + 85 = 225 kg

The total number of people = 3

∴ The average weight = 225/3 = 75 kg

B. PERCENTAGE

Percentage, traditionally denoted by the symbol '%', is an important method of representing fractions. Mathematically, a percentage is a fraction with the denominator equal to 100.

$$\text{Percentage} = \frac{\text{Value Obtained}}{\text{Base value}} \times 100$$

Consider the following table. Examples 2 to 6 clarify the basic concepts of percentages. These examples use data from the table below which shows the sales (in million dollars) of the 'Mango Fruit Company' and the 'Apple Fruit Company' over four years.

	1999	2000	2001	2002
Mango Fruit Company	25	30	40	60
Apple Fruit Company	50	45	60	50

Example 2: Out of the total sales in 2000, what percentage of sales belongs to the Mango Fruit Company?

Solution: Total sales in 2000 = 30 + 45 = 75 million dollars.

75 is the base value. We are calculating the percentage of sales of the Mango Fruit Company out of the total sales of 75 for the year 2000.

From the table, the sales of the Mango Fruit Company in 2000 = 30 million dollars.

$$\therefore \text{Percentage of total sales in 2000 which belong to the Mango Fruit Company} = (30/75) \times 100 = 40\%$$

C. PERCENTAGE INCREASE OR DECREASE

Percentage increase or decrease is defined as the numerical increase or decrease in the value of a certain quantity when expressed as a percentage of the original value.

Percentage increase or decrease

$$= \frac{\text{Increase or decrease in value}}{\text{Original (or base) Value}} \times 100$$

Example 3: What is the percentage growth in the sales of the Apple Fruit Company between the years 1999 and 2001?

Solution: From the table, we have,

Sales of the Apple Fruit Company in 2001 = 60 million dollars

Sales of the Apple Fruit Company in 1999 = 50 million dollars

$$\therefore \text{Increase in sales} = 60 - 50 = 10 \text{ million dollars}$$

Base (or original) Value = 50

Note: The base value is always the value with which we compare the change. In this case, we compare the change in sales of the Apple Fruit Company between 1999 and 2001 to the sales in the year 1999.

Therefore, the sales in 1999 i.e. 50 becomes the base value.

$$\therefore \text{Percentage growth} = (10/50) \times 100 = 20\%$$

Example 4: What is the percentage change in the sales of the Apple Fruit Company in the year 2002?

Solution: A percentage change is always calculated with respect to a certain base value. In a case like this, where no base value is given, the percentage change is generally assumed to be with respect to the value of the previous time span (month, year, decade, etc). Hence, in this case the percentage decrease should be found with respect to the value of the sales in 2001.

From the table, we have,

Sales of the Apple Fruit Company in 2002 = 50

Sales of the Apple Fruit Company in 2001 = 60

Note: The change in sales from 2001 to 2002 is $50 - 60 = -10$ million dollars.

Since this change is negative, this percentage change is also known as percentage decrease.

$$\therefore \text{The percentage change} = (-10/60) \times 100 = -16.67\%$$

Example 5: What is the percentage decrease in the sales of the Apple Fruit Company in the year 2000?

Solution: If a question explicitly states percentage decrease, the “-” sign is not required. In such a case, the change is expressed in absolute terms.

For the Apple Fruit Company,

Sales in 1999 = 50

Sales in 2000 = 45

Change in sales = $45 - 50 = -5$ million dollars

Decrease in sales = $50 - 45 = 5$ million dollars

$$\therefore \text{Percentage decrease in sales} = (5/50) \times 100 = 10\%$$

While the percentage decrease in this case is 10%, the percentage change is -10%.

D. DIFFERENCE IN PERCENTAGE CHANGE

The difference in percentage change is defined as the difference between two percentage changes calculated in the usual way. Percentage change is not a percentage but an absolute number since a percentage calculation always has a base value.

Example 6: What is the difference in percentage change in the sales of the Mango Fruit Company and the Apple Fruit Company in the year 2001?

Solution: We skip the steps dealing with calculation of percentage changes, which should be understood by this point. After those steps, we get,

Percentage change in sales for the Mango Fruit Company in 2001 = 33.33%

Percentage change in sales for the Apple Fruit Company in 2001 = 33.33%

Difference in percentage change = |Percentage change for the Mango Fruit Company - Percentage change for the Apple Fruit Company| = 33.33 - 33.33 = 0

III. APPROXIMATIONS/SMART CALCULATIONS

As most competitive exams consist of multiple choice questions, we can (in most cases) find the correct answer using approximate calculations. However, approximating is an art which can be mastered only through practice. If one approximates without keeping the following tips and strategies in mind, one might end up with the wrong answer.

**IMPORTANT:**

- Round off as few numbers as possible. When you round off a number, try to round it off to the nearest integer.
- Try to round off all the numbers similarly. This means that, if two numbers are to be rounded off, try to increase both of them or decrease both of them. Try to avoid rounding off one of the numbers by decreasing it and the rest of the numbers by increasing them or vice versa unless doing so genuinely simplifies calculations.
- Once you have arrived at an answer, list down all the numbers that you approximated and identify one or two numbers which have the greatest effect on the final answer. Check that the approximation in these numbers is as small as possible.
- Depending on whether most of the variables have been increased or decreased in the approximation, we can say whether the final answer should be more or less than the approximate answer. This step helps to eliminate a bunch of options.
- Do not make approximation a habit. It should only be used when the data has numbers or decimals large enough to accommodate approximation.
- Approximation works well when the answer options are well spread out. When answer options are very close to each other, an incorrect approximation may lead to the wrong option being marked.
- Remember common approximations like

$$\pi \approx 3.14, \pi^2 \approx 10, \sqrt{2} \approx 1.4, \sqrt{3} \approx 1.73 \text{ and } \frac{1}{\pi} \approx 0.32$$

Example 7: What is the value of 15.22% of 179?

Solution: We have 15.22% of 179 = $\frac{15.22 \times 179}{100} = 27.2438$ (Exact value)

Alternatively (using approximation methods):

100% of 179 = 179

10% of 179 = 17.9 (Dividing both sides of the above equation by 10) ≈ 18 (approximated)

1% of 179 = 1.79 (Dividing both sides by 10) ≈ 1.8 (approximated)

0.1% of 179 = 0.179 ≈ 0.18

$\therefore 15.22\%$ of 179 = $(10\% + 5\% + 0.2\%)$ of 179
 $\approx 18 + [1\% \times 5] + [0.1\% \times 2]$
 $\approx 18 + 9 + 0.36$
 ≈ 27.36 , which is off by only 0.4% of the actual answer

We thus see how this approach helps us to reach a reasonably accurate answer quickly. Here, we can calculate even faster by some smart observation. Since the value of 10% of 179 is already known, 5% of 179 should be half of it. Hence, calculating $1\% \times 5$ is not required.

Note: The approximate value is more than the exact value because we have approximated 17.9 to 18 (which is a higher value).

The above problem shows the implementation of the percentage approximation technique, which can be used in any problem involving percentages.

Example 8: What approximate percentage of 581 is 238?

(a) 44%

(b) 42%

(c) 47%

(d) 51%

Solution: We use the percentage approximation technique introduced above.

We know that 50% of 581 \approx 290. 238 is less than 290 and hence will be less than 50% of 581.

\therefore Option (d) can be eliminated.

100% of 581 = 581 \approx 580 (approximated)

10% of 581 \approx 58

1% of 581 = 5.81 \approx 5.8

\therefore 40% of 581 $\approx 4 \times 58 = 232$

Now, consider the answer options. 42% of 581 = $40\% + 2\% \approx 232 + 11.6 \approx 243.6$

As 42% itself is more than 238, we can say that 44% and 47% are much greater than 238.

Hence, 243.6 is the value closest to 238

\therefore Options (a) and (c) can also be eliminated.

Hence, **option b**.

Note: 232 is nearly 41% of 581 ($232 + 5.8 = 237.8$). But 41% is not listed in any of the answer options. From this example, we can conclude that it is not necessary for the answer options to be exact values. They may themselves be approximated values. This is often the case where data is expressed graphically and the exact value of a data point may be impossible to find.

A. PERCENTAGES EXPRESSED AS FRACTIONS

Since a percentage is nothing but a fraction with the denominator equal to 100, we can express any percentage as a fraction. It is almost always more convenient to work with fractions in equations, and so we should generally convert percentages in questions to fractions. In certain cases, fractions may be easier to work with. A decision on whether to use percentages or fractions should be done on a case to case basis.

For Example, Consider the percentage 50%. We are required to convert it to a fraction. We proceed as below.

$$50\% = \frac{50}{100} = \frac{1}{2}$$

So, if we say $1/2$, it is equivalent to saying 50%.

A few typical fraction-to-percentage conversions follow. Going through them should give one the confidence needed to do quick conversions. Note that all the examples deal with the reciprocals of natural numbers. This is intentional; knowing the reciprocals of the natural numbers can be very useful in doing calculations faster.

While this list is not exhaustive, it covers most of the commonly used fractions and their conversion in terms of percentages.

$\frac{1}{2} = 0.5$ or 50%	$\frac{1}{3} = 0.33$ or 33.33%	$\frac{1}{4} = 0.25$ or 25%
$\frac{1}{5} = 0.2$ or 20%	$\frac{1}{6} = 0.1667$ or 16.67%	$\frac{1}{7} = 0.1428$ or 14.28%
$\frac{1}{8} = 0.125$ or 12.5%	$\frac{1}{9} = 0.1111$ or 11.11%	$\frac{1}{10} = 0.1$ or 10%
$\frac{1}{11} = 0.0909$ or 9.09%	$\frac{1}{13} = 0.0769$ or 7.69%	$\frac{1}{17} = 0.0588$ or 5.88%

Example 9: What is 14.28% of 49?

Solution: 14.28% of 49 = $\frac{14.28 \times 49}{100} = 7$

Alternatively,

If we remember that 14.28% is equivalent to $\frac{1}{7}$, we can save a lot of time by immediately replacing 14.28% by $\frac{1}{7}$.

\therefore 14.28% of 49 = $49 \times \frac{1}{7} = 7$

Example 10: What is 23% of 247?

Solution: We can say that 23% is approximately equal to $(7.67 \times 3)\%$.

Looking at our table, we find that 7.67% is nothing but $\frac{1}{13}$.

\therefore 23% = $\frac{3}{13}$

\therefore 23% of 247 = $247 \times (\frac{3}{13}) = 57$

We can see from the examples above how the inter conversion between fractions and percentages can help simplify calculations.

IV. TABLES

In a typical table, data is organized in the form of rows and columns with the headings listing the parameter that the entire row or column represents. There are a variety of problems based on different types of tables.

Tables can be of the observational type where a table containing a large amount of data is given and the questions are based on certain values of parameters that simply have to be filtered from the table. These questions are essentially counting-based questions and take a form similar to "In how many of the given years was the sales more than x dollars?" and so on. Here, you just need to count the number of years where the condition given in the question is satisfied and mark the answer accordingly.

The other types of table-related questions are more analysis-based. Here the data may not be as voluminous but some working may be required to fill up the table or to analyse the table. These questions generally involve mathematical concepts such as ratio-proportion, percentage change and growth rates.

Example 11:

	2007	2008	2009	2010	2011
HUL	325	450	475	400	525
P & G	450	650	500	500	525
ITC	475	525	525	550	500

The table above shows the sales revenue for three FMCG companies (for a particular product category) over a period of 5 years. All the figures are in Rs. Crores.

Question 1: For how many years did P & G have the highest yearly sales in this category, among the given companies?

Solution: Among the given companies, P & G had the highest sales in 2008 (Rs. 650 crores) and in 2011 (Rs. 525 crores).

Thus, P & G had the highest yearly sales in the given category for 2 years.

Question 2: The combined sales of HUL and ITC in 2009 were what percentage of the sales of P & G in 2009?

Solution: The sales of HUL and ITC in 2009 were Rs. 475 crores and 525 crores respectively.

Therefore, their combined sales were Rs. 1000 crores.

Sales of P & G in 2009 were Rs. 500 crores.

$$\therefore \text{Required percentage} = \frac{1000}{500} \times 100 = 200\%$$

Question 3: What were the average sales of ITC over the given period?

Solution: The average sales of ITC over the given period are

$$\frac{475 + 525 + 525 + 550 + 500}{5} = \text{Rs. 515 crores}$$

Example 12: Consider the following information regarding the performances of a class of 1000 students in four different tests:

Tests	I	II	III	IV
Average marks	60	60	70	80
Range of marks	30 to 90	45 to 75	20 to 100	0 to 100

If a student scores 74 marks in each of the four tests, in which one of the following tests is her performance the best comparatively? (CSAT 2012)

- (a) Test I (b) Test II (c) Test III (d) Test IV

Solution: A student scored 74 marks. We can assume that this will be his best performance if he scored 74 marks in the test which has least highest marks and the least higher range of marks.

Clearly test II satisfies these criteria.

Hence, **option b.**

V. CASELETS

As their name suggests, caselets are **Small Cases**. A 'case' is nothing but a detailed description of a situation that is encountered almost everywhere in the business world, as the most convenient method to teach a person about a situation. Caselets are, thus, a snapshot of a situation in words.

Caselets consist of both Factual data and Numerical data interspersed in an unorganised fashion. In its natural form, it is inconvenient to derive useful information from it. To make it usable, a caselet has to be filtered and converted into a form that is more readable and useful, like a table.

Example 13: 2200 students of a college decided to go for a trip. When asked about preferences, 60% of the students said "Yes". Boys formed one-fourth of this group. 80 students said that were "Undecided". Within this lot, boys and girls were in the ratio 1:3. The number of girls who said "No" for the picnic was four times the number of girls who were "Undecided".

(Based on UPSC Prelims 2011):

Question 1: How many girls said “yes” for the picnic?

Solution: First, make a table as shown below:

	Boys	Girls	Total
Yes			
No			
Undecided			
Total			

Total students = 2200

Number of students with “Yes” = $0.6 \times 2200 = 1320$

Number of boys with “Yes” = $0.25 \times 1320 = 330$

\therefore Number of girls with “Yes” = $1320 - 330 = 990$

80 students were “Undecided”, out of which boys and girls were in the ratio 1 : 3

Thus, number of boys and girls who were undecided is 20 and 60 respectively.

Number of students who said “No” = $2200 - 1320 - 80 = 800$

Number of girls who said “No” = $4 \times$ number of girls who were undecided = $4 \times 60 = 240$

\therefore Total number of girls = $990 + 240 + 60 = 1290$

\therefore Total number of boys = $2200 - 1290 = 910$

\therefore Number of boys who said “No” = $910 - 330 - 20 = 560$

Thus, the completed table is as shown below:

	Boys	Girls	Total
Yes	330	990	1320
No	560	240	800
Undecided	20	60	80
Total	910	1290	2200

Thus, number of girls who said “Yes” = 990

Question 2: How many boys said “No” for the picnic?

Solution: Consider the final table obtained in the solution to the first question.

560 boys said “No” for the picnic.

Question 3: How many girls were asked about the picnic?

Solution: Consider the final table obtained in the solution to the first question.

A total of 1290 girls were asked about the picnic.

VI. CHARTS AND GRAPHS

A graph is a pictorial form of representing data. It is easier to observe trends and patterns using graphical data compared to tabular data. Also, a graph makes it convenient to depict a very large volume of data compared to a table which may become unwieldy.

A basic graph consists of two parameters; ‘Variables’ and ‘Axes’. Since a basic graph is two dimensional in nature it has two axes, the X and Y axes, on which the quantities to be measured and analysed are plotted. These quantities are also known as the variables. A graph depicts the value and behaviour of one variable with respect to the other variables. The variables in a graph are known as the dependent and independent variable. The data in a graph/chart can be represented in a tabular format and vice versa. The common methods of representation are:

1. Simple Line Graph
2. Stacked Line Graph
3. Simple Bar Graph
4. Cumulative Bar Graph

5. Percentage Bar Graph

6. Pie Chart

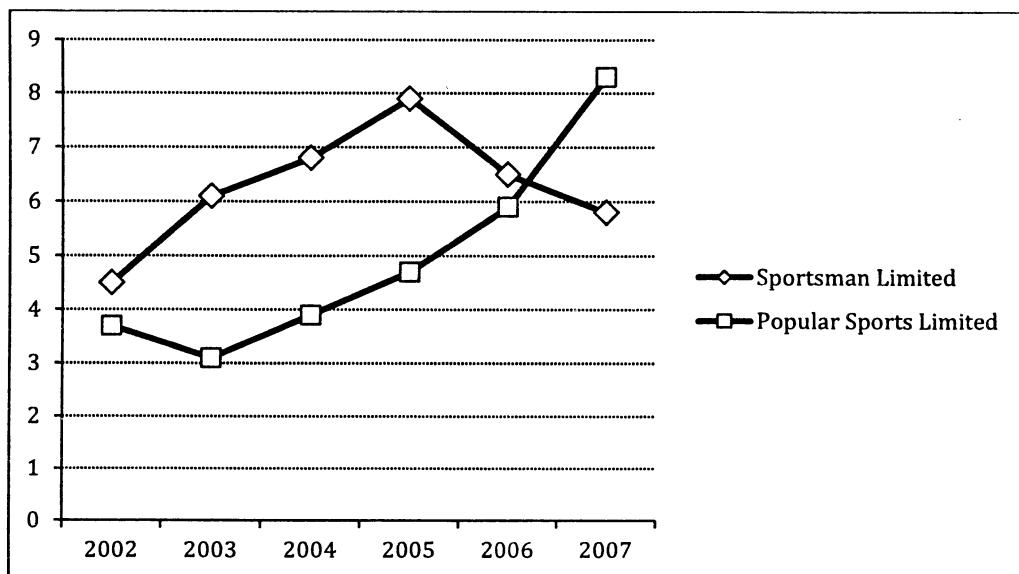
A line graph involves plotting of two variables with respect to each other on a two-dimensional scale. In a line graph, the variable that is used to compare the various parameters is generally plotted on the x -axis and is known as the independent variable. The actual value of each variable is known as the dependent variable and is plotted on the y -axis. The values plotted for one variable are joined by line segments, thereby giving this graph the name "Line Graph". Consider the tabular data given below. Use this table to represent the same data in various formats to gain a better understanding of data representation and appreciate the unique features of each form of data representation.

The table below provides the sales revenues of two sports goods manufacturing companies. All the figures are in INR million.

	2002	2003	2004	2005	2006	2007
Sportsman Limited	4.5	6.1	6.8	7.9	6.5	5.8
Popular Sports Limited	3.7	3.1	3.9	4.7	5.9	8.3

A. SIMPLE LINE GRAPH

Since the revenues of two companies are to be compared for each year, the base for comparison i.e. the year range from 2002 to 2007 is plotted on the x -axis. This is the independent variable as its value is not dependent on the value of the revenue. For both the companies, the value of the revenue is plotted as a point on the y -axis against each year. Since the point to be plotted for revenue for a particular year depends on the year for which it is being done, the revenue becomes the dependent variable. Once the values for a particular company are plotted, the points are joined by straight line segments. This representation of the tabular data is known as a "Simple Line Graph".



Features of the graph: The data is represented using dots or any other geometrical symbols on a scale represented on the x -axis or the horizontal axis, and the y -axis or the vertical axis.

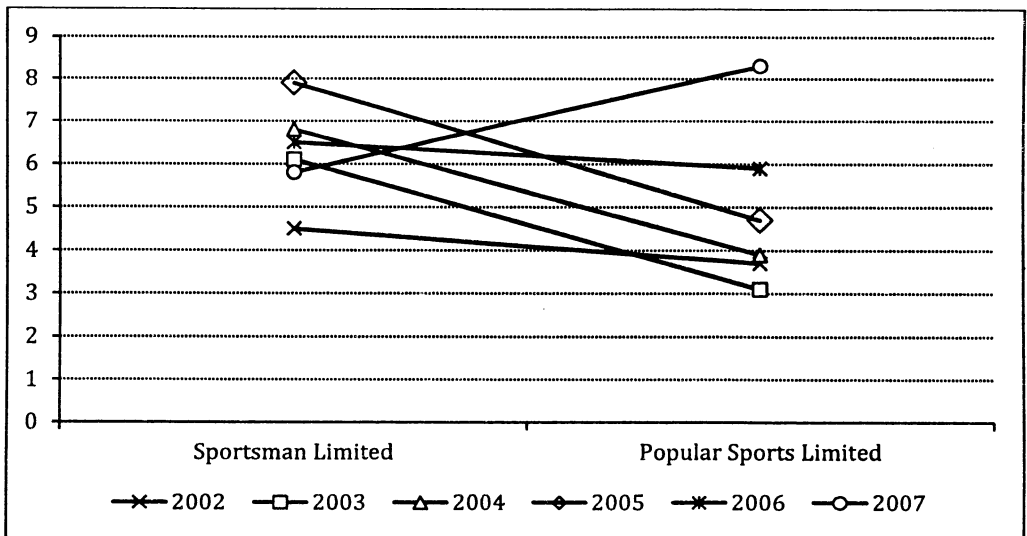
The labels associate a variable with the manner of its representation on the graph. In this case, the "diamonds" represent the sales revenue of Sportsman Limited and the "squares" represent the sales revenue of Popular Sports Limited.

Advantages: For the company Sportsman Limited, the line connecting 2002 and 2003 has the greatest slope (defined as ‘the angle which the line makes with the positive x-axis’). This means that the sales increase for this company was the maximum in this period. Similarly, for the company Popular Sports Limited, the greatest increase is between the years 2006 and 2007. Observe that the revenue for Popular Sports Limited overtook the revenue for Sportsman Limited only after 2006.

The biggest advantage of a line graph over tabular data is that it allows one to observe trends more easily compared to a table. The conclusions above were arrived at without any calculations. Even voluminous data can be handled with ease using a graph. For instance, if the revenues for 10 companies were to be compared instead of 2, or if the revenue for these 2 companies was to be compared over a period of 20 years instead of 6, it would still be easy to read the data from the simple line graph compared to the table.

Limitations: The biggest limitation of a line graph is that it may not be possible to find the exact value of the data point in a lot of cases. For instance, consider this question “What is the sales revenue of Popular Sports Limited in 2003?” From the table, the required value is INR 3.1 million. However, from the line graph, it is difficult to say whether the value is 3.1, 3.2, 3.15 or 3.1444 and so on. Multiple readings of the same data may yield multiple answers thereby leading to large differences in analysis.

Theoretically, the line graph can also be formed by representing the companies, instead of the years, on the x-axis. In this case, the graph would look as shown below:



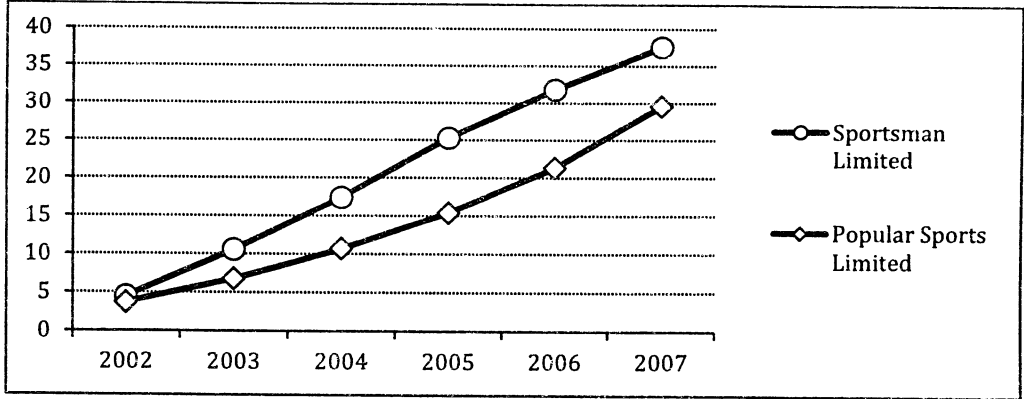
Representing the line graph in this manner is useful if the number of years for which the data is to be compared for the two companies is very large. However, as is evident above, it is not easy to understand the trend of the revenue for a particular company over the years. For instance, if one wants to find the year in which the revenues for Sportsman Limited started decreasing or the year in which the revenues for Popular Sports Limited started increasing, it is easier to do so if the years are represented on the x-axis.

B. STACKED LINE GRAPH

A Stacked Line Graph is also called a Band Diagram. A band diagram is identical in appearance to a Line Graph. It differs from a Line Graph in that the values of the variable (generally on the y-axis) are cumulative. This means that each value plotted on the cumulative axis is the sum of all values of the cumulative variable up to and including that point.

Band diagrams are useful when the total value of a variable over a period of time (or for a range of time) is required. For instance, if one needs to calculate the total sales for a company up to a given year, it is easier to do so using a Band diagram compared to a simple line graph.

The tabular data given earlier can be represented as a band diagram as shown below:

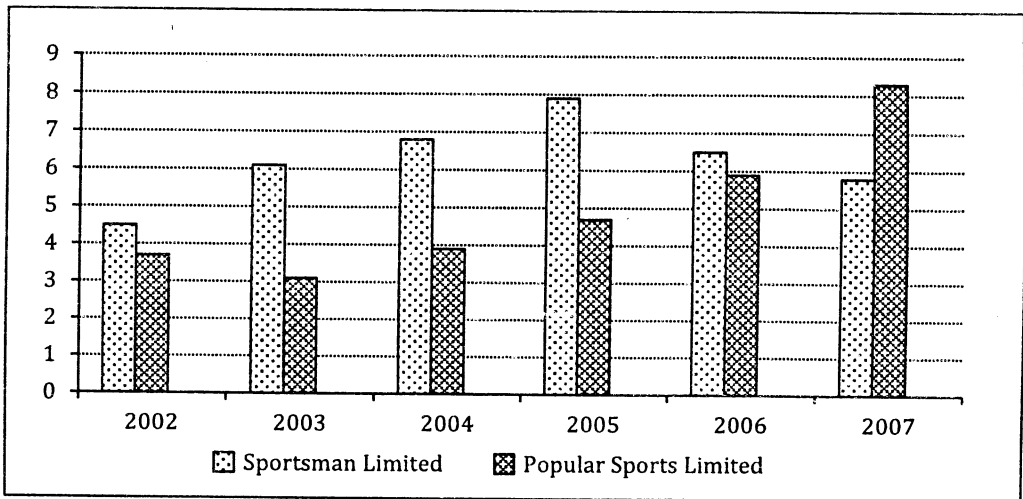


A Band Diagram gives the cumulative values for every successive year. Thus, if we were to calculate the sales revenue of Sportsman Limited in 2005, we would have to deduct the revenue shown up to 2005 from the revenue shown up to 2006. A Band Diagram usually either increases or decreases throughout due to its cumulative nature. However, if the data points consist of both positive and negative values, this may not be the case. For instance, if a band diagram showing growth rates is drawn, it may decrease if a few values of positive growth rate are followed by a negative growth rate.

C. SIMPLE BAR GRAPH

A Bar Graph can be visualised as a Line Graph where each plotted value is joined to the x-axis. Whenever data is represented as a bar parallel to one of the axes such that the height (or length, depending on which axis the bar is parallel to) of the bar is equal to the value of the data point, we get a bar graph. A Simple Bar Graph is one where the value of each data point is represented as a separate bar.

A simple bar graph for the tabular data given earlier is as shown:



Advantages: A bar graph is useful when multiple dependent variables are to be compared with respect to a single independent variable. For instance, if a company wanted to compare the profits

from 6 of its products in various years, the revenue for each product in 2002 would be plotted as a bar next to each other. This would enable the company to find out which product yields the maximum and minimum revenue, or which product is consistently the highest revenue earner and so on.

Like a line graph, a bar graph is better suited to analysing a trend or pattern compared to tabular data and can handle large volumes of data easily.

Compared to a line graph, a bar graph is easier to visualise. For instance, if the production figures of a company for 5 years are given, it is easier to visualise the production for a given year as a stack of products kept one above the other up to the value of the production rather than just a point on the line graph.

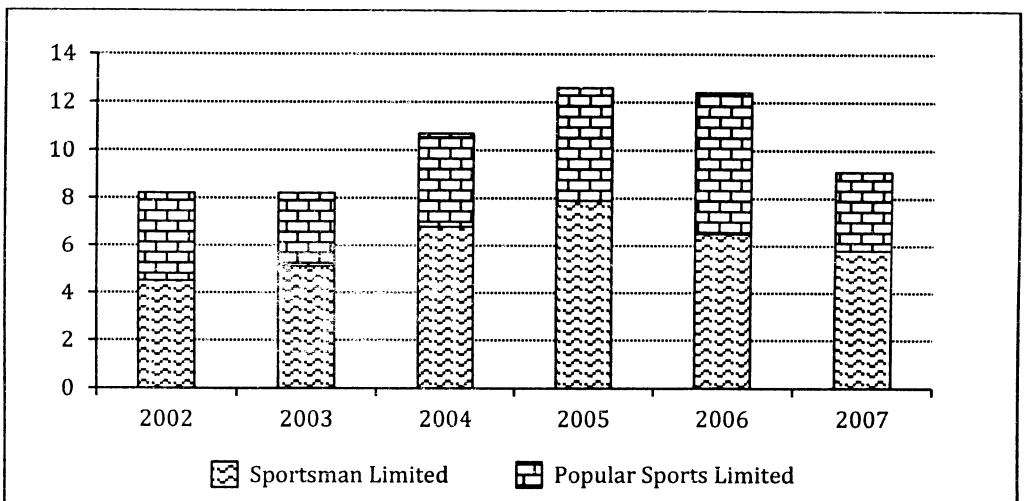
Limitations: For very large volumes, a bar graph may tend to get cluttered. Hence, in cases with a large number of data points, a line graph may be better than a bar graph.

Like a line graph, it may not always be possible to find the exact value of the data point in a bar graph, thereby reducing the accuracy of the analysis.

A bar graph is better to work with if a trend or pattern is to be observed over a period of time.

D. CUMULATIVE BAR GRAPH

In a cumulative bar graph, the individual bars for a certain independent variable are stacked on top of each other. The tabular data seen earlier is represented as a cumulative bar graph below:



Consider the year 2003. The revenue for Sportsman Limited and Popular Sports is 6.1 and 3.1 million respectively. Hence, in the cumulative bar chart, the revenue for Popular Sports Limited is drawn from the top of the bar for Sportsman Limited. Hence, the bar for Popular starts at 6.1 million and ends at 9.2 million. The revenues for each year are plotted in the same manner.

Advantages: Like a Band Diagram, a cumulative bar graph provides the total data for a given independent variable. In the figure above, the top of the cumulative bar for a year indicates the total sales revenue of Popular Sports Limited and Sportsman Limited for that year. If these were the only two sports goods manufacturing companies, one could say that the total value of the sports goods manufacturing industry is represented by the top end of the bar.

Cumulative Bar Graphs are very useful when it comes to voluminous data (for example, data involving a large number of companies), as the number of side-by-side bars is reduced.

A cumulative bar graph is also useful if the number of points can be classified into categories and grouped together. For instance, if a company XYZ manufactures 4 soap brands, 3 shampoo brands

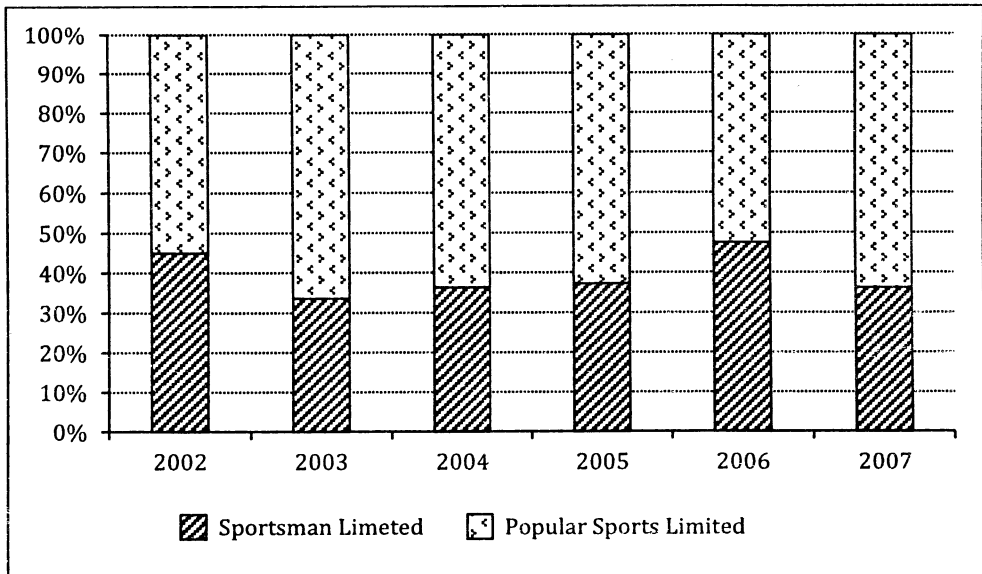
and 2 oil brands, it easier to compare the revenues earned from soaps with respect to the revenue from shampoos and oils using a cumulative bar graph compared to a standard bar graph.

Limitations: Apart from the inherent limitations of a bar graph, a cumulative bar graph also involves greater calculations. If there were 5 companies represented above, finding the revenue figures for individual companies would have been much more tedious and error prone.

E. PERCENTAGE BAR GRAPH

A Percentage Bar Graph is a cumulative graph where the maximum value of each category is converted to 100.

The values in this graph are represented as a percentage of the total category value. For example, in the diagram below, (again based on the tabular data) the sales revenue of Sportsman Limited in 2002 was approximately 54.8% of the total Revenue for both the companies combined.



Advantages: This type of graph saves a lot of time if the contribution of various categories to a whole is to be found. For instance, for the soap and shampoo manufacturing company XYZ mentioned earlier, the contribution of the soap division compared to the shampoo division and the oil division can be found easily using this graph.

Limitations: Since the data points are expressed in terms of percentages, calculating the actual values may become a tedious process, especially if the percentages are non-standard values such as 61.89% or 89.22% and so on. As can be seen above, the absolute value for any data point cannot be obtained till the total value of each of the bars is mentioned. This is done by either giving the values in a table or mentioning the values at the top of the bar.

F. PIE CHART

A Pie Chart is a very commonly used method to depict the fractional contribution of parts to a whole (in other words, the breakup of a whole into multiple categories). In a Pie Chart, the fractional value of each of the parts is represented as a sector of the circle. The area of the portion is proportional to the fractional value of the part.

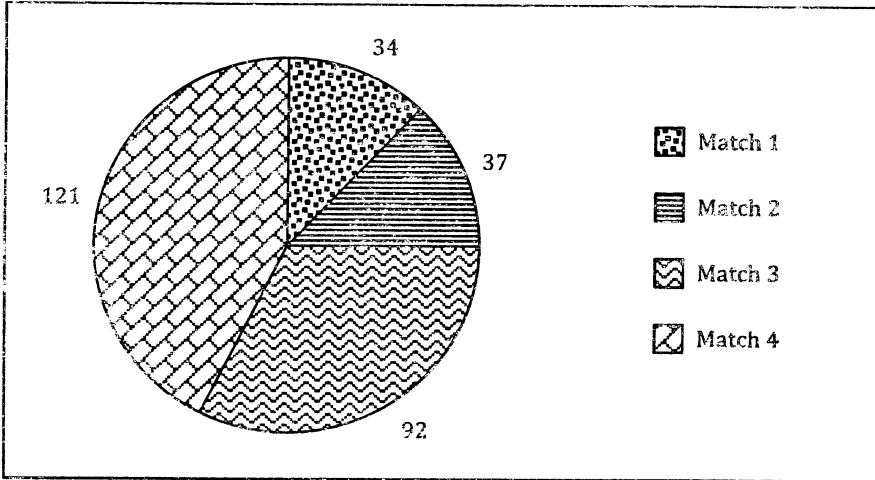
Data in a Pie Chart can be represented in the following three ways:

i. Data represented as an absolute value:

Consider the scores made by batsman Dhoni against Australia in 4 matches:

Match 1 – 34
 Match 2 – 37
 Match 3 – 92
 Match 4 – 121
 Total runs – 284

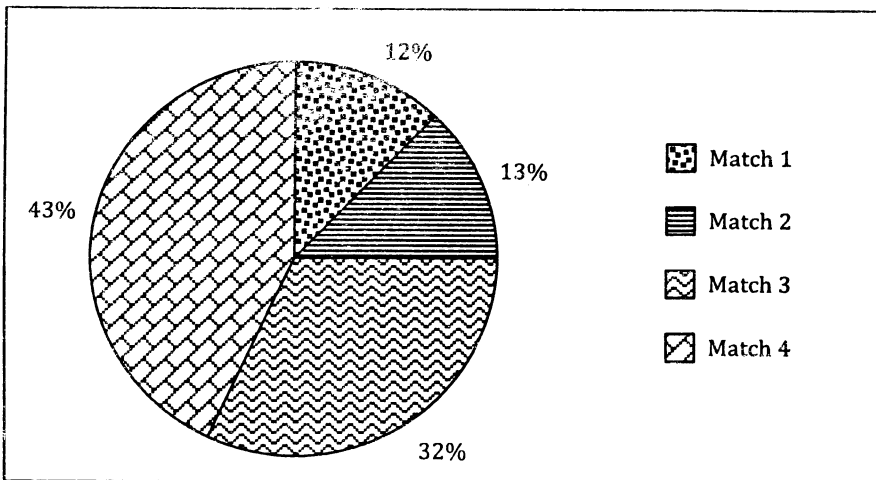
The pie chart below represents Dhoni's scores in his four matches against Australia.



ii. Data represented as a percentage:

First find the total runs scored by the batsman and then find out what percentage of the total runs were scored by him in each match. After this, we convert the information into a pie chart.

When data is expressed in terms of a percentage, the sum of the values in the pie chart is always equal to 100.



In the above case, the number of runs scored in each match can be calculated only if the total number of runs scored in all the matches taken together is known. Hence, the total value of the data should be known to read a pie chart expressed in terms of percentages. (In some special cases, the total value may not be required.)

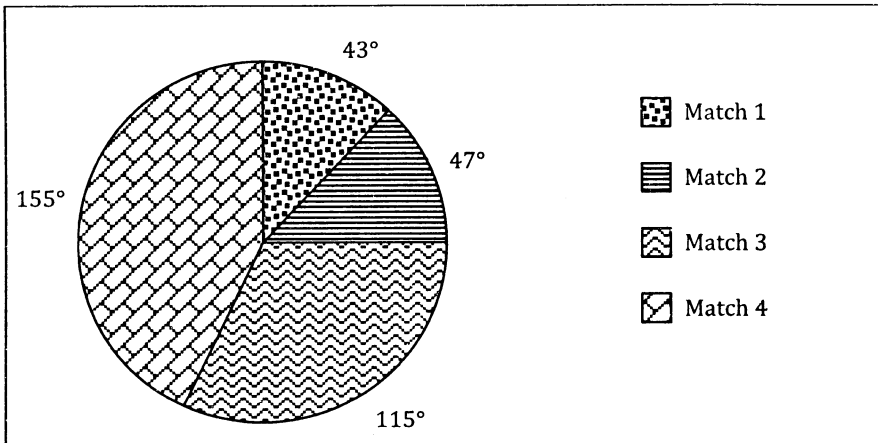
iii. Data represented in degrees:

Here, the data points are represented in terms of degrees of a circle. The mathematical basis for this is as follows:

A complete circle (in a pie chart) represents 100%. The same circle also represents 360° geometrically. Thus, 100% = 360°

∴ 1% = 3.6°

Now convert each percentage into degrees using the above relationship. Hence, we get the following figure.



Since this method of representation is also based on percentages, the actual value of each data point can be found only if the total value of the entire data is known.

For instance, if Dhoni’s score in Match 3 is required, given that his total score in all matches combined is 288, it can be found either using percentages or degrees.

Match 3 is represented as 115° out of 360°.

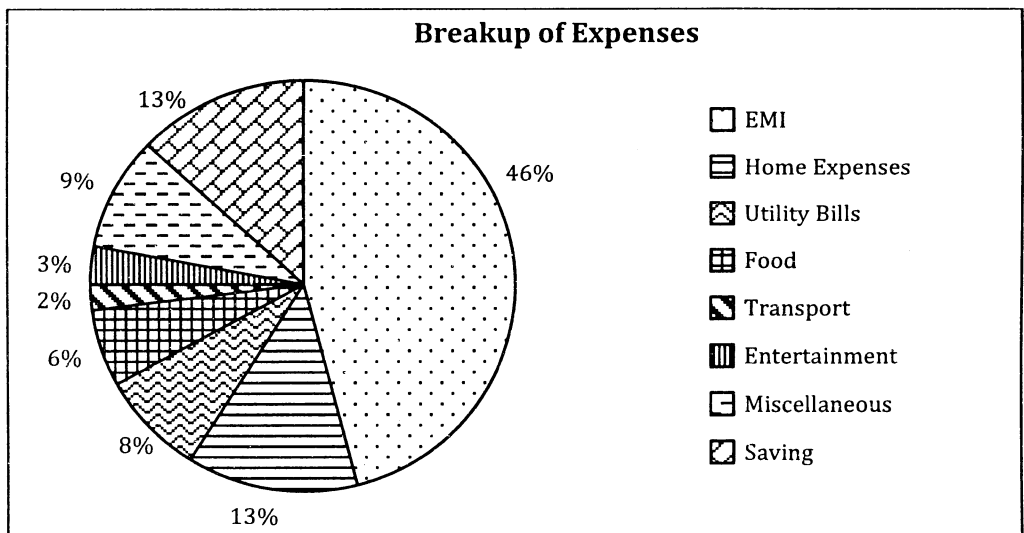
360° is equivalent to his total runs i.e. 288 runs.

∴ Dhoni’s score in Match 3 = $(288/360) \times 115^\circ = 92$ runs

Also, in terms of percentages, the score in Match 3 corresponds to 32% of the total. Hence, if 100% corresponds to 288 runs, we need to find the corresponding value for 32%

∴ Dhoni’s score in Match 3 = $(288/100) \times 32 = 92.16 = 92$ runs

Example 14:



The pie chart above shows the expense breakup of a person out of his monthly salary, which is Rs. 80,000. Answer the questions that follow.

Question 1: What is the difference between the amount that the person pays under “Utility Bills” and the amount that he pays under “Miscellaneous”?

Solution: Observe that “Utility Bills” and “Miscellaneous” are 8% and 9% respectively of the person’s monthly salary.

Therefore, amount paid as “Utility Bills” = $0.08 \times 80000 = \text{Rs. } 6,400$

And, amount paid as “Miscellaneous” = $0.09 \times 80000 = \text{Rs. } 7,200$

\therefore Difference = $7200 - 6400 = \text{Rs. } 800$

Note: It is not necessary to calculate the exact values. The difference between the amount paid in these two categories is simply $(9 - 8)\%$ i.e. 1% of Rs. 80,000 i.e. Rs. 800.

Question 2: If EMI, Home Expenses and Utility Bills are considered as “Mandatory Expenses”, how much does the person as “Mandatory Expenses” every month?

Solution: Since EMI, Home Expenses and Utility Bills are “Mandatory Expenses”, the category of “Mandatory Expenses” forms $46 + 13 + 8 = 77\%$ of the total monthly expenses.

\therefore Required expenditure = $0.77 \times 80000 = \text{Rs. } 61,600$.

Question 3: Assuming that the monthly income of this person remains constant; and if the person stops all entertainment expenditure and saves this amount as well, what is the new savings amount of the person?

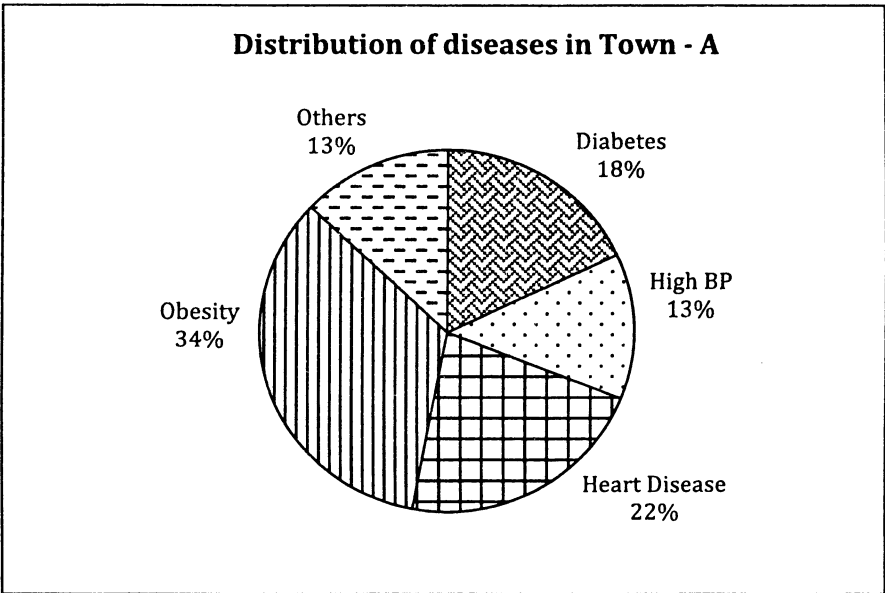
Solution: Since the monthly income remains constant, the monthly income of the person is still Rs. 80,000.

Since the person does not spend anything on entertainment and saves this money as well, savings now form $(13 + 3)\%$ i.e. 16% of his monthly income.

\therefore New savings = $0.16 \times 80000 = \text{Rs. } 12,800$

Example 15: Directions for the following 2 (two) items:

The following pie charts show the break-up of disease categories recorded in the patients from two towns, Town A and Town B. Pie charts plot the disease categories as percentage of the total number of patients. Based on these, answer the two items that follow the charts. (CSAT 2011)



Question 1: Which of the two towns has a higher number of persons with Diabetes?

- (a) Town A (b) Town B (c) Same in Town A and Town B (d) No inference can be drawn

Solution: The given pie charts give disease categories as percentage of total number of patients. We do not know the total number of patients in the two towns. So it is not possible to find the number of diabetes patients in towns A and B.

Hence, **option d**.

Question 2: What can we say about persons with more than one disease from these graphs?

- (a) There are likely to be persons with more than one disease in Town A.
- (b) There are likely to be persons with more than one disease in Town B.
- (c) There are likely to be persons with more than one disease in both Towns A and B.
- (d) No inference can be drawn.

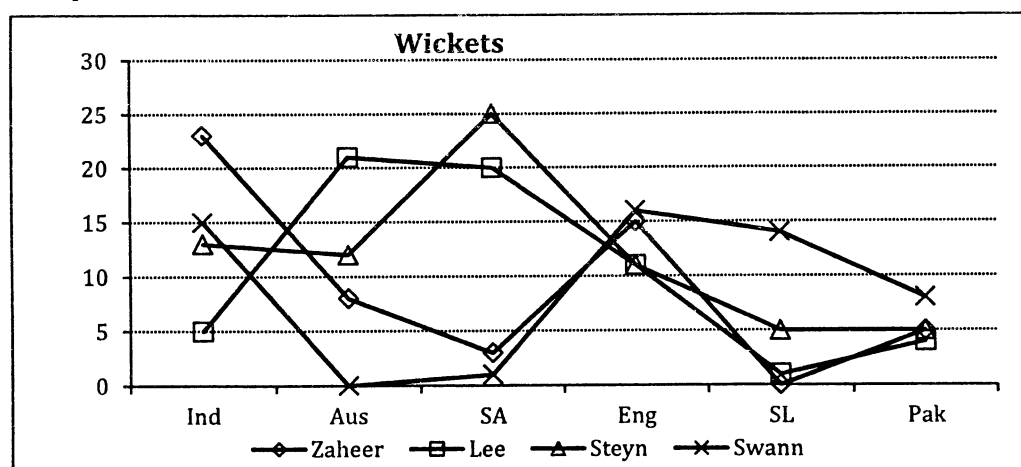
Solution: Since, the numbers in town B add up to more than 100%, we can conclude that Town B definitely has patients with more than one disease.

In town A, the numbers add up to exactly 100%.

Hence, no one in town A has more than one disease.

Hence, **option b**.

Example 16:



The line graph above shows the number of wickets taken by 4 different bowlers in 6 different countries in the previous year.

Question 1: Who took the most wickets in the previous year?

Solution: Total wickets taken by each bowler are:

$$\text{Zaheer} = 23 + 8 + 3 + 15 + 0 + 5 = 54$$

$$\text{Lee} = 5 + 21 + 20 + 11 + 1 + 4 = 62$$

$$\text{Steyn} = 13 + 12 + 25 + 11 + 5 + 5 = 71$$

$$\text{Swann} = 15 + 0 + 1 + 16 + 14 + 8 = 54$$

Thus, Steyn took the most wickets in the previous year.

Question 2: If Ind, Pak and SL are considered the subcontinent, which bowler took the least wickets in the subcontinent?

Solution: Total wickets taken by each bowler in the subcontinent are:

$$\text{Zaheer} = 23 + 0 + 5 = 28$$

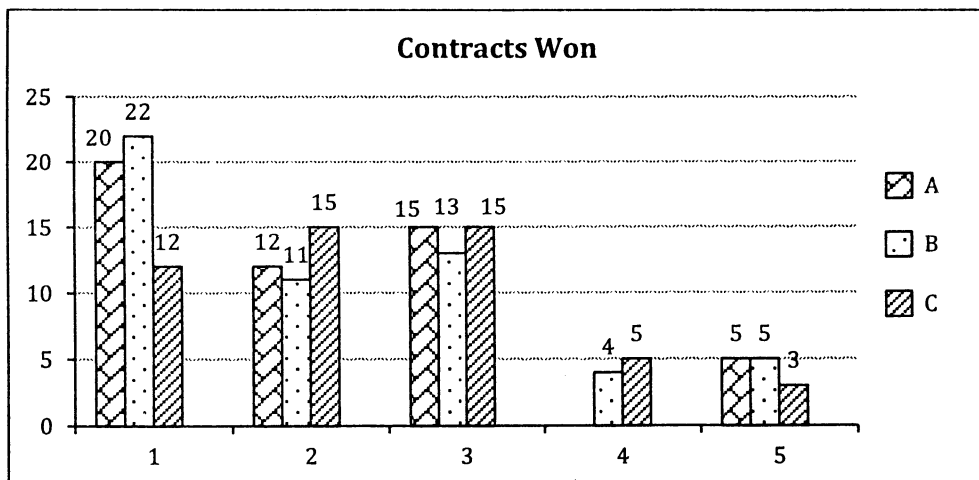
$$\text{Lee} = 5 + 1 + 4 = 10$$

$$\text{Steyn} = 13 + 5 + 5 = 23$$

$$\text{Swann} = 15 + 14 + 8 = 37$$

Thus, Lee took the least wickets in the subcontinent.

Example 17:



The bar graph below shows the number of contracts won by 3 companies A, B and C in 5 different years.

Question 1: How many times did the number of contracts won by company A exceed the average number of contracts won by it over the given period?

Solution: Company A won 20, 12, 15, 0 and 5 contracts in years 1, 2, 3, 4 and 5 respectively. Thus, average number of contracts won by company A over the given period is

$$\frac{20 + 12 + 15 + 0 + 5}{5} = \frac{52}{5} = 10.4$$

Thus, company A won more contracts than its average in 3 years (years 1, 2 and 3 where it won 20, 12 and 15 contracts respectively).

Question 2: How many contracts were given out in years 2 and 3 together; assuming that all the contracts that were given out in these years were won by companies A, B and C only?

Solution: Assuming that all the contracts that were given out in these years were won by companies A, B and C only, the total number of contracts given out is:

$$12 + 11 + 15 + 15 + 13 + 15 = 81$$

TEST 1

Answer questions 1 to 5 based on the following information: The table below shows the number of buildings constructed in various cities in a state over a few years.

Years City	2005	2006	2007	2008	2009	2010
Allahabad	1200	1350	1410	1492	1520	1600
Chennai	1020	1050	1065	1088	1135	1170
Kolkata	1340	1365	1400	1425	1475	1520
Mumbai	1824	1950	2045	2060	2180	2350
Bangalore	1642	1752	1840	1965	2010	2254
Pune	1610	1740	1810	1960	2064	2210

1. The total number of buildings constructed across all the given cities in 2006 from what approximate percentage of the total number of buildings constructed across the same cities in 2009?

- (a) 78.66% (b) 82.55% (c) 85.44% (d) 88.66% (e) 90.44%

2. What is the difference between the total number of buildings constructed in 2005 and 2008 across all cities?
 (a) 1354 (b) 1294 (c) 1454 (d) 1346 (e) 1406
3. What is the total number of buildings constructed across all 6 cities over the given period?
 (a) 42567 (b) 45787 (c) 47787 (d) 46787 (e) None of the above
4. What is the percent increase in the number of buildings constructed in Mumbai in 2007 over the previous year?
 (a) 3.35% (b) 3.87% (c) 4% (d) 4.27% (e) 4.87%
5. What is the ratio of the total number of buildings constructed in Allahabad to the total number of buildings constructed in Pune over the given period?
 (a) 542 / 729 (b) 249 / 328 (c) 221 / 296 (d) 1027 / 1485 (e) None of the above

Answer questions 6 to 10 based on the following information:

School	A		B		C		D		E	
	App.	Pass.	App.	Pass.	App.	Pass.	App.	Pass.	App.	Pass.
2006	415	221	278	264	745	448	624	320	684	368
2007	384	205	290	270	712	426	674	380	625	354
2008	325	286	312	272	768	321	690	455	614	286
2009	430	336	325	276	815	756	716	648	576	272
2010	412	390	345	290	845	714	728	596	542	354
2011	365	340	334	305	932	628	744	686	522	104

6. What is the ratio of the total number of students who appeared for the examination in 2008 to the total number of students who appeared for the examination in 2009?
 (a) 291 : 298 (b) 301 : 318 (c) 673 : 715 (d) 381 : 398 (e) None of these
7. During 2010, which school had the highest percentage of students that passed the examination?
 (a) A (b) B (c) C (d) D (e) E
8. For school C, which year showed the lowest passing percentage?
 (a) 2006 (b) 2007 (c) 2008 (d) 2011 (e) None of these
9. What was the overall percentage of number of students who passed in terms of the number of students who appeared from all the schools together in 2011?
 (a) 61.71% (b) 78.41% (c) 66.23% (d) 71.21% (e) None of these
10. What is the approximate ratio of the average number of students passed from schools B and D respectively over the given period?
 (a) 71 : 85 (b) 28 : 51 (c) 15 : 33 (d) 46 : 59 (e) None of these

TEST 2

Answer questions 11 to 14 based on the following information: An institute offers a degree with specializations available in Science, Commerce and Arts. 35% of the total students in the institute are girls. The number of boys studying Commerce in the institute is 416 which is 40% of the total number of boys in the institute. 45% of the girls in the institute study Arts. The number of boys and girls studying Commerce is in the ratio of 4 : 1. 25% of the boys in the institute study Science.

11. How many girls study Science in this institute?
(a) 104 (b) 194 (c) 252 (d) 354 (e) None of these
12. The number of girls studying Commerce is what percent of the number of boys studying Science?
(a) 15% (b) 25% (c) 30% (d) 40% (e) 45%
13. What is the ratio of the number of boys studying Arts to the number of girls studying Arts?
(a) 3 : 2 (b) 91 : 73 (c) 13 : 9 (d) 123 : 109 (e) 61 : 54
14. What is the total number of students in the institute?
(a) 1600 (b) 1540 (c) 1640 (d) 1800 (e) 1720

Answer questions 15 to 18 based on the following information: In a city XYZ, all the people read some newspaper every day. 5478 people like to read only Times of India. 1420 people like to read only Economic Times and 2684 people like to read only Hindustan Times. 2060 people like to read only DNA and 4686 people like to read only The Hindu. 4062 people like to read only Employment News. 2466 people like to read Times of India as well as Employment News. 1540 people like to read Economic Times as well as Employment News. 3542 people like to read Times of India as well as The Hindu.

15. The total number of people reading Times of India forms what percent of the total number of people reading a newspaper?
(a) 32% (b) 41% (c) 45% (d) 22% (e) 36%
16. The total number of people reading Employment News forms what percent of the total number of people reading some newspaper?
(a) 19% (b) 24% (c) 29% (d) 34% (e) 39%
17. The total number of people reading only Hindustan Times and only DNA together forms what percent of the total number of people reading some newspaper?
(a) 15% (b) 11% (c) 23% (d) 17% (e) 19%
18. The total number of people reading only The Hindu forms what percent of the total number of people reading some newspaper?
(a) 7% (b) 9% (c) 13% (d) 15% (e) 17%

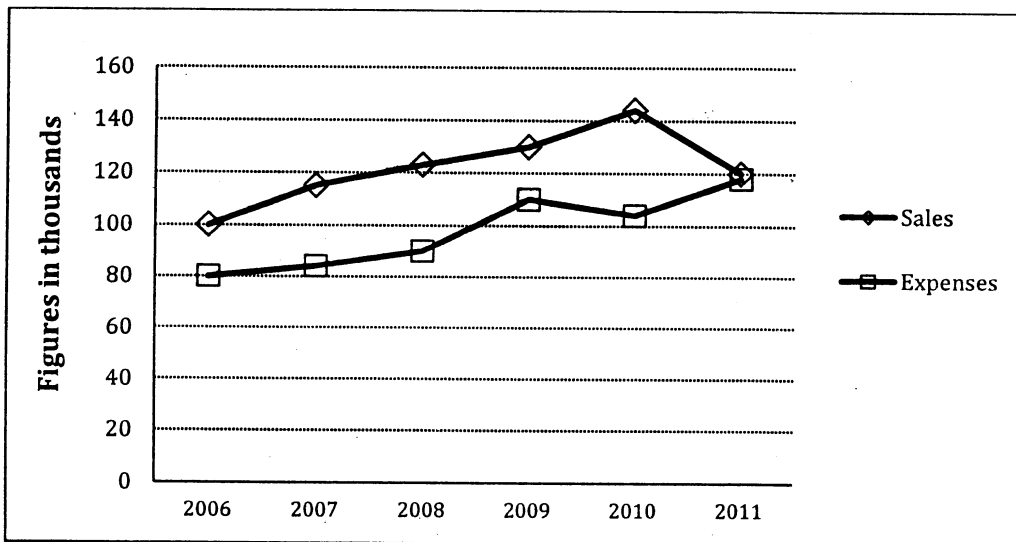
Answer questions 19 to 22 based on the following information: In a school consisting of 2400 children, the ratio of girls to boys is 7 : 5 respectively. All the children have taken different classes as per their hobby viz. chess, badminton, table-tennis and carom. 30 percent of the boys take table-tennis classes. The number of girls taking badminton classes is three – fifth of the number of boys taking the same. One-fourth of the girls take carom classes. The total number of students taking carom classes is 650. One-fifth of the boys take chess classes and the remaining boys take badminton classes. The girls taking chess classes are thrice the number of boys taking the same. The remaining girls take table-tennis classes.

19. What is the ratio of the number of girls taking table-tennis classes to the number of boys taking badminton classes?
(a) 33 : 30 (b) 3 : 2 (c) 33 : 20 (d) 8 : 5 (e) 16 : 11

20. The number of girls taking carom classes is what percent of the total number of children in the school?
 (a) 14.58% (b) 12% (c) 9.54% (d) 16% (e) 20%
21. What is the total number of children taking badminton classes?
 (a) 450 (b) 800 (c) 320 (d) 650 (e) 530
22. What is the number of girls taking table-tennis classes?
 (a) 600 (b) 120 (c) 330 (d) 440 (e) 350

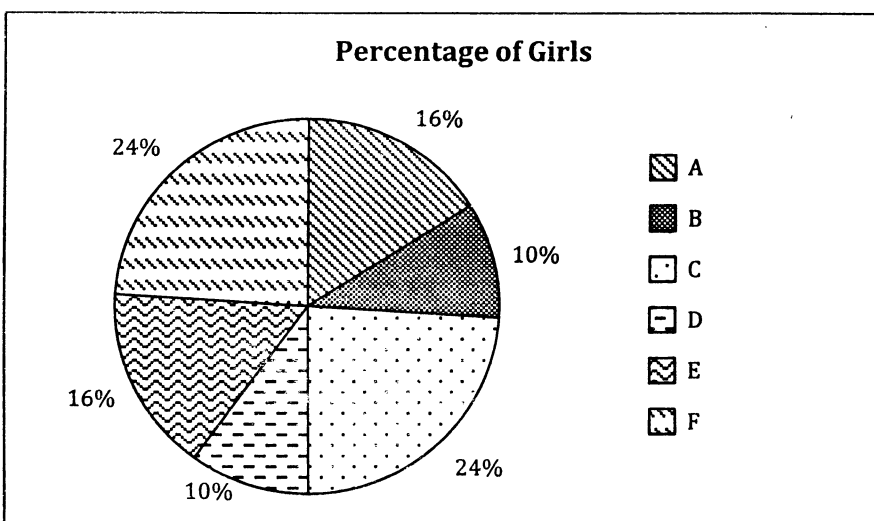
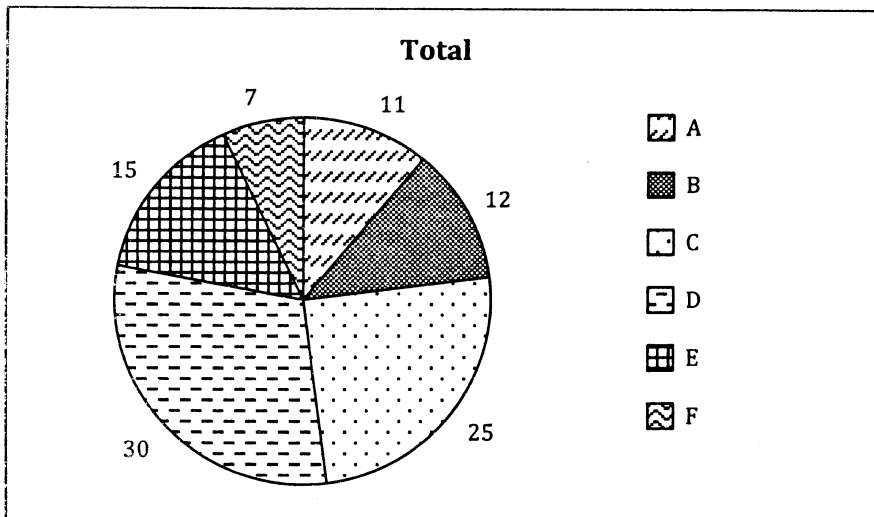
TEST 3

Answer questions 23 to 27 based on the following information:



23. In which year is the profit maximum?
 (a) 2011 (b) 2010 (c) 2007 (d) 2006 (e) 2008
24. What is the ratio of the expenses in 2008 to that in 2010?
 (a) 6 : 7 (b) 7 : 6 (c) 7 : 5 (d) 5 : 7 (e) None of these
25. What is the difference between the profit in 2006 and the profit in 2009?
 (a) 10 (b) 5 (c) 20 (d) 15 (e) 0
26. What is the overall profit (in thousands) over the given period?
 (a) 110 (b) 130 (c) 150 (d) 160 (e) 170
27. The sales in 2008 form what percentage of the sales in 2006?
 (a) 110 (b) 105 (c) 115 (d) 112.5 (e) 125

Answer questions 28 to 32 based on the following information:



There are 2500 students in all sections combined.

28. How many boys are there in section D?

- (a) 700 (b) 650 (c) 660 (d) 675 (e) 680

29. What is the angle subtended by the number of students in section B?

- (a) 43.2° (b) 43° (c) 42.8° (d) 45° (e) 46°

30. What is the ratio of the number of girls in section C to that in section E?

- (a) 2 : 5 (b) 5 : 2 (c) 4 : 3 (d) 3 : 4 (e) None of these

31. The number of boys in section F is what approximate percentage of the number of girls in section A?

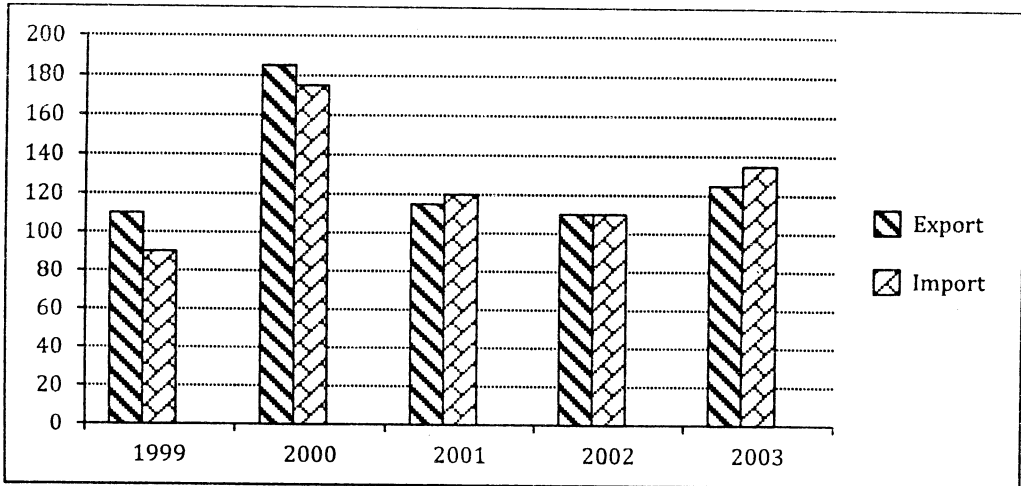
- (a) 350 (b) 320 (c) 300 (d) 330 (e) 340

32. What is the difference between the number of students in section D and section E?

- (a) 350 (b) 300 (c) 400 (d) 325 (e) 375

TEST 4

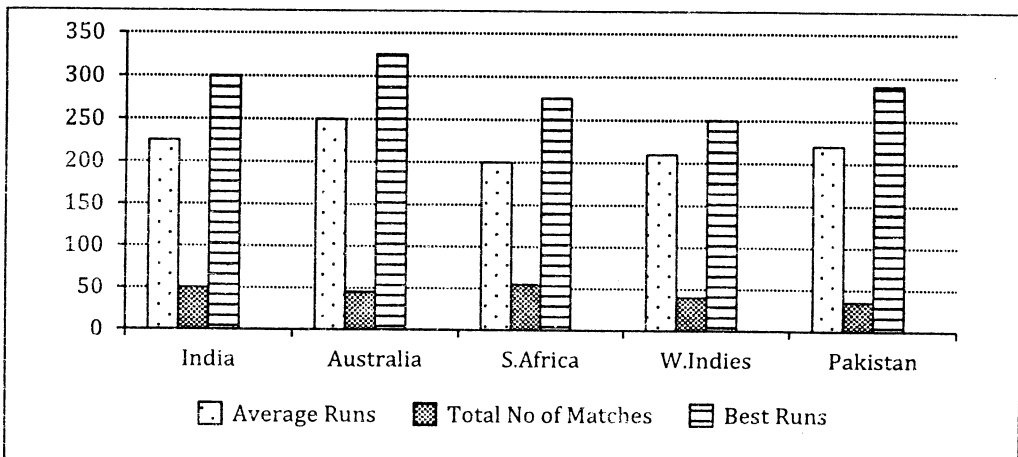
Answer questions 33 to 37 based on the following information:



The given graph shows the exports and imports of a country for the period 1999-2003.

33. What is the maximum difference between exports and imports of the same year?
 (a) 18 (b) 19 (c) 21 (d) 12 (e) 20
34. What is the ratio between exports in the year 1999 and exports in the year 2003?
 (a) 25 : 22 (b) 21 : 26 (c) 26 : 21 (d) 22 : 25 (e) None of these
35. The imports in 2000 form what percentage of exports in 2003?
 (a) 120 (b) 130 (c) 140 (d) 145 (e) 135
36. What is the difference between sum of exports in the year 1999 and 2000 and sum of imports in the year 2002 and 2003?
 (a) 55 (b) 50 (c) 60 (d) 65 (e) 45
37. What is the total difference between exports and imports over these years?
 (a) 15 (b) 18 (c) 12 (d) 21 (e) 10

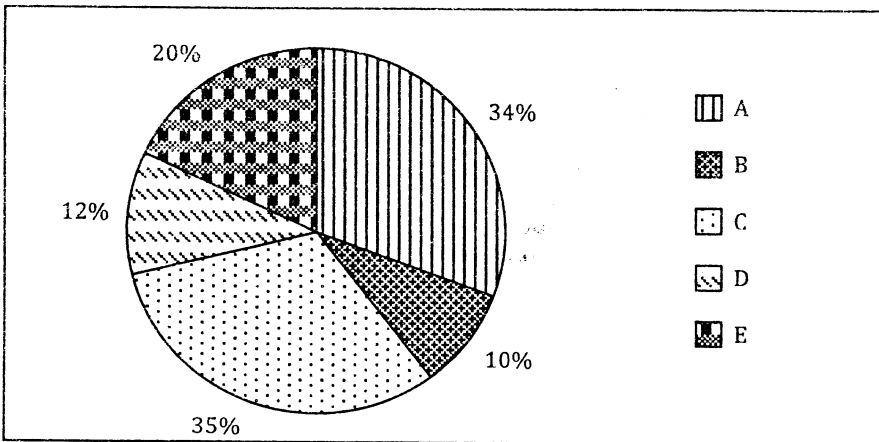
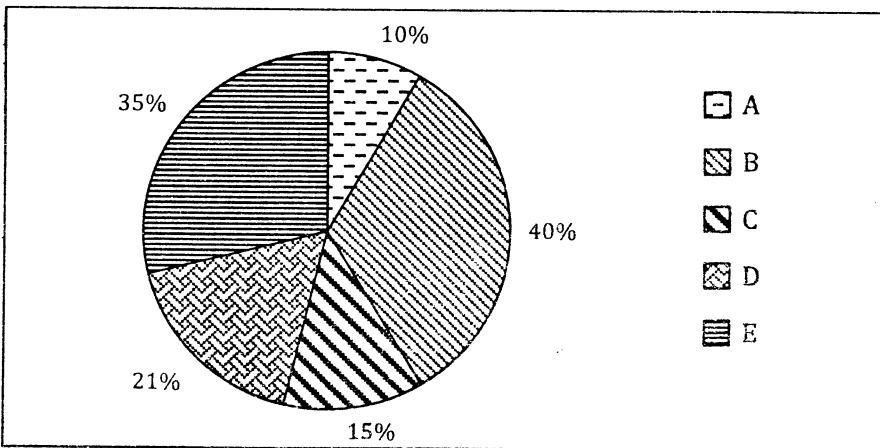
Answer questions 38 to 42 based on the following information:

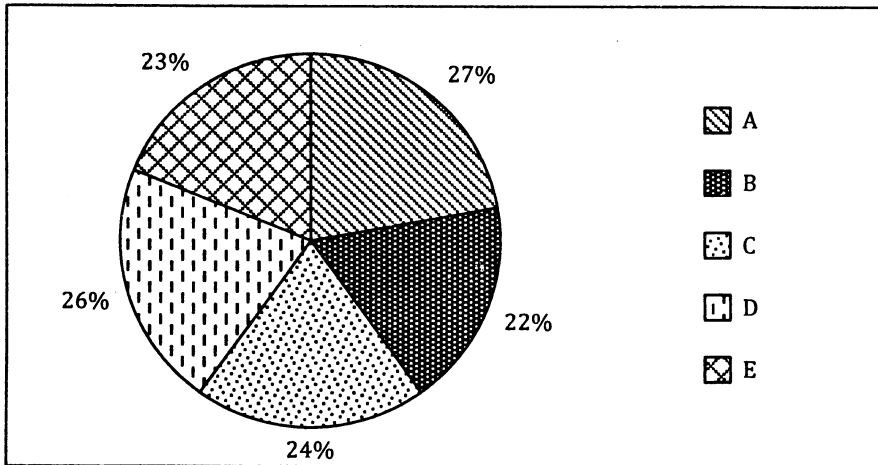


38. What is the difference between the total runs scored by India and Pakistan?
 (a) 3500 (b) 3550 (c) 3600 (d) 3400 (e) 3450
39. What is the ratio of the total runs made by Australia and W.Indies?
 (a) 75 : 56 (b) 56 : 75 (c) 76 : 55 (d) 55 : 78 (e) None of these
40. Which country has the minimum total runs?
 (a) India (b) Australia (c) W.Indies (d) S.Africa (e) Pakistan
41. Which country has the lowest difference between average runs and best runs?
 (a) India (b) Australia (c) S.Africa (d) W.Indies (e) Pakistan
42. What is the total number of runs scored by S.Africa?
 (a) 11050 (b) 11100 (c) 11000 (d) 10050 (e) 10090

TEST 5

Answer questions 43 and 44 based on the following information: There are 5 cricket teams -> A, B, C, D, E & all play all 3 forms of the game i.e.T20, ODI & Tests. The same pool of players is available for all forms of the game. After the contract signing the following graphs emerged:





The figures given in percentage terms are the percentage of total number of cricketers contracted to the team.

43. There are a set of players who do not leave one particular team irrespective of format. If they are the maximum possible (for one team), which team can they belong to?
 (a) A (b) B (c) D (d) E
44. If a player can play in a maximum of 2 teams, then which format has the maximum number of players in more than 1 team?
 (a) T20 (b) ODI (c) Tests (d) Both ODI and Tests

Answer questions 45 and 46 based on the following information:

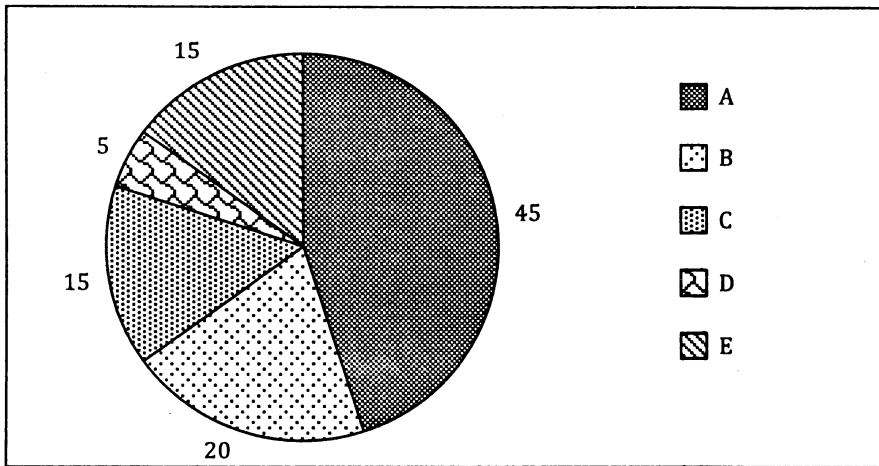
* Avg no. of (goal)	Team A	Team B	Team C	Team D
	3	2	1.5	4
Range of goals	0-4	1-4	1-3	1-6

* - per player per match.

Further it is known that Ronaldo of Team A scored an average of 1.5 goals for Team A, Maradona of team B scored an average of 2 goals for Team B, Raul of Team C scored an average of 2.5 goals for Team C, Roberto of team D scored an average of 5 goals for Team D.

45. If this data is for all the players (all teams have equal number of players) then who has done the best for his team?
 (a) Ronaldo (b) Maradona (c) Raul (d) Roberto
46. There are 4 players in Team A, 5 players in Team B, 6 players in Team C and 4 players in Team D. Is the average of Raul greater than that of all the other players combined (given that all played the same number of matches)?
 (a) Yes (b) No (c) The averages are equal (d) Cannot be determined

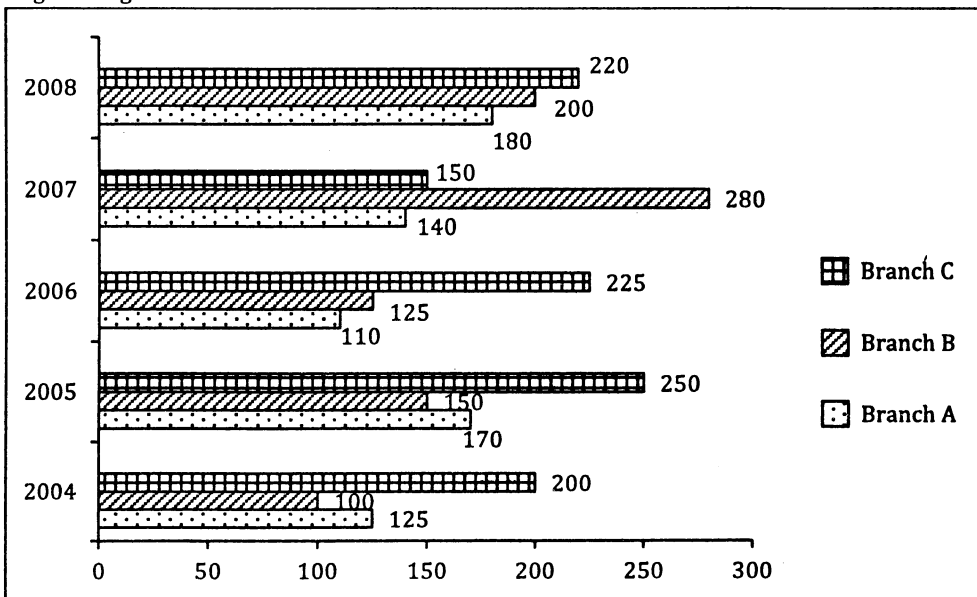
47. All figures given below are in percentage terms. If 50% of A shift to B, 33.33% of C shift to D & E is closed & its share is equally distributed amongst the rest then what is the final share of B?



- (a) 44.25% (b) 45.25% (c) 46.25% (d) None of these

Answer questions 48 to 50 based on the following information:

The graph below shows the number of students studying in branches 'A', 'B' and 'C' of 'XYZ College of Engineering' from 2004 to 2008.



48. What is the difference in percentage change in the number of students of branch A and branch C in the year 2008?

- (a) 28.57 (b) 46.67 (c) 64.29 (d) 18.1

49. By what percentage is the total number of students studying in branch B greater than those in branch C, in the year 2007?

- (a) 86.67% (b) 87.67% (c) 86.76% (d) 86%

50. What is the percentage change in the number of students of branch B for the year 2006?

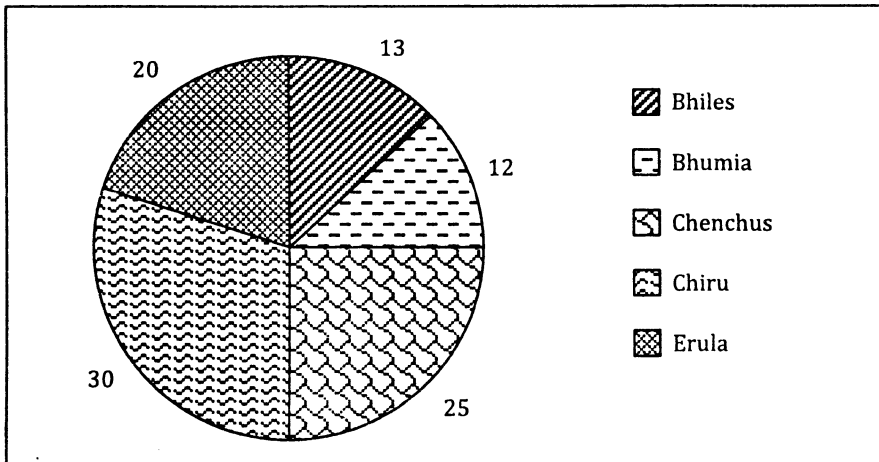
- (a) 17% (b) 16.67% (c) - 16.67% (d) - 17%

TEST 6

Answer questions 51 and 52 based on the following information: The following table gives the annual sales (Rs. crores) for five different companies P, Q, R, S and T for six consecutive years 1991, 1992, 1993, 1994, 1995 and 1996.

Companies	Annual Sales (Rs. Crores)					
	1991	1992	1993	1994	1995	1996
P	864	900	875	1000	975	970
Q	785	1040	980	1060	980	1200
R	620	780	820	910	940	1000
S	695	790	740	800	850	900
T	740	850	520	980	460	900

51. The average annual sales (in Rs. crores) of company Q during the given period was
 (a) 986.5 (b) 1060 (c) 1007.5 (d) 995.5
52. The sales of company T in 1995 are approximately what percentage of the sales of company P in 1992?
 (a) 51% (b) 40% (c) 47% (d) 60%
53. The diagram below shows the distribution of wealth among the different tribes of the country. Assuming that the category is exhaustive and there is an equal number of people in each category (tribes); by how much is the poorest category of tribes poorer than the richest category? (in %)



- (a) 150% (b) 60% (c) 45% (d) None of these

54.

Year	Sales of units	
	Dairy milk	3 Star
2009	64	32
2010	71	34
2011	76	46
2012	80	54

If the percentage change in sales of units for 3 Star from 2009 to 2010 is the expected % change for Dairy milk from 2012 to 2013, what are the expected sales of Dairy Milk in 2013?

- (a) 83 (b) 90 (c) 85 (d) None of the above

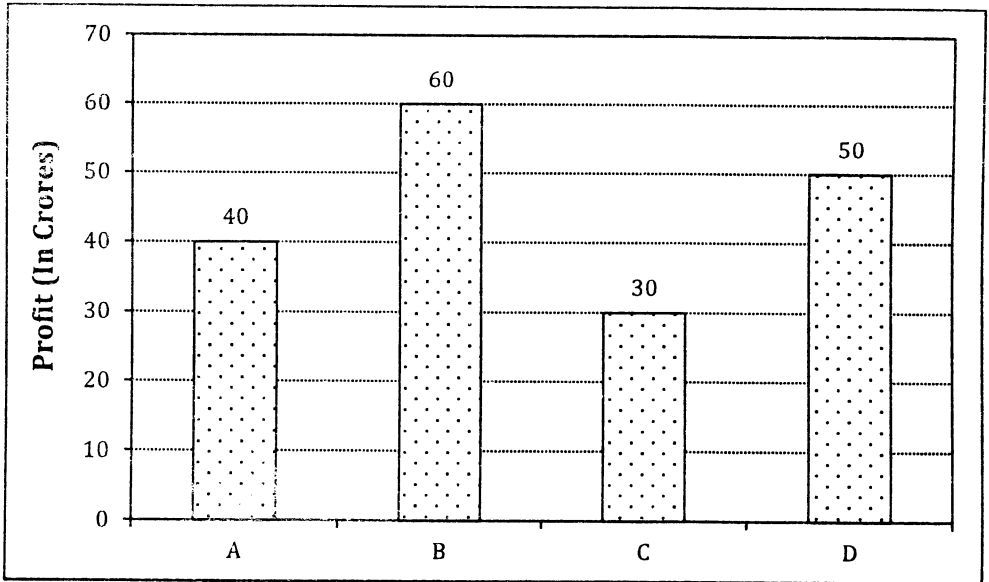
55.

Company	Revenues	
	2001	2002
Dapple	730	795
Damdung	165	230
Dony	837	902
Dhilips	444	509
Denq	139	204

The table gives the revenue of 5 companies for 2 consecutive years. The ratio of the revenue of 2001 to 2002 is highest for?

- (a) Dapple (b) Damdung (c) Denq (d) Dony

56.



The profit of company C is what percentage of the average profit of all the companies taken together?

- (a) 66.67% (b) 60% (c) 50% (d) None of these

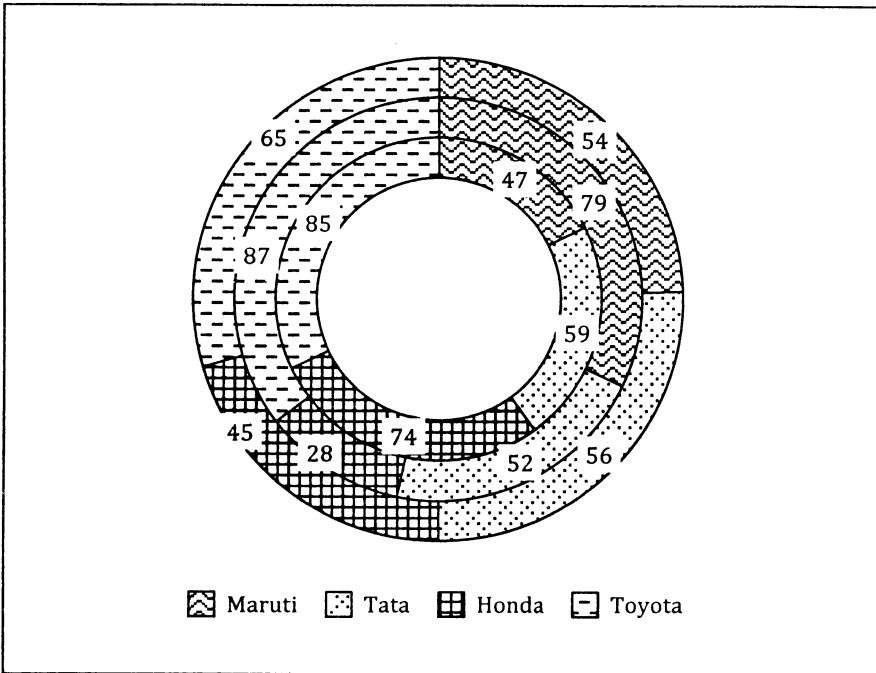
57.

Price	Vehicle Type			
	Ferrari	Volvo Bus	Railways	Auto
Price/ Km	85	60	30	40

If the ratio of the number of people travelling by various means of transportation (in the order given in the table) is 3 : 2 : 4 : 1, what is the average price per person/ Km?

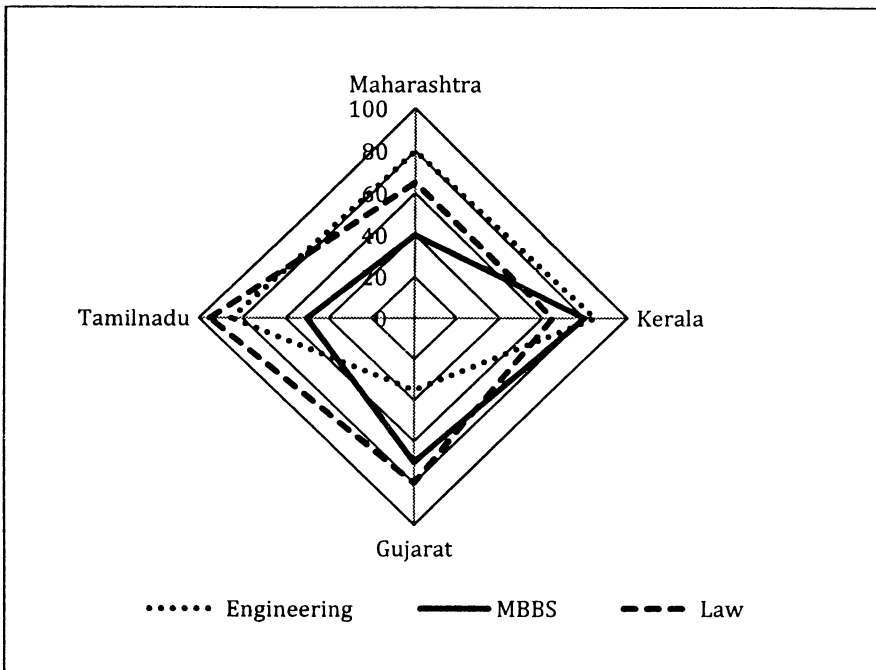
- (a) 50 (b) 54 (c) 51.5 (d) 53.5

58. The graph below represents the number of cars (in lakhs) sold by four companies in the first quarter of 2013. The inner-most circle represents the sales in January while the outer-most circle represents the sales in March; by how many lakhs (in terms of number of cars sold) did the sales in March fall as compared to the sales in January?



(a) 20 (b) 45 (c) 65 (d) 35

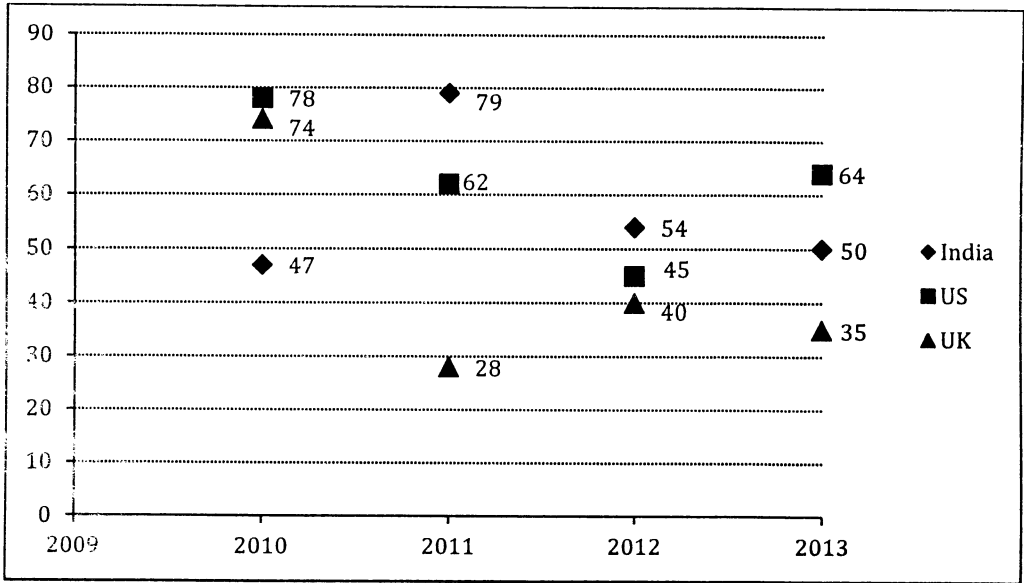
59. The graph below represents the number of students from four states (in lakhs) who opt for various streams of education.



What is the ratio of the number of engineering students from Maharashtra to MBBS students from Kerala to Law students from Gujarat?

- (a) 2 : 1 : 2 (b) 1 : 2 : 1 (c) 1 : 1 : 1 (d) 1 : 2 : 3

60. The graph below represents the number of tourists (in lakhs) who visited three countries - India, US and UK - during the period 2010 - 2013.



Which year saw the highest number of total visitors across all three countries?

- (a) 2010 (b) 2011 (c) 2012 (d) 2013

Answer questions 61 to 65 based on the following information: Study the two figures given below and answer the five items that follow:

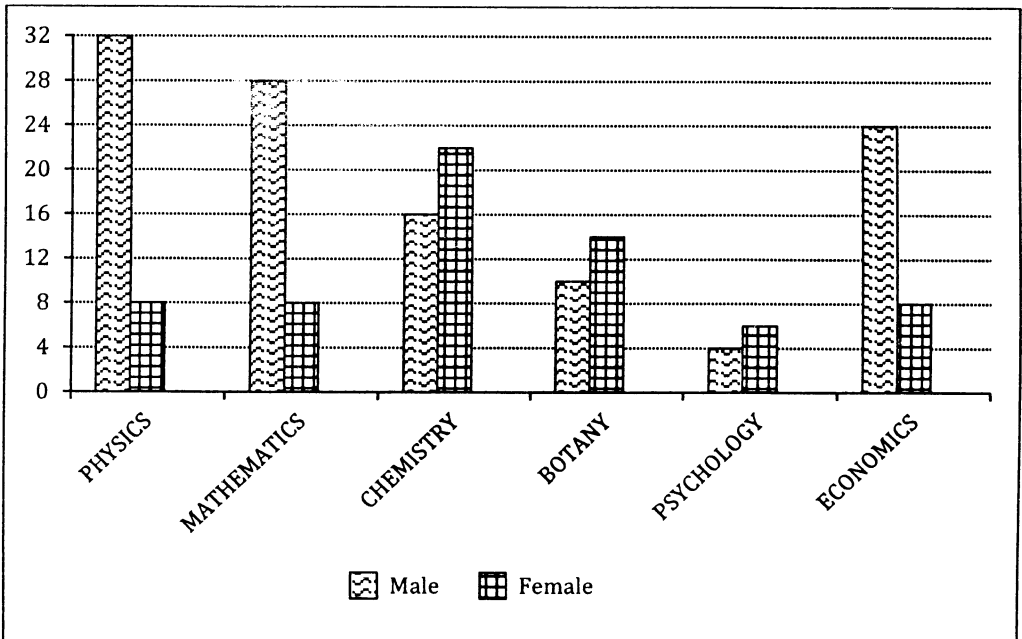


Figure 1: Number of professors in selected disciplines in a University by sex

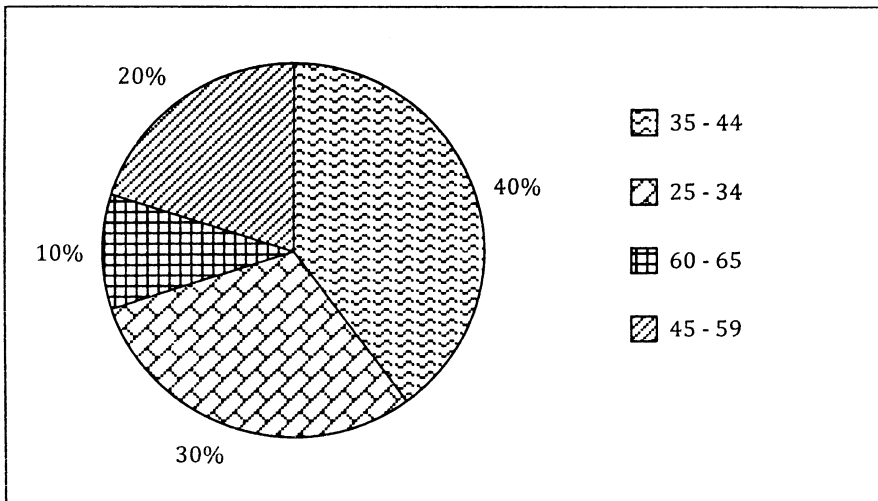


Figure 2: Age of Physics Professor
[UPSC 2013]

61. How many Physics professors belong to the age group 35 – 44?
 (a) 18 (b) 16 (c) 14 (d) 12
62. Which one of the following disciplines has the highest ratio of males to females?
 (a) Physics (b) Mathematics (c) Chemistry (d) Economics
63. What percentage of all Psychology professors are females?
 (a) 40 % (b) 50 % (c) 60 % (d) 70 %
64. If the number of female Physics professors in the age group 25 – 34 equals 25 % of all the Physics professors in that age group, then what is the number of male Physics professors in the age group 25 – 34?
 (a) 9 (b) 6 (c) 3 (d) 2
65. If the Psychology professors in the University constitute 2 % of all the professors in the University, then what is the number of professors in the University?
 (a) 400 (b) 500 (c) 600 (d) 700

2

Graphs and Diagrams

I. GRAPHS

Graphs are used to represent data. Having data in the form of diagrams helps us understand relationships between two interdependent quantities.

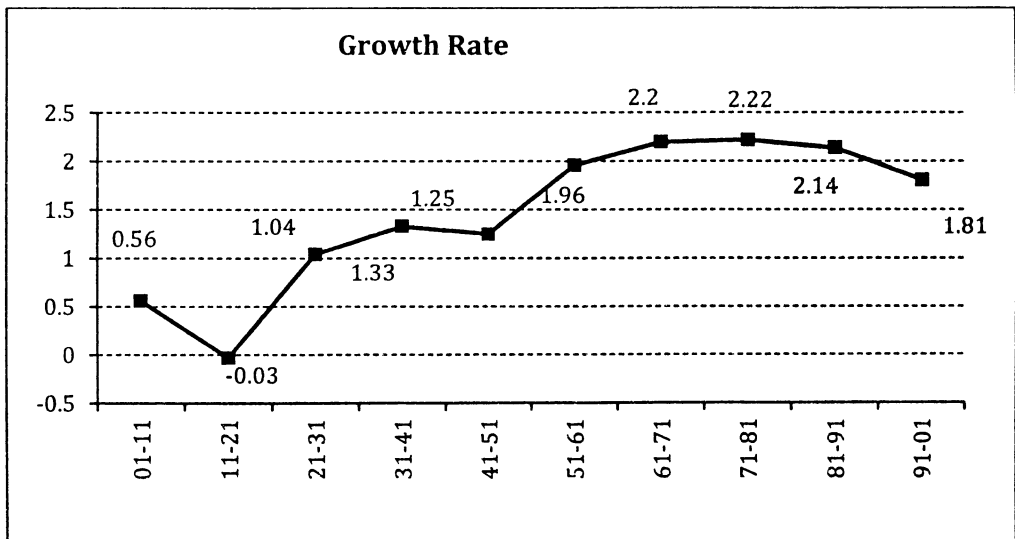
Graphs are used extensively in various fields. They help us study rates, analyse and compare data, and predict trends.

II. STRUCTURE OF A GRAPH

A typical graph has the following:

1. A horizontal axis on which the **independent variable** is plotted.
2. A vertical axis on which the **dependent variable** is plotted.
3. Data

Let's look at an example of a graph. The following graph shows the growth rate in percentage of a country's population for the years ranging from 1901 to 2001.



The horizontal axis shows the decades. The vertical axis, which is calibrated with equidistant values, shows the growth rate. The data points show the growth rate of population in every decade.

Example 1: State whether each of the following statements based on the preceding graph is true or false.

1. The population has dropped only once in the given decades.
2. The trend shows that India's population is dropping since 1981-1991.
3. In the given decades the population is highest in 1991-2001.

Solution: Consider each statement.

1. A negative growth rate denotes a drop in population. So statement 1 is **True**.
2. The growth rate is dropping since 1981-1991. However, it is still positive indicating that the population has been growing. So statement 2 is **False**.
3. The population has been growing since 1921. It is obvious that the highest population is in 1991-2001. So statement 3 is **True**.

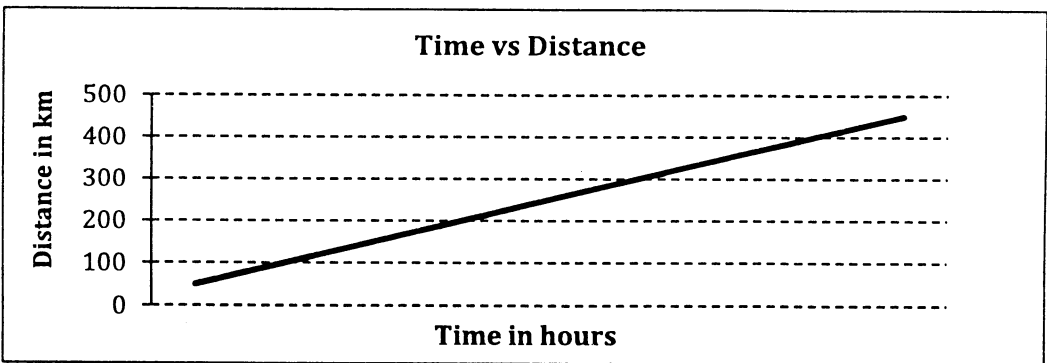
III. DIRECT AND INVERSE VARIATION

Two quantities may have a **direct** or **inverse** or no relationship. Quantities are said to be varying directly if an increase (of magnitude) in one causes an increase (of magnitude) in the other. Quantities are said to be varying inversely if an increase (of magnitude) in one causes a decrease (of magnitude) in the other and vice versa. We know that distance = speed × time. Consider a car running at a constant speed of 50 kmph. The table below shows the distance covered as the number of hours increase.

Speed (kmph)	Time (hours)	Distance (km)
50	1	50
50	2	100
50	3	150
50	4	200
50	5	250
50	6	300
50	7	350
50	8	400
50	9	450

As the number of hours increase, the distance covered increases. Conversely, as the distance covered increases, the time required for the journey increases. In other words, for a constant speed, time and distance are **directly proportional**.

We plot the values in the table in the graph below.



The nature of the graph shows a direct relation between the two quantities, time and distance.

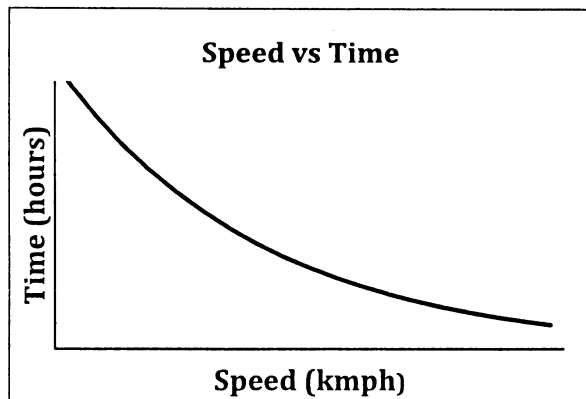
REMEMBER:

- Any straight line passing through the origin shows a direct variation between the two quantities plotted.
- If x and y are directly proportional, $x = ky$, where k is a constant.

Now, if the car has to run a distance of 500 km, an increase in speed will reduce the time required for the journey. Speed and time are thus **inversely proportional**. The table below shows the time required for different values of speed.

Distance	Speed	Time
500	100.00	5.00
500	87.50	5.71
500	58.62	8.53
500	51.29	9.75
500	34.36	14.55
500	20.14	24.82
500	17.62	28.37
500	11.81	42.35
500	10.33	48.40
500	9.04	55.31

The corresponding graph is as follows:



Any graph of this nature shows an inverse relationship.

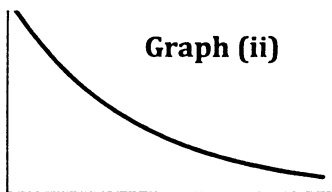
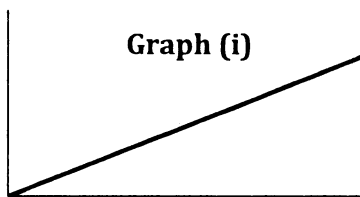
 **REMEMBER:**

If x and y are inversely proportional, $x = k/y$, where k is a constant.

Example 2: Match the relations 1, 2, 3 and 4 given below with the correct graphs from (i) and (ii).

1. Circumference and radius
2. Density and volume
3. Temperature of oxygen and volume
4. Altitude and temperature

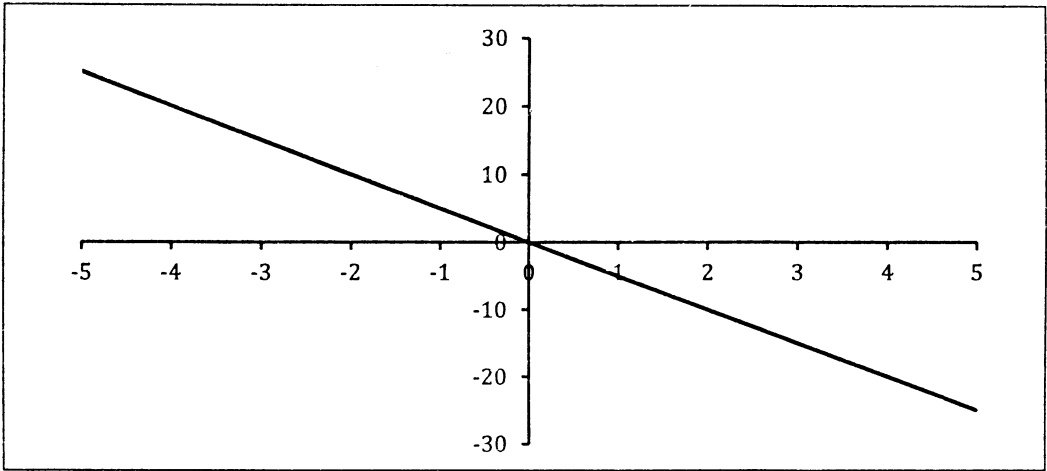
Graphs (i) and (ii) are given below.



Solution:

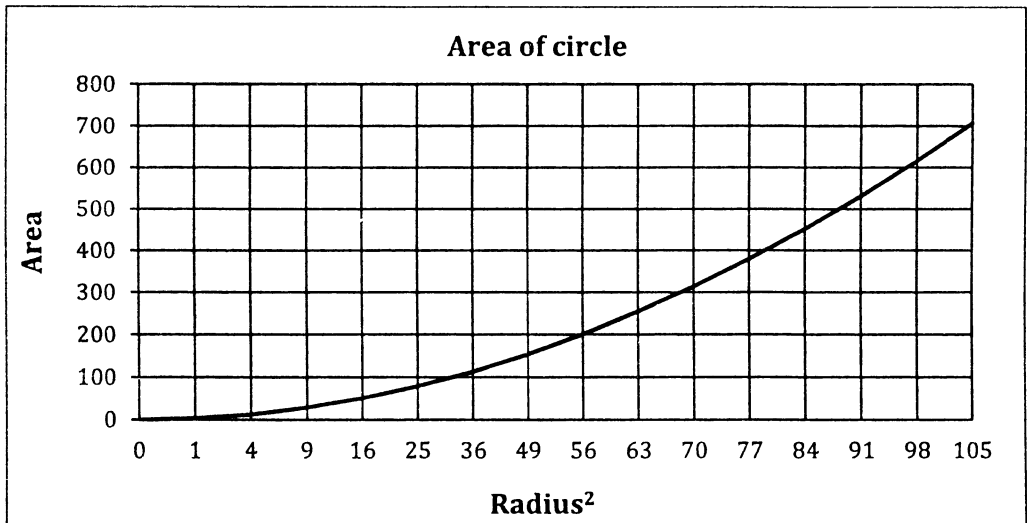
1. Circumference is directly proportional to radius. So graph (i) is the correct graph for this relation.
2. Volume is inversely proportional to density. So graph (ii) is the correct graph.
3. Temperature and volume are directly proportional. So graph (i) is the correct graph.
4. Altitude and temperature are inversely proportional. So graph (ii) is the correct graph.

Now let us analyse some more graphs and state whether they show a direct or an inverse variation or neither.



This graph is a straight line and passes through the origin. However, we can see that as value of the independent variable increases, the value of the dependent variable decreases and vice versa. So is it a direct variation or not? We see that as the *magnitude* of the independent variable increases, the *magnitude* of the dependent variable also increases, albeit in the negative direction and vice versa. So it is a direct variation. In this case $x = ky$ with the value of k being negative.

The next graph shows the area of a circle against the square of its radius. Is this a direct variation?

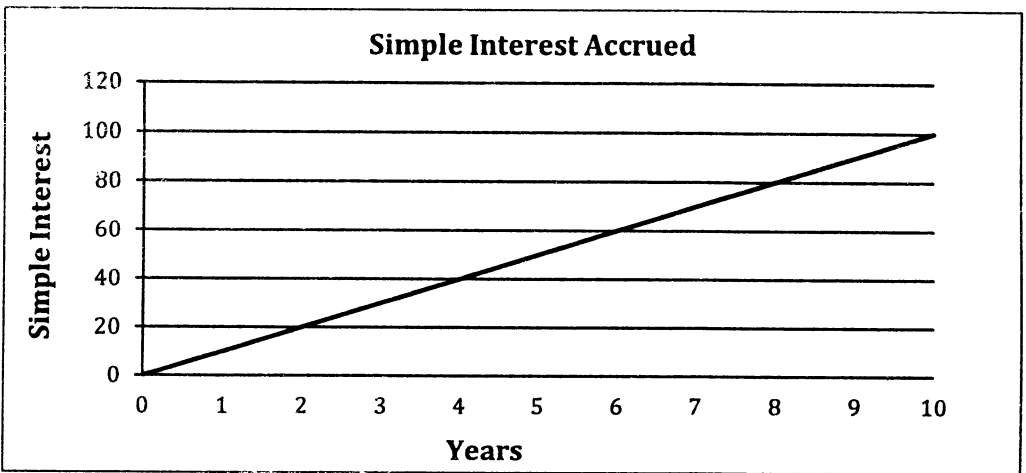
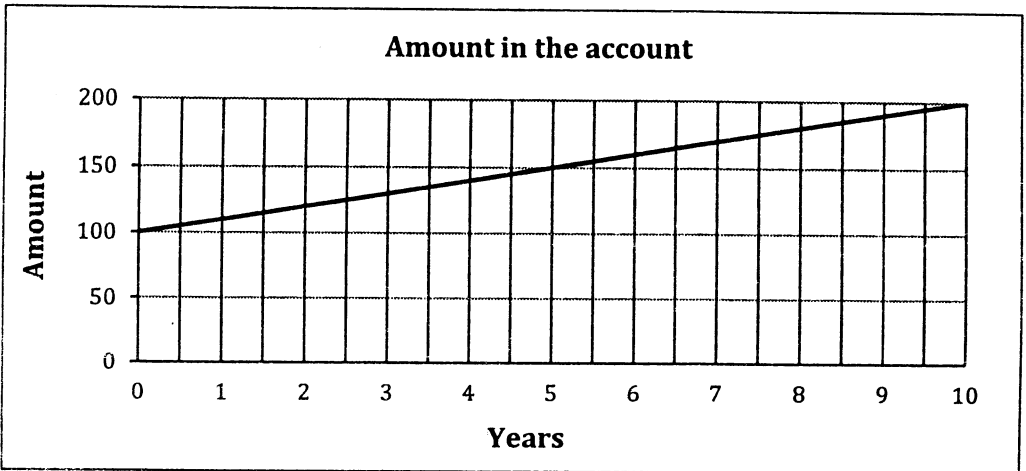


$\text{Area} = \pi r^2$ denotes a direct proportion between the area and square of radius. But then why is the graph not a straight line? This is because of the fact that the values on the x -axis are not equally spaced.

Now suppose that an amount of Rs. 100 has been kept in a bank account for 10 years at a simple interest of 10% per annum. We know that

$$\text{Simple Interest} = \text{Principal} \times \text{Period} \times \frac{\text{Rate}}{100}$$

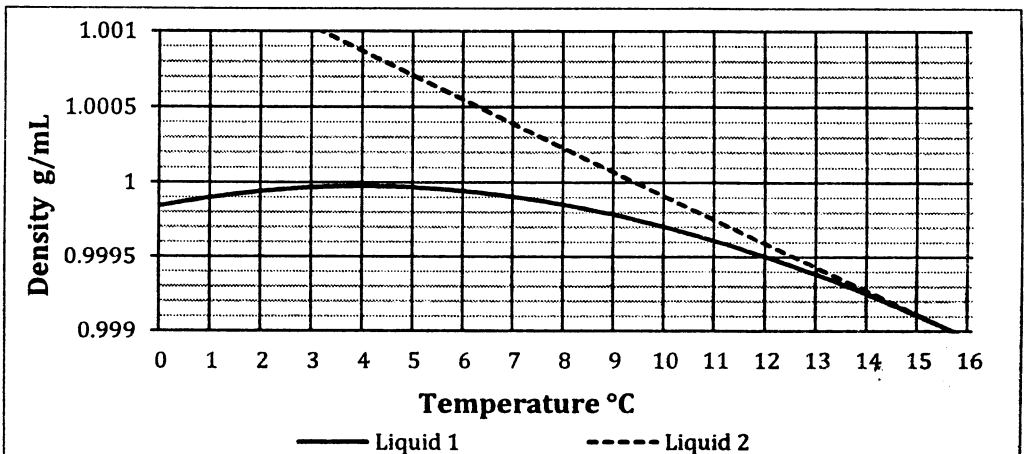
Thus, when principal and rate are constant, Simple Interest and Period are directly proportional. The first graph below shows the amount in the bank account every year. The second graph shows the simple interest accrued.



In the first graph, we can see that at $x = 0, y$ has a value of 100. So, $y = kx$ is not true. Graphically, it means that the graph does not pass through the origin. It is however a straight line graph. So we can say that the amount shows a positive relation with the number of years, but this is neither a direct nor an inverse variation.

The second graph is a direct variation.

Example 3: The following curve represents the volume-density graph of two liquids. With reference to the graph, which of the following statements is not true?



1. The density of liquid 1 is maximum at 4°C .
2. The density of liquid 1 is directly proportional to temperature below 4°C .
3. The density of liquid 2 falls as temperature rises.
4. The density of liquid 2 is not proportional to temperature.

Solution: Statements 1 and 3 are very clearly true. Statement 4 is also true as the graph of the density of liquid 2 against temperature is a straight line, but does not pass through the origin. Statement 2 is false as the graph of density of liquid 1 against temperature is not a straight line and does not pass through the origin.

IV. INTERPRETATION OF GRAPHS

Graphical representation of data is very often more readable than a tabular or verbal representation. It is easier to understand data, analyse trends and draw conclusions using graphs. Graphs are used frequently in all walks of life and hence it is very important for any professional to be able to correctly interpret graphs.

Let us state some conventions that all graphs should follow unless stated otherwise.

1. The horizontal axis (or the **x-axis**) has the independent variable which increases as you go right and decreases as you go left.
2. The vertical axis (or the **y-axis**) has the dependent variable, which increases as you go up and decreases as you go down.
3. The point of intersection of the horizontal and vertical axes is the **origin**. This may denote either of the following:
 - The point where both the dependent and independent variables become zero.
 - The point where both the dependent and independent variables attain their smallest possible values.
 - The point where both the dependent and independent variables attain the smallest values that the graph intends to show.

The meaning of the origin may be clearly stated or may have to be understood in context of the graph.

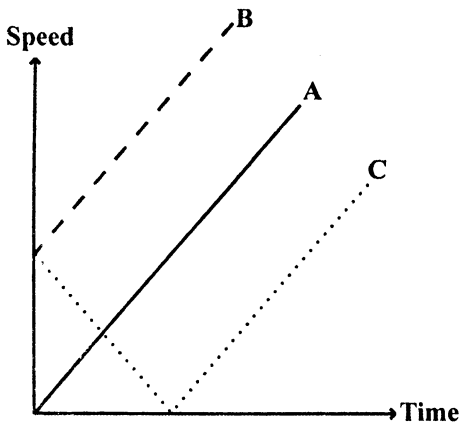
We will reiterate the facts in the previous section because they will be used extensively in the examples that follow:

1. A direct variation means the magnitude of y increases as the magnitude of x increases.
The steeper the graph is, faster the dependent variable grows in magnitude. Or in other words, greater the slope is, greater the rate of increase of y .
2. An inverse variation means that the magnitude of y decreases as the magnitude of x increases and vice versa.
3. There can be variations which are neither direct nor inverse.

Example 4: Three persons standing on the top of a very high tower throw metal balls into the air.

1. Person 1 just drops the ball down.
2. Person 2 throws the ball down with non-zero speed.
3. Person 3 throws the ball up with non-zero speed.

The speed - time graphs of the balls thrown by the three are as below.



The correct order of the graphs for the balls thrown by person 1, person 2 and person 3 respectively is:

- (a) (B, A, C) (b) (B, C, A) (c) (A, C, B) (d) (A, B, C)

Solution: Person 1 drops the ball.

∴ Speed in the beginning should be zero.

∴ A corresponds to person 1.

Person 2 throws the ball down. The ball has some speed in the beginning and then it goes on increasing.

∴ Graph B corresponds to person 2.

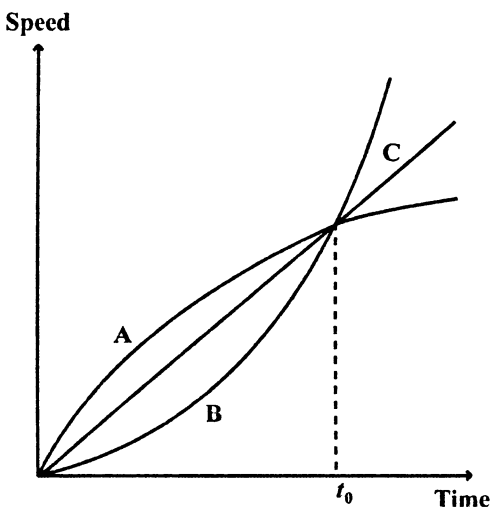
Person 3 throws the ball up. So there is some speed initially. When the ball reaches the highest point that it can reach, it has a speed of zero. It then starts falling down with increasing speed.

∴ Graph C corresponds to person 3.

Hence, **option d**.

Example 5: Which of the assertions 1, 2 and 3 are correct regarding a race between A, B and C, velocity – time graph for which is shown.

1. A leads C and C leads B until time t_0 .
2. If the race continues for long enough, eventually B will lead C and C will lead A.
3. After time t_0 speed as well as acceleration of B is the greatest among the three.



- (a) 1, 2, and 3 are all correct (b) Only 1 and 3 are correct.
 (c) Only 2 and 3 are correct. (d) Only 1 is correct.

Solution: Distance covered is given by the area under the speed-time curve.

- ∴ Until time t_0 , A leads C and C leads B.
- ∴ 1 is correct.

Acceleration is given by the slope of the speed-time curve.

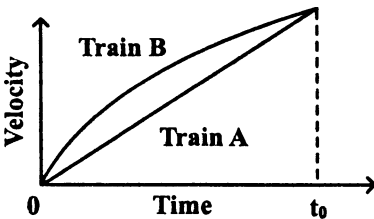
- ∴ Acceleration of B is the greatest after time t_0 .
- ∴ 3 is also correct.

Note that eventually, area under curve B will be greater than that under C and area under C will be greater than that under A.

- ∴ 2 is also correct.

Hence, **option a**.

Example 6: Consider the following Velocity - Time graph. It shows two trains starting simultaneously on parallel tracks. (CSAT 2011)



With reference to the above graph, which one of the following statements is **not** correct?

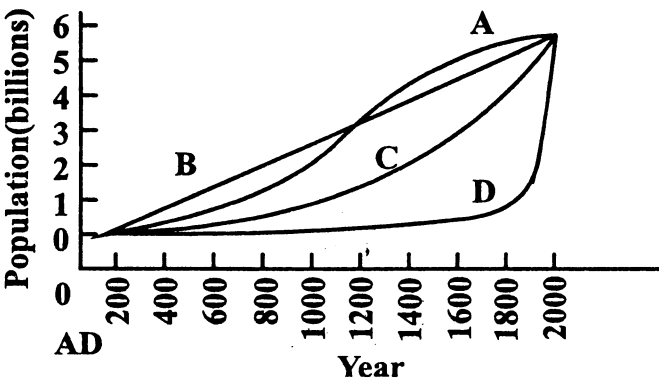
- (a) Train B has an initial acceleration greater than that of train A.
- (b) Train B is faster than Train A at all times
- (c) Both trains have the same velocity at time t_0
- (d) Both trains travel the same distance in time t_0 units.

Solution: Velocity = $\frac{\text{Distance travelled}}{\text{time taken}}$

As the velocity of train B is higher than train A, we can say that both the trains do not travel the same distance in the same time.

Hence, **option d**.

Example 7: The following figure has four curves namely A, B, C and D. Study the figure and answer the item that follows. (CSAT 2011)



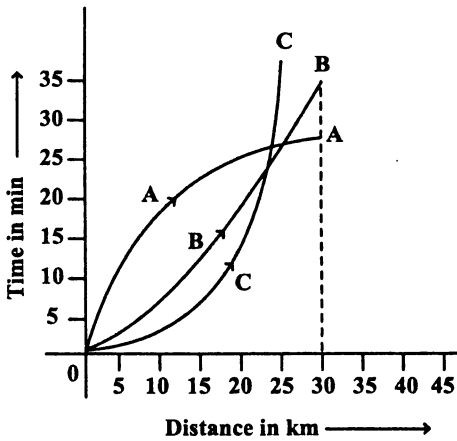
Which curve indicates the exponential growth?

- (a) A
- (b) B
- (c) C
- (d) D

Solution: Only graph C represents exponential growth.

Hence, **option c**.

Example 8: Consider the following distance -time graph. The graph shows three athletes A, B and C running side by side for a 30 km race. (CSAT 2011)



With reference to the above graph, consider the following statements:

1. The race was won by A.
2. B was ahead of A up to 25 km mark.
3. C ran very slowly from the beginning.

Which of the statements given above is/are correct?

- (a) 1 only (b) 1 and 2 only (c) 2 and 3 only (d) 1, 2 and 3

Solution: Time require to complete 30 km was lowest for A.

Hence, A won the race.

Hence, statement 1 is correct.

Till 25 km, B was faster than A. Hence, statement 2 is also correct.

In the initial part of the race, C covered greater distance than A and B. So C had the highest speed.

Hence, statement 3 is false.

Hence, **option b.**

V. GRAPHS AND FUNCTIONS

Functions are a way of assigning a rule deciding how changes in an independent variable cause corresponding changes in a dependent variable. Hence, some graphs are also functions. If x is the independent variable and y is the dependent variable of a function f , then the function is denoted algebraically by $y = f(x)$.

For example, we can express the area of a circle (y) as a function of its radius x as $y = \pi x^2$

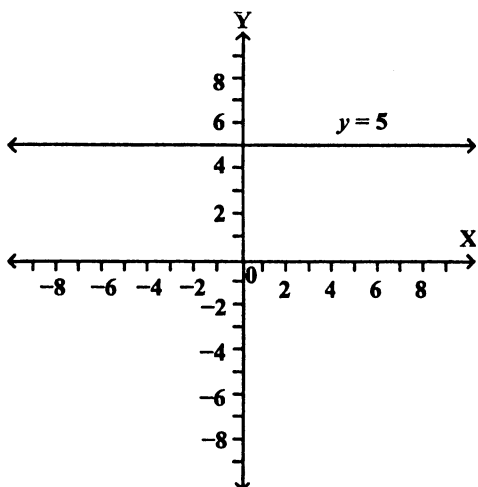
Following are the graphs of some standard functions:

A. CONSTANT FUNCTION

A constant function is of the form $f(x) = c$, where c is a real number.

The value of this function remains the same irrespective of what the input is. Hence, it is called a constant function.

The following is the graph of the constant function $f(x) = 5$.



The graph of any constant function $f(x)$ is similar to the graph above.

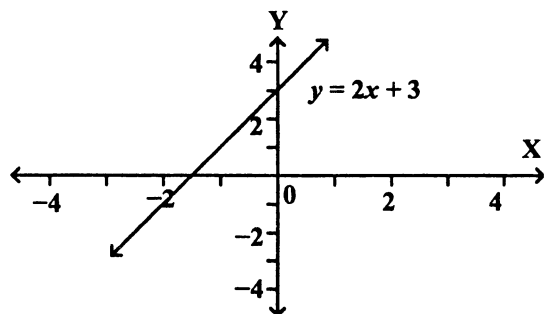
B. LINEAR FUNCTION

A **linear function** is of the form $f(x) = ax + b$, where a, b are real numbers.

The graph of a linear function is a straight line, which intersects the x -axis at $-b/a$ and y -axis at b .

For $b = 0$, the graph of a linear function will pass through the origin.

The graph of function $f(x) = 2x + 3$ is as follows:

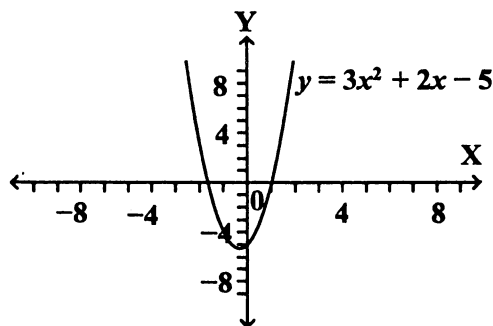


C. QUADRATIC FUNCTION

A **quadratic function** is of the form $f(x) = ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$.

The graph of a quadratic function of the form $f(x) = ax^2 + bx + c$ is a parabola whose major axis is parallel to the y -axis.

The graph of the function $f(x) = 3x^2 + 2x - 5$ is as follows:

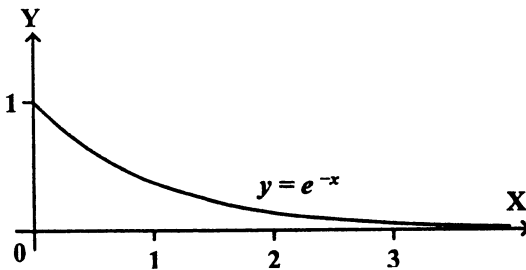
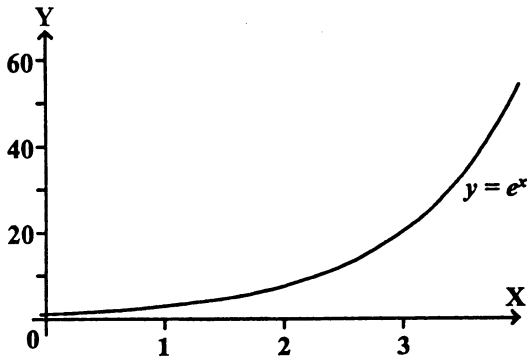


D. EXPONENTIAL FUNCTION

The function $f(x) = e^x$ is called the **exponential function**.

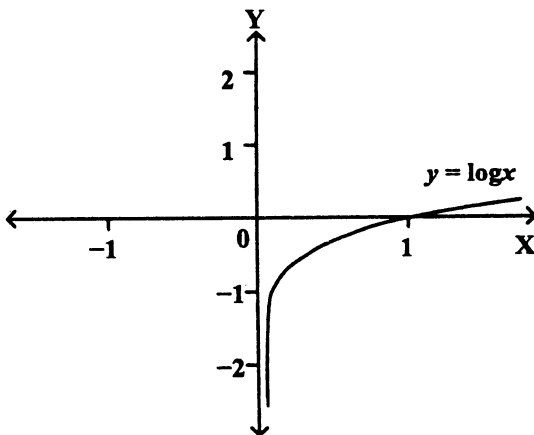
e is called the Euler's number and it has an approximate value of 2.718.

The graphs of the functions $f(x) = e^x$ and $f(x) = e^{-x}$ are as follows:



E. LOGARITHMIC FUNCTION

The function $f(x) = \log x$ ($x > 0$) is called the **logarithmic function**. The base of the log function, when not mentioned, is 10. Logarithm to the base e is known as the natural logarithm and denoted by $\ln(x)$ or $\log_e x$.

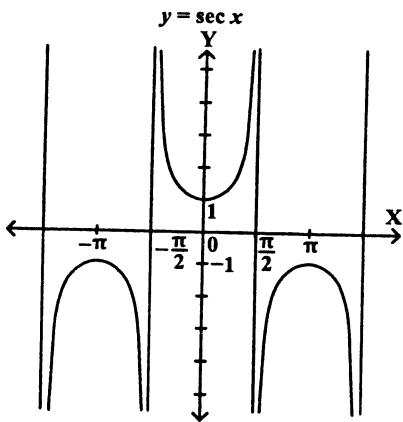
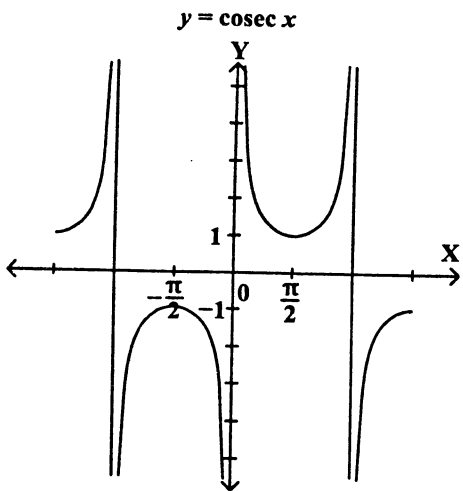
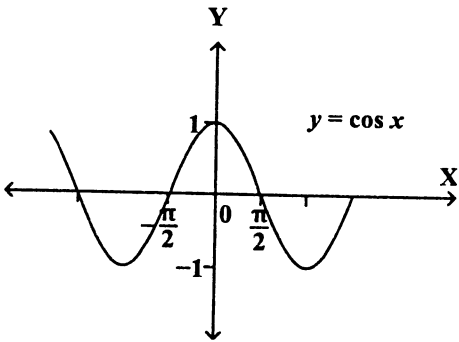
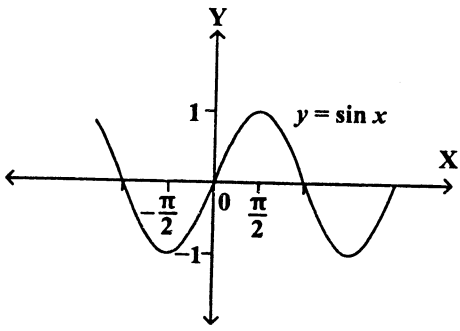


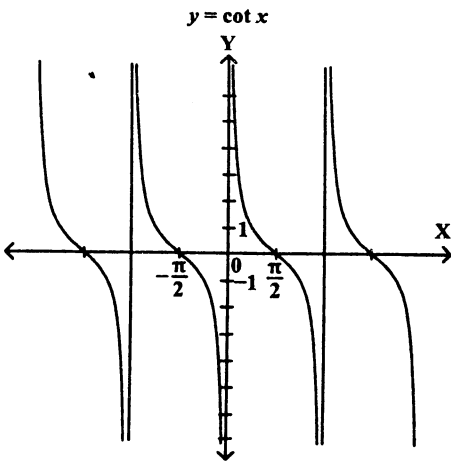
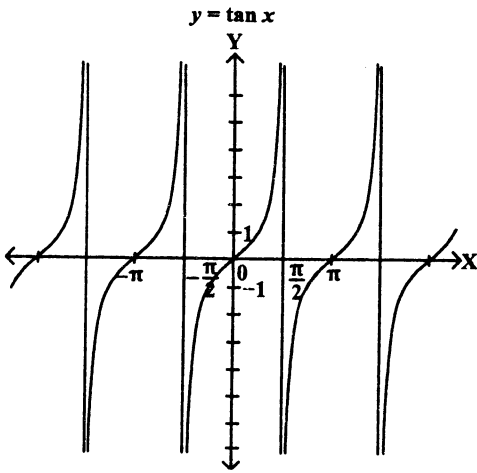
This is the graph of the logarithmic function to the base 10. If the base changes, the graph also changes accordingly.

F. TRIGONOMETRIC FUNCTION

Trigonometric functions are related to the angles and sides of triangles. The basic trigonometric functions are $\sin x$, $\cos x$, $\tan x$, $\operatorname{cosec} x$, $\sec x$ and $\cot x$. Their graphs follow.

All of these functions are **periodic functions**. Periodic functions repeat their values after fixed intervals. So their graphs look the same after fixed intervals.





VI. EVEN AND ODD FUNCTIONS

Any real valued function $f(x)$ can be classified into **even**, **odd** or **neither even nor odd** type of functions. Even and odd functions are functions which satisfy particular symmetric relations with respect to the x and y axis.

A. EVEN FUNCTION

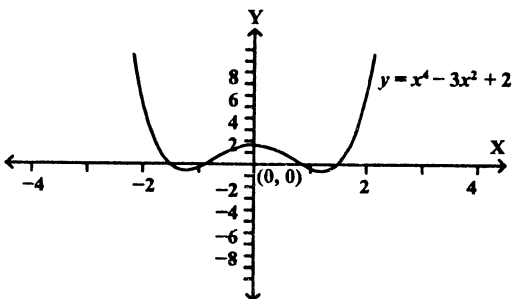
Let $y = f(x)$ be a function such that x, y are real numbers.

If the graph of $f(x)$ is symmetric about the y -axis i.e. it remains the same even when reflected about the y -axis, then $f(x)$ is called an even function.

For an even function $f(x)$,

$$f(x) = f(-x) \text{ for all } x \in \mathbb{R}.$$

Consider the function $f(x) = x^4 - 3x^2 + 2$. Its graph is as below.



It is seen that the graph is symmetric about the y -axis. So $f(x)$ is an even function. For an even function, the left half of the plane appears as the mirror image of the right half of the plane.

We can also verify this using $f(x) = f(-x)$

$$\text{Here, } f(-x) = (-x)^4 - 3(-x)^2 + 2 = x^4 - 3x^2 + 2 = f(x)$$

Other examples of even functions are $\cos x$, $|x|$, x^{2n} ; $n \in \mathbb{N}$.

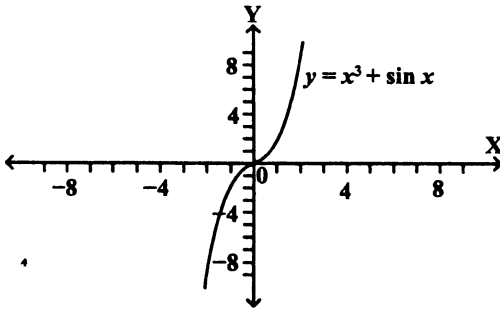
B. ODD FUNCTION

If the graph of $f(x)$ is symmetric about the origin i.e. it remains the same even after rotation of 180° about the origin, then $f(x)$ is called an odd function.

For an odd function $f(x)$,

$$f(x) = -f(-x) \text{ for all } x \in \mathbb{R}.$$

Consider the function $f(x) = x^3 + \sin x$. Its graph is as follows.



It is seen that the graph is symmetric about the origin. So $f(x)$ is an odd function. For an odd function the inverted left half of the graph looks like the mirror image of the right half of the graph.

$$\text{Here } f(-x) = (-x)^3 + \sin(-x) = -x^3 - \sin x = -f(x)$$

Other examples of odd functions are $\sin x$, $\tan x$, x^{2n+1} ; $n \in \mathbb{N}$

REMEMBER:

- The sum, difference, product, or division of any two even functions is also an even function.
- The sum or difference of any two odd functions is an odd function.
- Division or product of an even number of odd functions is an even function whereas that of an odd number of odd functions is an odd function.

So, an odd function raised to an even power is an even function whereas an odd function raised to an odd power is an odd function.

C. NEITHER EVEN NOR ODD FUNCTIONS

There are functions that satisfy the conditions of neither even nor odd functions. Such functions are said to be neither even nor odd functions.

For example:

$$f(x) = x^3 + 1$$

$$\therefore f(x) = (-x)^3 + 1 = -x^3 + 1$$

$$\therefore f(x) \neq \pm f(-x).$$

Thus, it is neither an even nor an odd function.

REMEMBER:

- The only function that is both even and odd is the function defined by $f(x) = 0$ for all real x .
- Even function \times Odd function \equiv Odd function
- Even function/Odd function \equiv Odd function

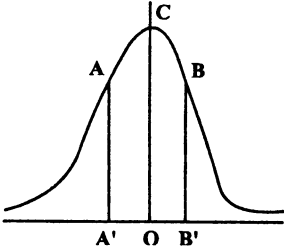
- Even function \pm Odd function \equiv Neither even nor odd function

Example 9: An even function, lying entirely above the x -axis, encloses an area of 1 sq. unit. A' and B' are two points on the x -axis and are equidistant from the origin. A and B lie on the curve such that $A'A$ and $B'B$ are perpendicular to the x -axis.

The area enclosed from $-\infty$ to AA' is 0.3 units. What is the area enclosed by the function from AA' to BB' ?

- (a) 0.4 sq. units (b) 0.5 sq. units (c) 0.7 sq. units (d) Cannot be determined

Solution: As the function is even, it is symmetric about the y -axis. We can draw the function assuming some symmetrical shape as follows.



The total area enclosed = 1 sq. unit,

Area from $-\infty$ to $AA' = 0.3$ sq. units.

\therefore By symmetry, area from BB' to $\infty = 0.3$ sq. units.

\therefore Area from AA' to $BB' = 1 - 0.3 - 0.3 = 0.4$ sq. units.

Hence, **option a**.

VII. SHIFTING AND SCALING OF FUNCTIONS

In this section we will study the changes that occur in shapes of graphs when the independent (x) or dependent (y) variables are **shifted** or **scaled**.

Shifting is nothing but moving a graph from one position to another corresponding to changes in the two variables.

Scaling is altering the shape of the graph corresponding to changes in the two variables.

A. SHIFTING

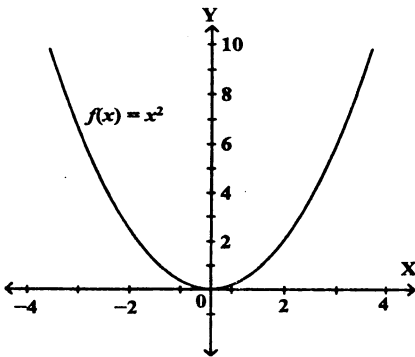
Consider the function $f(x) = x^2$.

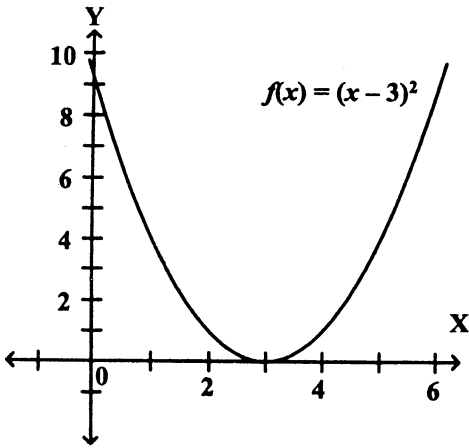
The graph of this function is a parabola that opens upward and passes through the origin.

Horizontal Shifting

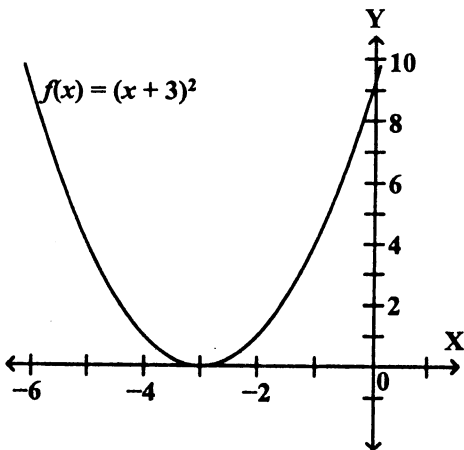
Now we study what happens to this parabola when we replace x by $(x \pm c)$, or when $f(x)$ changes to $(x \pm c)^2$.

This kind of **translation** is called **horizontal shifting** of a graph.





When we replace x by $(x - c)$; $c > 0$, the graph shifts c units to the right.



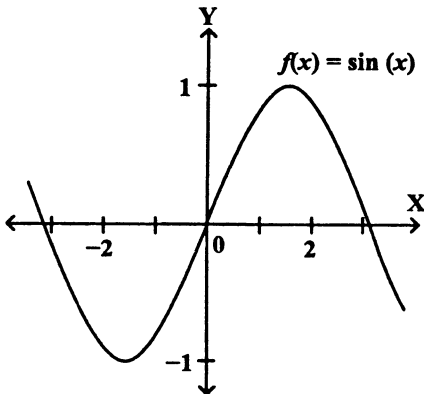
When we replace x by $(x + c)$; $c > 0$, the graph shifts c units to the left.

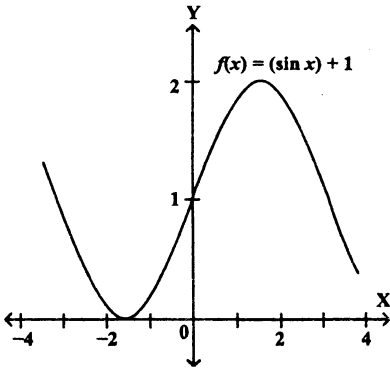
Note that changing x to $(x - c)$; $c > 0$, is equivalent to shifting the origin to $(-c, 0)$ which in turn is equivalent to shifting the graph to the right by c units. Similarly, changing x to $(x + c)$; $c > 0$, is equivalent to shifting the origin to $(c, 0)$ or shifting the graph to the left by c units.

Vertical Shifting

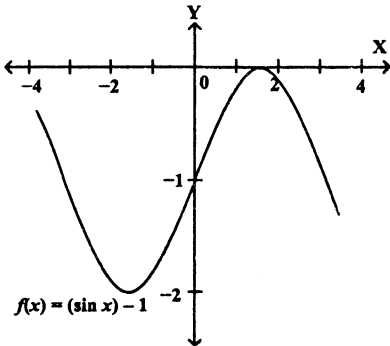
The translation in which $f(x)$ changes to $f(x) \pm c$ is called **vertical shifting**.

Consider the function $f(x) = \sin x$





When we replace $f(x)$ by $f(x) + c$; $c > 0$, the graph moves c units upwards.



When we replace $f(x)$ by $f(x) - c$; $c > 0$, the graph moves c units downwards.

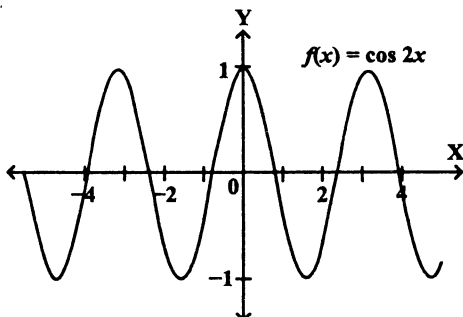
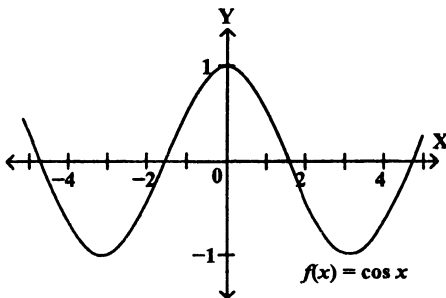
Thus shifting is a type of translation in which only the position of the graph changes but the shape and size of the graph remain the same.

B. SCALING PROPERTY

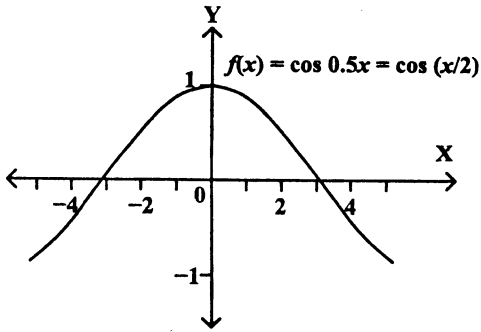
Horizontal Scaling

The translation in which $f(x)$ changes to $f(cx)$ or $f(x/c)$, is called **horizontal scaling**.

Consider the function $\cos x$.



When we replace $f(x)$ by $f(cx)$, where $c > 1$, the graph gets compressed horizontally.

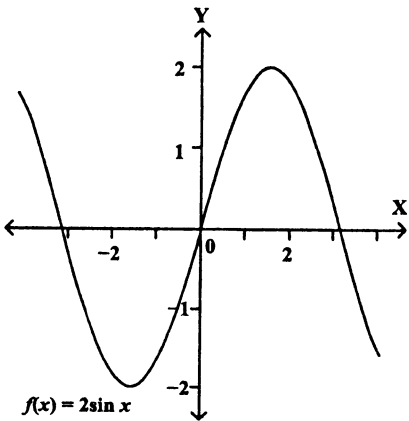
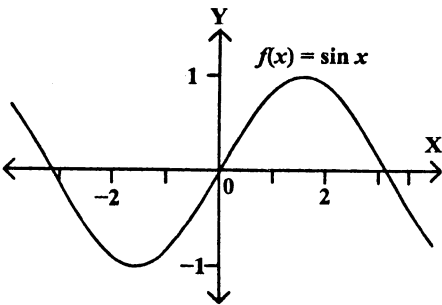


When we replace $f(x)$ by $f(x/c)$, where $c > 1$, the graph gets stretched horizontally.

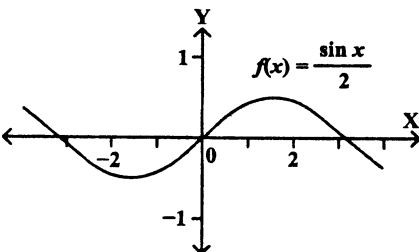
Vertical Scaling

The translation in which $f(x)$ changes to $cf(x)$, is called **vertical scaling**.

Consider the function $f(x) = \sin x$



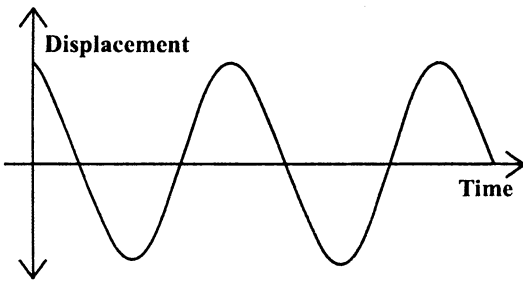
When $f(x)$ changes to $cf(x)$, $c > 1$, the graph gets stretched vertically.



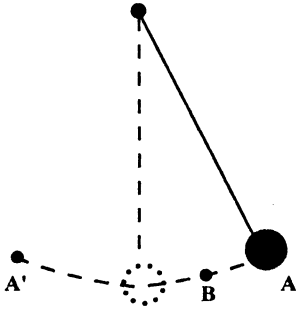
When $f(x)$ changes to $cf(x)$, $0 < c < 1$, the graph gets compressed vertically.

Thus scaling is a type of translation in which the shape of the graph changes.

Example 10: The following graph shows the displacement of a simple pendulum against time.

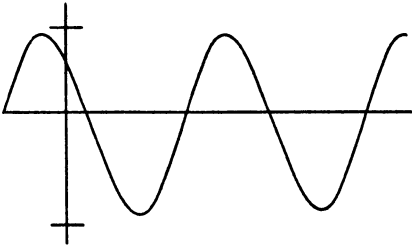


In the graph, initially, the pendulum is at an extreme position A.

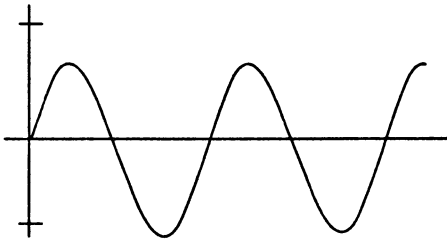


If the initial position of the pendulum is B (moving towards, A'), then the displacement follows which of the following graphs?

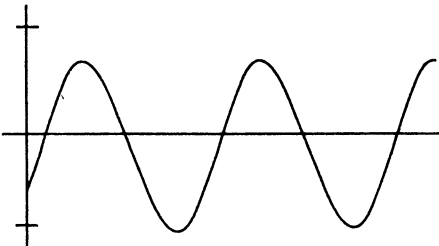
(a)



(b)



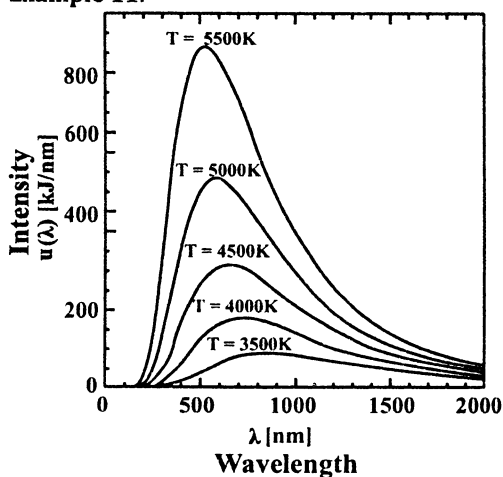
(c)



(d) None of these

Solution: The initial position shifts by a distance of $AB = \delta$, say.
 $\therefore x$ changes to $x + \delta$. So $f(x)$ changes to $f(x + \delta)$, which is shown in option a.
Hence, **option a**.

Example 11:



This graph shows the variation of the intensity of black body radiation as a function of wavelength for various absolute temperatures.

Detailed studies have revealed that the product of the absolute temperature and wavelength corresponding to the maximum intensity (λ_{max}) at this temperature is a universal constant (b) called the Wien's displacement constant.

On the basis of this information, identify the incorrect statement among the following four:

- (a) For a particular wavelength, the intensity of black body radiation increases with temperature.
- (b) The intensity of radiation for extremely small and extremely large wavelengths is negligibly low.
- (c) The wavelength corresponding to maximum intensity (λ_{max}) decreases as the temperature decreases.
- (d) On every isotherm, there lies exactly one point, the tangent at which is horizontal.

Solution: As given, $\lambda_{\text{max}} \times T = b$; where b is the Wien's constant.

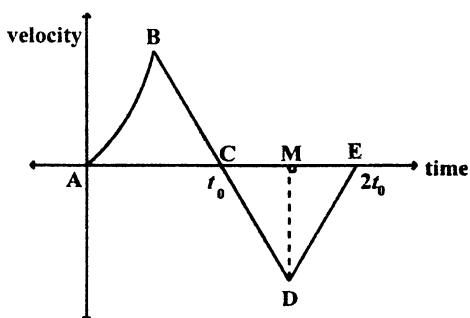
Thus, when the temperature decreases, the wavelength corresponding to the maximum intensity increases.

Hence, statement (c) is incorrect.

The other statements are seen to be correct from the nature of the graph.

Hence, **option c**.

Example 12: A particle moves along the x -axis, starting from the origin at time $t = 0$. The velocity-time graph of the particle is as follows.



At $t = 2t_0$, the particle is back at the origin.
 The curves BC, CD and DE are linear while AB is not.
 Identify the incorrect statement among the following:

- (a) The particle undergoes constant acceleration in the interval D - E.
- (b) The particle is applied an increasing accelerating force in the interval A - B.
- (c) The particle would end up at the origin even if the curve joining points D and E was a quadratic.
- (d) If the area enclosed by the curve ABC is A_0 the length of the perpendicular DM is $\frac{2A_0}{t_0}$

Solution: Acceleration is defined as the rate of change of velocity.

Using this, we can easily conclude that the first two statements are correct.

Since the particle ends up at the origin at time $t = 2t_0$, the net distance travelled is zero, and the area under the velocity-time curve between the time interval 0 to $2t_0$ should be zero.

Hence, the area of ΔCDE is also A_0 .

Hence, length of the perpendicular DM is $\frac{2A_0}{t_0}$.

So, statement (d) is also correct.

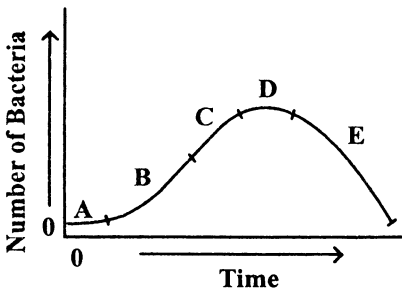
If the points D and E were joined by a quadratic curve, the area under the graph would be greater than or lesser than A_0 when the curvature is concave upward or convex upward respectively.

So, statement (c) is incorrect.

Hence, **option c**.

Example 13: Read the passage given below, study the graph that follows and answer the three items given below the figure.

During a party, a person was exposed to contaminated water. A few days later, he developed fever and loose motions. He suffered for some days before going to a doctor for treatment. On starting the treatment, he soon became better and recovered completely a few days later. The following graph shows different phases of the person's disease condition as regions A, B, C, D and E of the curve. (CSAT 2011)



Question 1: Which region/regions of the curve correspond/corresponds to incubation phase of the infection?

- (a) A only
- (b) B only
- (c) B and C
- (d) No part of the curve indicates the incubation phase

Solution: The person started recovering soon after he started treatment. This means that after starting treatment, the number of bacteria decreased. This must be region E.

The time when he suffered is thus from B to D, where the bacteria and hence the symptoms increased.

A must be the period immediately after being exposed to contaminated water, or the incubation period of the infection, when he did not show any symptoms.

Hence, **option a**.

Question 2: Which region of the curve indicates that the person began showing the symptoms of infection?

- (a) A (b) B (c) C (d) D

Solution: Part B of the curve indicates that the person began showing symptoms of infection. Hence, **option b**.

Question 3: Which region of the curve indicates that the treatment yielded effective relief?

- (a) C (b) D (c) E (d) The curve does not indicate the treatment

Solution: Region E indicated that the treatment yielded effective relief. Hence, **option c**.

VIII. DIAGRAMS

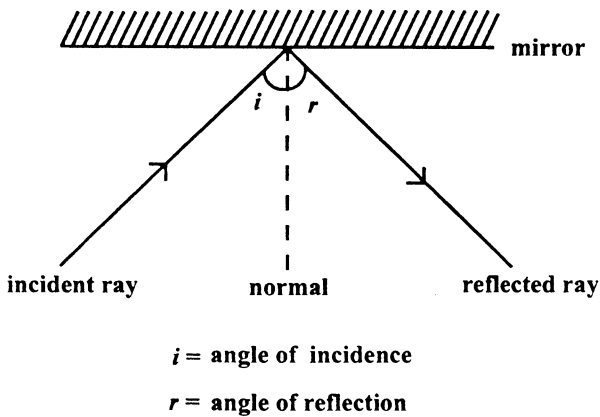
We all have encountered diagrams in school in subjects like science, geography and maths. Diagrams, like graphs, are used to represent some information. Again, it is important to be able to read diagrams correctly and extract relevant information from them.

In this section, we will see some applications of diagrams.

A. MIRRORS AND RAY DIAGRAMS

When a ray of light strikes a plane mirror, it gets reflected. The reflected ray follows the following rules called the laws of reflection:

1. The incident ray, the reflected ray and the normal to the surface at the point of incidence lie in the same plane.
2. The incident ray and the reflected ray make equal angles with the normal.
3. The incident and reflected rays lie on the opposite sides of the normal.

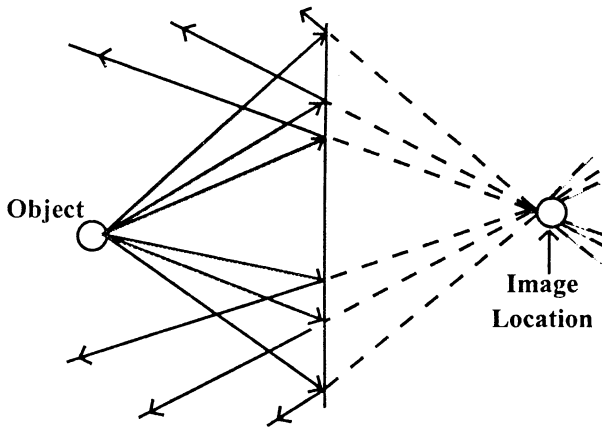


Ray Diagrams

A **ray diagram** shows the path that a source of light follows. Directions of rays of light are indicated using arrows pointing in the correct directions. The lines behind the mirror where the reflected rays appear to converge are shown by dashed lines.

Formation of an image

Light rays from an object placed in front of a mirror are incident on the mirror and get reflected from the surface of the mirror according to the laws of reflection. The reflected rays appear to converge at a point behind the mirror, where the image is seen.



The image formed in a plane mirror has the following properties.

1. It is upright and has the same size as the object.
2. It is laterally inverted. For example, 'b' appears as 'd' in a mirror.
3. It is at the same distance from the mirror as the object, except that it is on the other side of the mirror.
4. It is a virtual image as light rays do not actually converge where the image is.

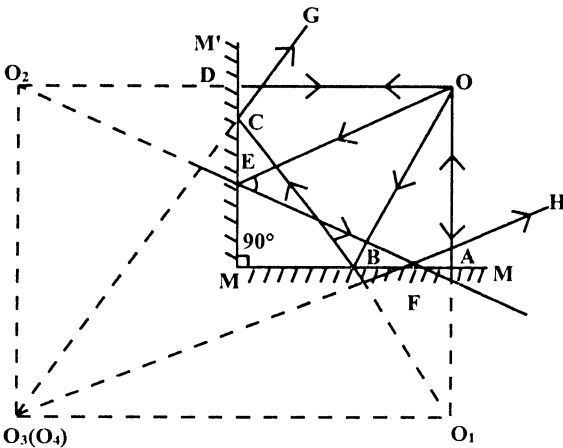
Number of images

Two mirrors placed next to each other at an angle produce two or more images due to multiple reflections. The number of images formed is given by the following formula:

$$\frac{360^\circ}{(\text{angle between the mirrors})} - 1$$

Thus, two mirrors placed at 90° form 3 images, two mirrors placed at 60° form 5 images and so on.

The following ray diagram shows the formation of images on two mirrors kept next to each other at an angle of 90°. O is the object.



Example 14: Two mirrors are placed with their reflecting surfaces making an angle of 90° with each other. Which of the following statements is true?

1. A man standing in front of the mirrors will see four reflections of himself.
2. Exactly one of the images of a flag kept in front of the mirror is not laterally inverted.

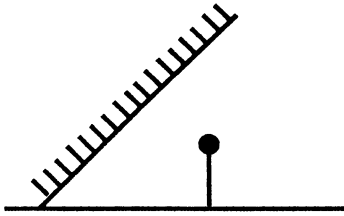
- | | |
|---------------------------|-----------------------------|
| (a) Only 1 is true | (b) Only 2 is true |
| (c) Both 1 and 2 are true | (d) Neither 1 nor 2 is true |

Solution: Two mirrors kept at 90° create three images. So statement A is not true.

The flag will have one laterally inverted image in each of the two mirrors and another image which will be formed due to the reflection of the image of the first mirror in the second and the image of the second in the first. This image will be a laterally inverted image of another laterally inverted image and hence will not be laterally inverted (or will be a true image).

Hence, **option b**.

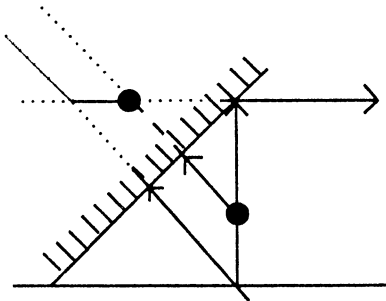
Example 15: A man is positioned in front of a plane mirror which makes an angle of 45° with the horizontal.



How does the man see himself in the mirror?

- (a) (b) (c) (d)

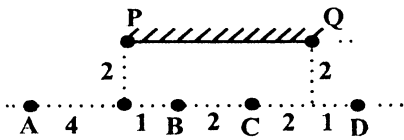
Solution: The ray diagram is as follows:



Thus the man sees himself as in option c.

Hence, **option c**.

Example 16:



PQ is a mirror with length 5 meters.

4 persons A, B, C, D are standing on a line that is parallel to PQ and at a distance of 2 meters from mirror PQ.

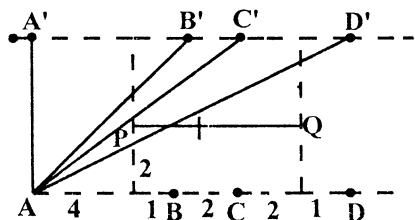
Exact positions of A, B, C, D are as shown in the diagram above.

How many people can A see in the mirror?

- (a) 0 (b) 1 (c) 2 (d) 3

Solution: It is important to note that person A can see only those people in the mirror, the line joining whose mirror image and A intersects the mirror PQ.

We draw the diagram approximately to scale.

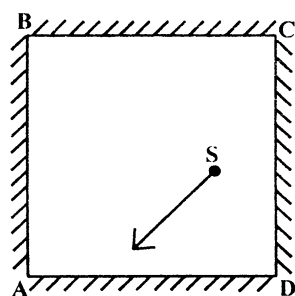


We can see that the line joining A and the mirror image of B does not intersect the mirror PQ and hence A cannot see B in the mirror.

Based on the above logic, A cannot see C but can see D in the mirror. Needless to say, A cannot see himself in the mirror.

Hence, **option b**.

Example 17:

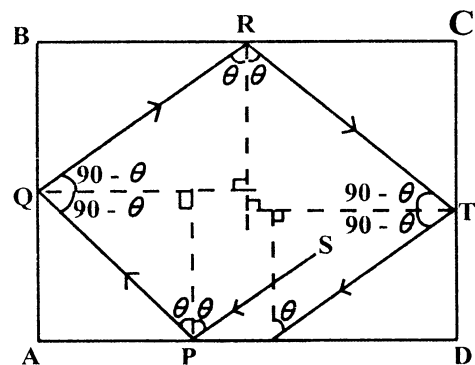


Four perpendicular plane mirrors AD, AB, BC and CD are placed such that they form an enclosed structure. (Assume them to be sufficiently large).

A beam of light strikes the mirror AD in the direction S (as shown in the figure). Find the direction of the beam of light after the 4th reflection.

- (a) Parallel to S and in the direction as S
- (b) Perpendicular to S
- (c) Parallel to S and in the direction opposite to S
- (d) We cannot determine the exact direction

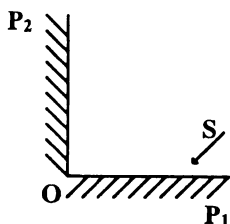
Solution:



Let us assume that the beam reflects at point P first, then Q, then R and at T, such that points P, Q, R and T lie on AD, AB, BC and CD respectively.

In the beginning, the beam makes an angle of θ with the normal at P then angle $90 - \theta$ after second reflection with the normal at Q, then angle θ with the normal at R and then angle $90 - \theta$ with normal at T. Thus after the fourth reflection, the beam makes an angle of θ with the normal to mirror AD, which is parallel to the direction of S and in the same direction as that of S.
Hence, **option a.**

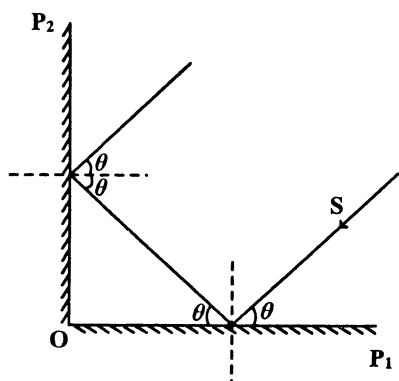
Example 18: Consider the figure given below and answer the item that follows: (CSAT 2011)



In the figure shown above, OP_1 and OP_2 are two plane mirrors kept perpendicular to each other. S is the direction of a beam of light falling on the mirror OP_1 . The direction of the reflected beam of light from the mirror OP_2 will be

- (a) Perpendicular to the direction S .
- (b) At 45° to the direction S .
- (c) Opposite and parallel to the direction S .
- (d) At 60° to the direction S .

Solution:



The fundamental law of reflection states that the angle of incidence is equal to the angle of reflection.

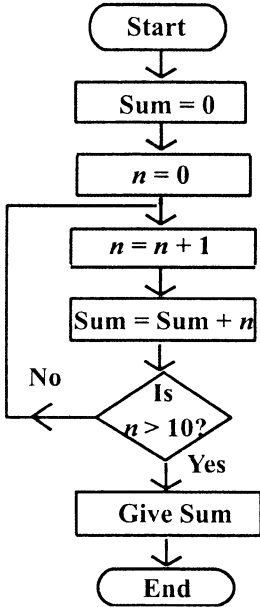
Using this principle, we plot the path of the ray.

We observe that both the incident and reflected rays make equal angles with the line OP_1 and make the same angle with the normal to the line OP_2 . Thus, they are parallel and oppositely directed.

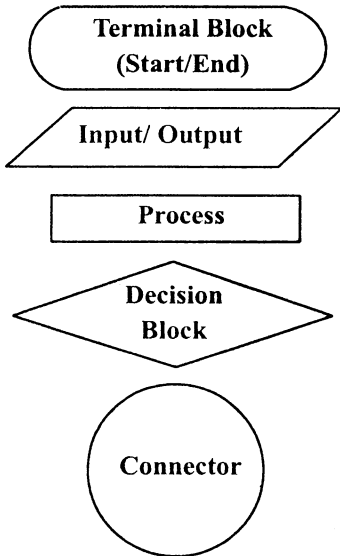
Hence, **option c.**

B. FLOWCHARTS

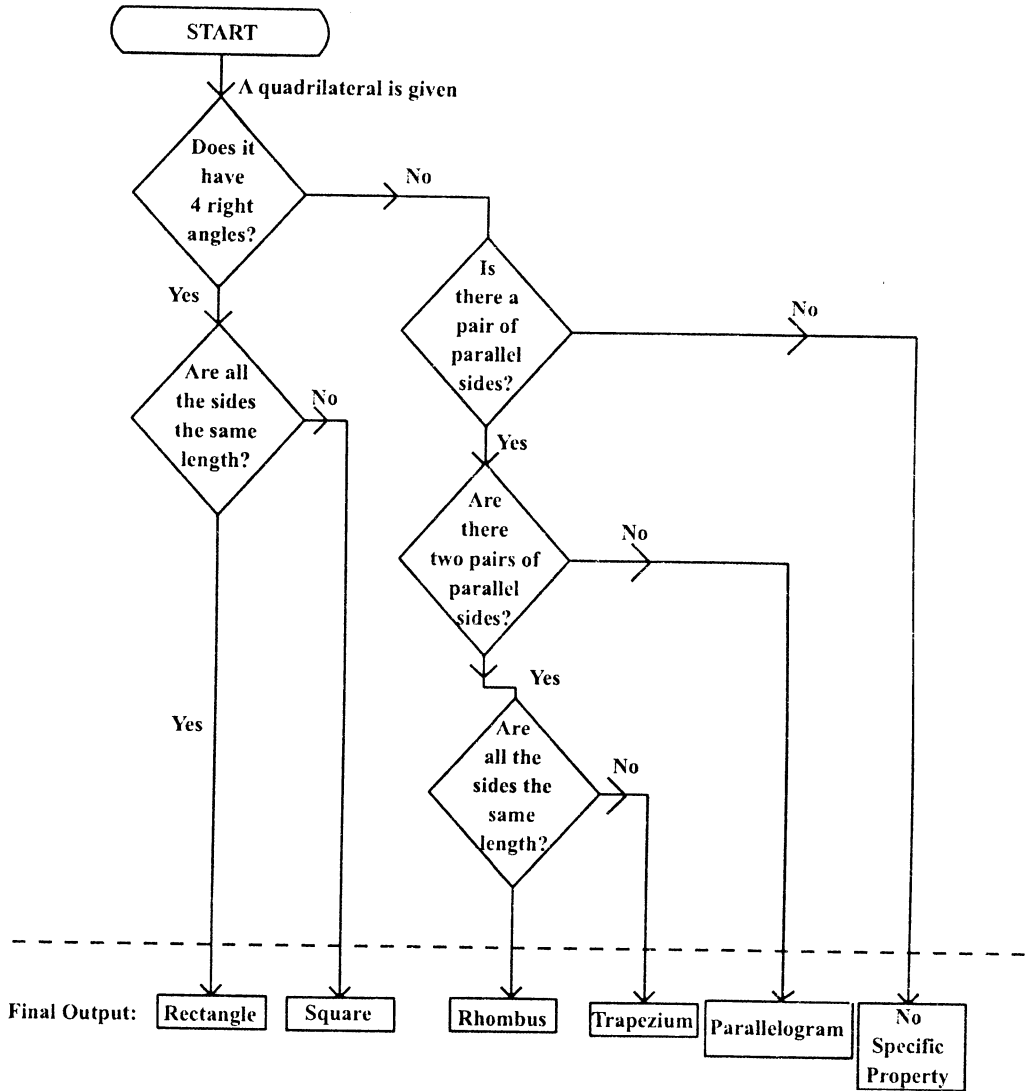
Flowcharts are diagrams used to represent processes. They give step by step solutions to problems. A simple flowchart to find the sum of 10 natural numbers is as follows:



The different symbols commonly used in flowcharts are as follows:



Example 19: The flowchart below helps identify the nature of a given quadrilateral.



Which of the following should be done to correct the flowchart?

1. Rectangle should be replaced by square and square should be replaced by rectangle.
2. Rhombus should be replaced by trapezium and vice - versa.
3. Trapezium should be replaced by parallelogram and vice - versa.
4. Rhombus should be replaced by square and vice - versa.

- (a) Only statement (4) is correct (b) Only statements (1) and (2) are correct
 (c) Only statements (1) and (3) are correct (d) More than 2 statements are correct.

Solution: When a quadrilateral has all right angles and all sides equal, then it is a square. When only opposite sides are equal then it is a rectangle.

Hence, statement (1) is correct.

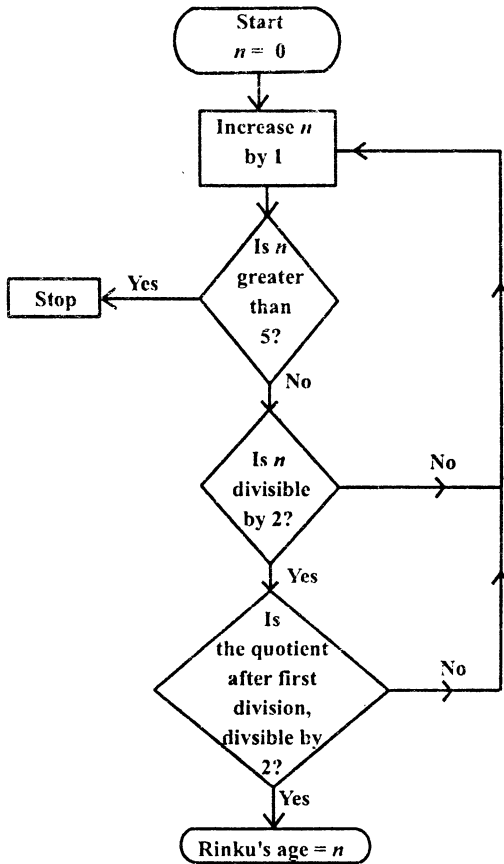
When a quadrilateral has only one pair of sides parallel, it is a trapezium and when it has both pairs parallel with or without having all sides equal it is a parallelogram.

Hence statement (3) is correct.

No other statements are correct.

Hence, **option c.**

Example 20: It is Rinku's n^{th} birthday today. However her absentminded uncle who is abroad does not know how old Rinku is. He however knows that she must be within 1 to 5 years old (both inclusive). Rinku's brother Chinku plans to let his uncle know how old Rinku is in a unique way. What is Rinku's age?



(a) 1

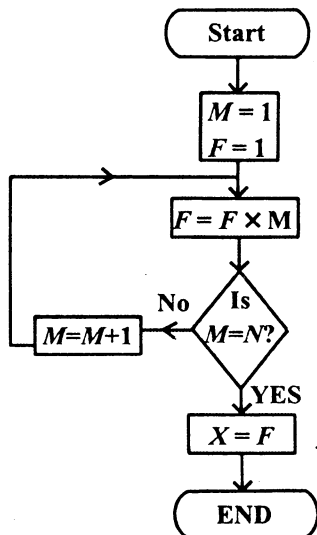
(b) 2

(c) 4

(d) 5

Solution: Let us start with option (a), $n = 1$
 When $n = 1$; n is not divisible by 2. Hence it is not Rinku's age.
 When $n = 2$; n is divisible by 2 and quotient is 1.
 This quotient is not divisible by 2 and hence 2 is not Rinku's age.
 When $n = 4$; n is divisible by 2 and quotient is 2.
 This quotient is divisible by 2 and hence Rinku's age is 4.
 Hence, **option c.**

Example 21:



In the above flow chart, what will be the value of X when we put, $N = 4$?

- (a) 16 (b) 10 (c) 24 (d) 4

Solution: Let us start with $M = 1$ and $F = 1$

$F = 1 \times 1 = 1$... (\because for $M = 1$, we have $F = 1$)

and $M \neq N$... ($\because N = 4$ and $M = 1$)

$\therefore M = M + 1 = 1 + 1 = 2$

Now,

$F = 2 \times 1 = 2$... (\because For $M = 2$, we have $F = 2$).

and $M \neq N$... ($\because N = 4$ and $M = 2$)

$\therefore M = M + 1 = 2 + 1 = 3$.

Now,

$F = F \times M$

$\therefore F = 2 \times 3$

$\therefore F = 6$... (\because For $M = 3$ we have $F = 6$).

and $M \neq N$... ($\because N = 4$ and $M = 3$).

$\therefore M = M + 1 = 3 + 1 = 4$

Now,

$F = F \times M$

$\therefore F = 6 \times 4$

$\therefore F = 24$... (\because For $M = 4$ we have $F = 24$)

and $M = N$... ($\because N = 4$ and $M = 4$)

$\therefore X = F = 24$

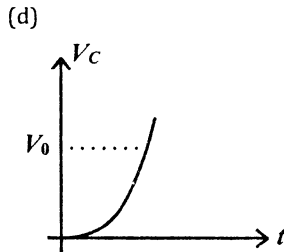
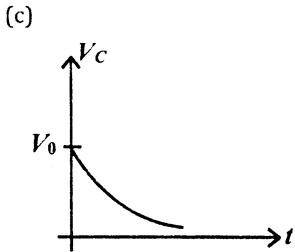
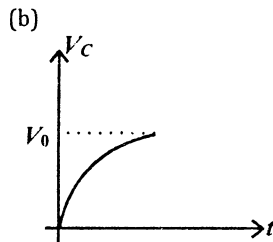
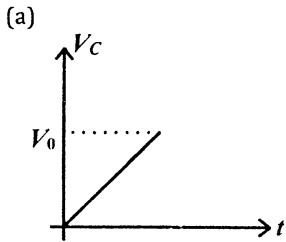
Hence, option c.

C. OTHER USEFUL DIAGRAMS

Population Pyramids

A **population pyramid** is a graphical representation of the distribution of various age groups in a given group of people. Population pyramids can be drawn for cities, countries, regions, communities etc.

A typical population pyramid has two graphs. The y-axis has the age. The x-axis shows the number of males on one side and the number of females on the other.



Solution: As we can see from the equation,

$$V_c = V_0 (1 - e^{-t/RC}),$$

At $t = 0$, V_c should be equal to 0. Hence, option (c) is eliminated. Option (a) is a linear graph and hence cannot be the answer. Hence, option (a) can be eliminated as well.

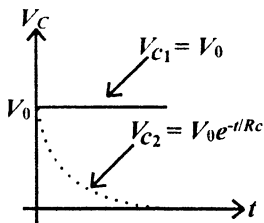
We are left with options (b) and (d).

$$V_c = V_0 (1 - e^{-t/RC})$$

$$V_c = V_0 - V_0 \cdot e^{-t/RC}$$

$V_{c1} = V_0$ is a constant graph. $V_0 e^{-t/RC}$ has the nature of an exponential function.

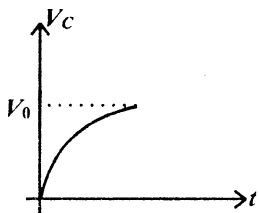
So, $V_{c2} = V_0 e^{-t/RC}$ can be drawn as given below.



Required equation $V_c = V_{c1} - V_{c2}$.

That means we need to subtract the graph of V_{c2} from V_{c1} .

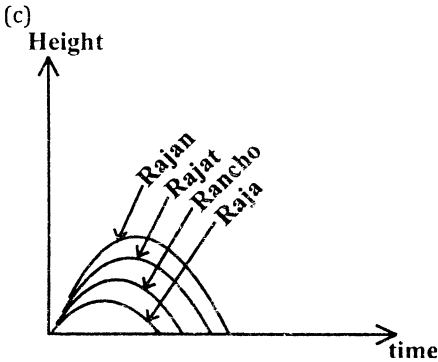
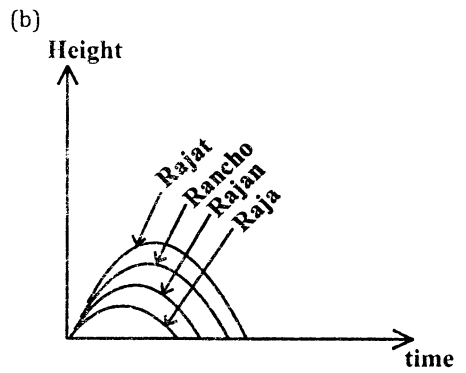
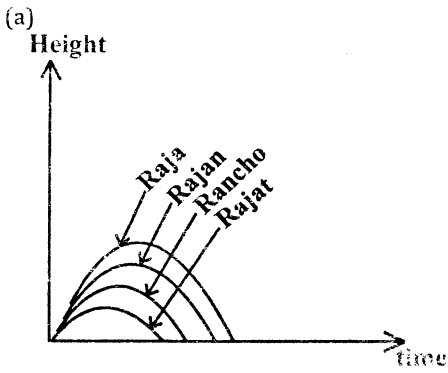
After subtraction we get,



Hence, **option b**.

Example 24: 4 brothers Raja, Rajan, Rancho and Rajat throw a stone vertically upwards with different velocities. It is known that the magnitude of velocities with which the stones are thrown is in the following order:

Raja throws the stone with a velocity greater than Rajan, who in turn throws the stone with a velocity greater than Rancho, who in turn throws the stone with a velocity greater than Rajat. Which of the following graphs depicts the heights attained by the stones thrown by each of them? Neglect the heights of the brothers.



(d) Cannot be determined

Solution: It is given that magnitudes of velocities of stones thrown by them are in the order:

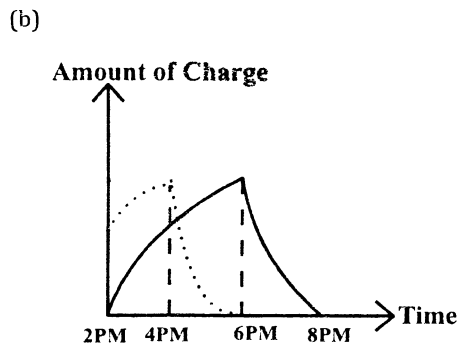
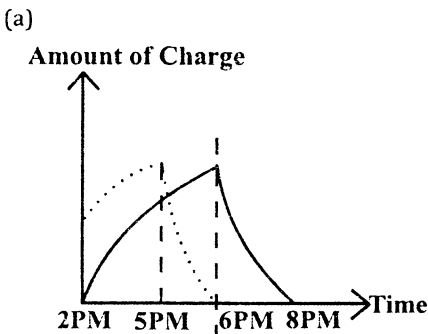
$$V_{Raja} > V_{Rajan} > V_{Rancho} > V_{Rajat}$$

Now, greater the magnitude of the velocity, more time is spent in the air and more the time spent in the air, more is height attained.

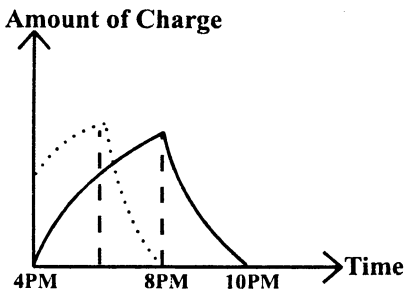
Hence, **option a.**

Example 25: Suresh has a new phone which takes 4 hours to charge and 2 hours to discharge. Yesterday he started charging it at 4 pm. After it charged completely he used the phone immediately and the phone started discharging.

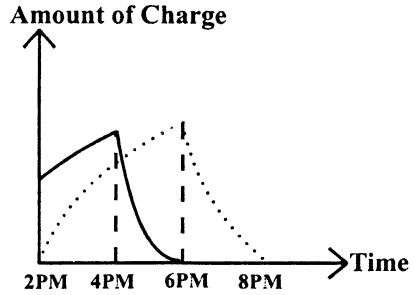
Today, he decides to charge the phone at 2 pm and then use it immediately after it charges fully. How will the graph look like for yesterday and today? The dotted line shows the charging and discharging for today and the unbroken line shows the same for yesterday. (Assume the phone had zero charge initially on both the occasions.)



(c)



(d)

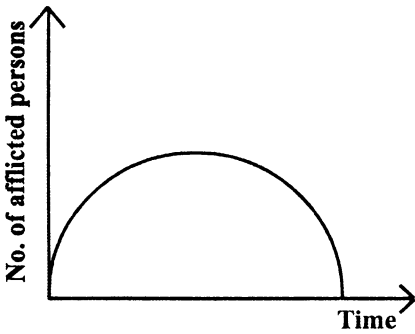


Solution: Yesterday the phone started charging at 4pm. That means it had zero charge at 4pm yesterday. In the options only option (c) has such a situation. Hence, **option c.**

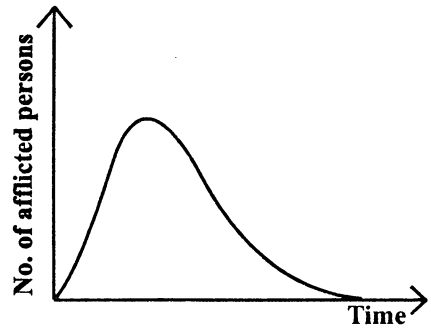
TEST 1

1. The jolly island city of Tahili witnessed a sudden and rapid outbreak of an epidemic. The biomedical labs got into action immediately and came up with a very cheap drug that cured the patient of this affliction completely in a matter of time. Ultimately, the disease was completely eradicated from the city. Which of the following curves is a fairly accurate representation of the impact of the epidemic?

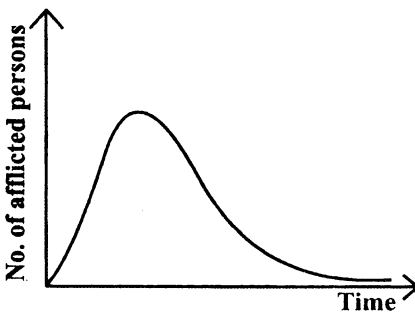
(a)



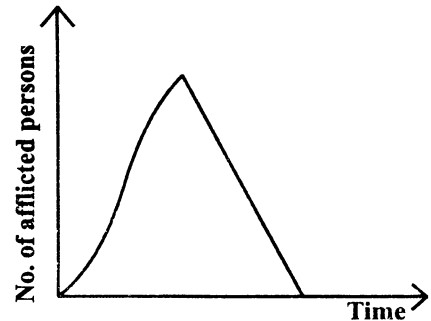
(b)



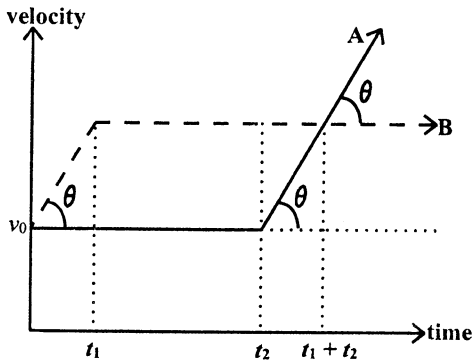
(c)



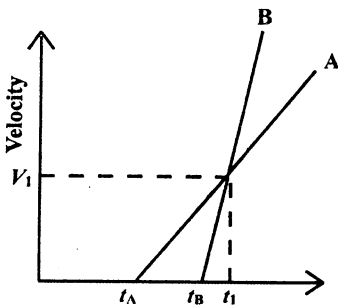
(d)



2. Two particles, that can move only along the x -axis, start together from the origin at $t = 0$. With the help of the following graph, determine the time when they meet again?



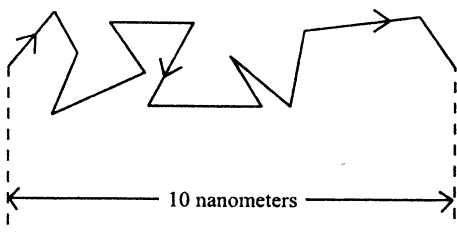
- (a) $2(t_1 + t_2)$ (b) $(t_1 + t_2) + \frac{4t_1 t_2}{(t_1 + t_2)}$
 (c) $t_1 + t_2 + \sqrt{2t_1 t_2}$ (d) They never meet again
3. The relative velocity of a particle A with respect to another particle B is defined as the velocity of A as seen by (measured by) B.



It is known that A and B start at the origin and can move only along the x -axis. With the help of the above graph, determine which of the following alternatives is incorrect:

- (a) The distance between A and B at $t = t_1$ is $\frac{v_1(t_B - t_A)}{2}$
 (b) Relative velocity of A with respect to B at $t = t_B$ is $\frac{2v_1 t_B}{t_1 - t_A}$
 (c) A is ahead of B for $t \leq t_1$.
 (d) Relative velocity of B with respect to A is equal to the relative velocity of A with respect to B at $t = t_1$.

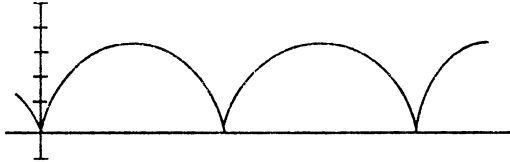
- 4.



The curve shown above indicates the path followed by an electron in an interval of 2 seconds inside a conducting material when a potential difference is applied across its ends. What is the average velocity of the electron in this time interval from point A to point B?

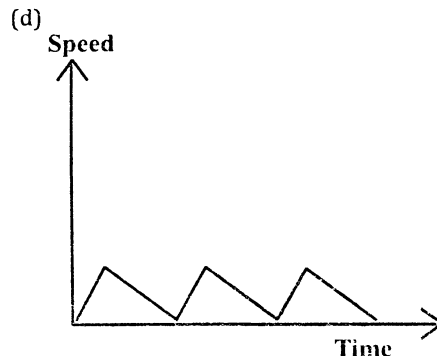
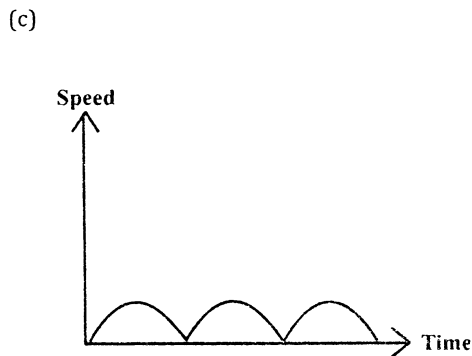
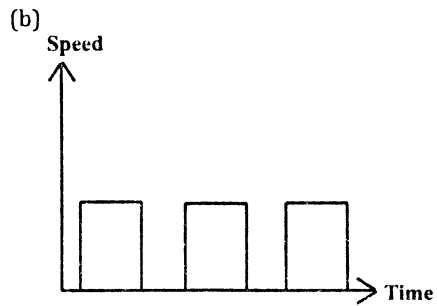
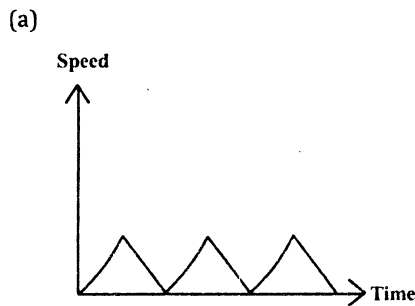
- (a) 5 nanometers per second (b) 10 nanometers per second
 (c) 7.5 nanometers per second (d) Cannot be determined

5. A circular wheel of radius 2 cm is undergoing pure rotation (i.e., the wheel does not slip on the surface while rolling). At some point, a grain of sand gets stuck on the rim of the wheel. The following curve shows the path followed by the grain while the wheel rolls on the ground with a uniform velocity. This curve is known as a cycloid.

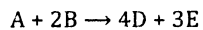


What is the horizontal distance travelled by the wheel till the grain touches the ground again?

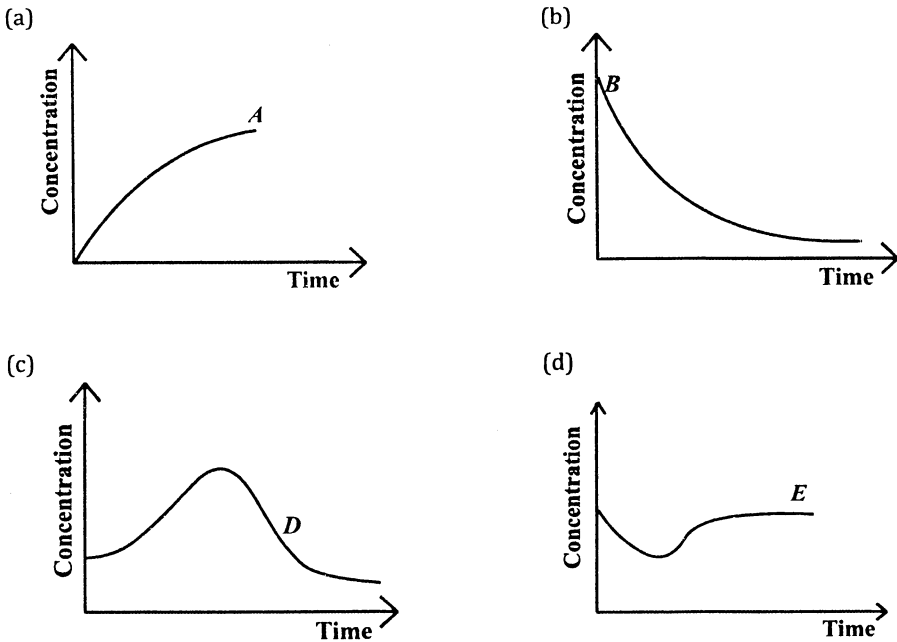
- (a) 4 cm (b) $2e^2$ cm (c) $2(1 + \sqrt{3})$ cm (d) 4π cm
6. Which of the following curves best describes the motion of the lowest point on the pendulum of a clock?



7. A chemical reaction is represented by the following equation:

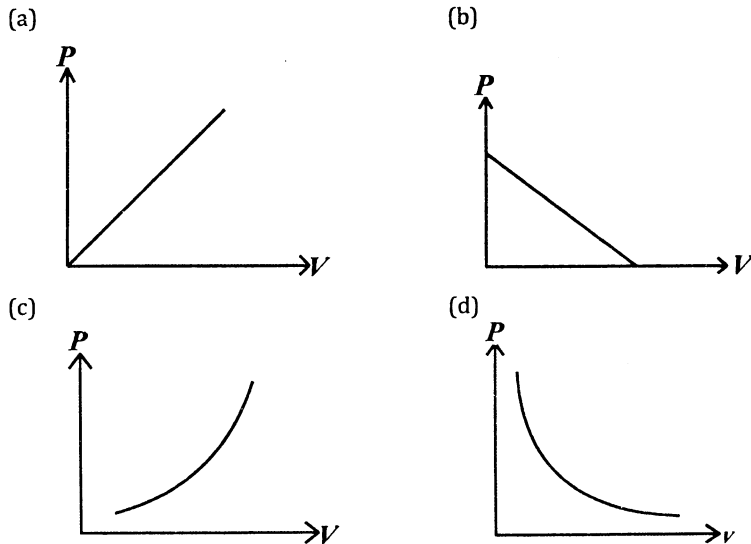


Which of the following graph gives the correct variation of the amount of each of these substances in the reaction mixture as a function of time?



8. The Boyle's Law in Thermodynamics states that: 'For a fixed amount of an ideal gas at a fixed temperature, the pressure P and volume V are inversely proportional.'

Which of the following represents a correct relationship between P and V ?



9. The ideal gas equation in Thermodynamics gives the following relationship between the pressure P , volume V and temperature T for a fixed amount of an ideal gas: $PV = nRT$.

Here, n and R are constants.

An isothermal process is one in which the temperature of the system remains constant while the other variables may change.

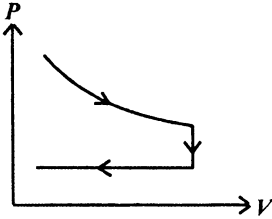
An isobaric process is one in which the pressure of the system remains constant.

An isochoric process is one in which the volume of the system remains constant.

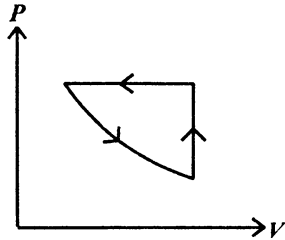
An adiabatic process is one in which no heat is exchanged between the system and the surroundings.

An ideal gas at a certain initial temperature T undergoes isothermal expansion. This is then followed by an isochoric process which is further followed by isobaric compression so that the final temperature of the gas is T . Which of the following graphs shows these transitions correctly?

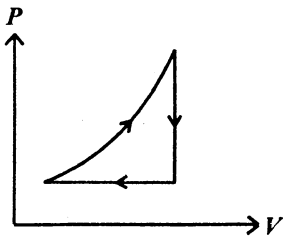
(a)



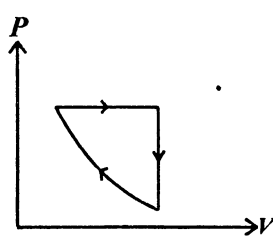
(b)



(c)



(d)



10. When a ray of light passes from one medium into another, it changes its direction. According to Snell's Law, the ratio of the sines of the angles of incidence and refraction is equivalent to the reciprocal of the ratio of the refractive indices (μ) of the two media.

The angle of incidence (i) is defined as the angle made by the incident ray with the normal to the boundary separating the two media. Similarly, the angle of refraction (r) is defined as the angle made by the refracted ray with the normal.

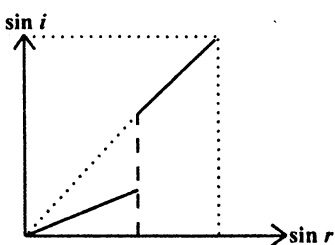
For example, the refractive index of a typical glass sample is 1.5 and that of air is 1.

Snell's law implies that when a ray of light passes from a denser medium (one with a higher refractive index) to a rarer medium (one with a lower refractive index), it bends away from the normal.

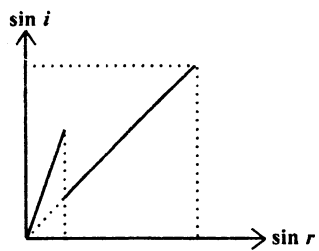
When light travels from a denser medium to a rarer medium, Snell's law seems to require in some cases (whenever the angle of incidence is large enough) that the sine of the angle of refraction be greater than one. This is, of course, impossible and the light in such cases is completely reflected by the boundary, a phenomenon known as total internal reflection. The largest angle of incidence which still results in a refracted ray is called the critical angle; in this case the refracted ray travels along the boundary between the two media.

Which of the following graphs correctly represents the relationship between the angle of incidence and the angle of refraction/reflection for a ray travelling from glass (refractive index $= \mu = 1.5$) into air?

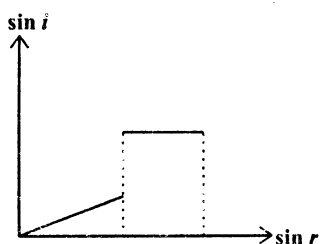
(a)



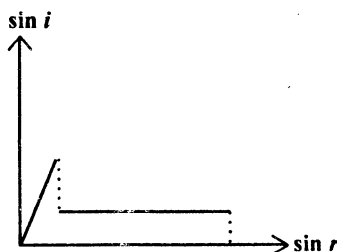
(b)



(c)

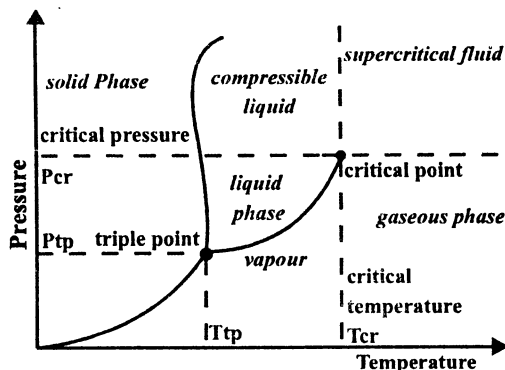


(d)



TEST 2

11. The following graph shows the phase diagram for water.



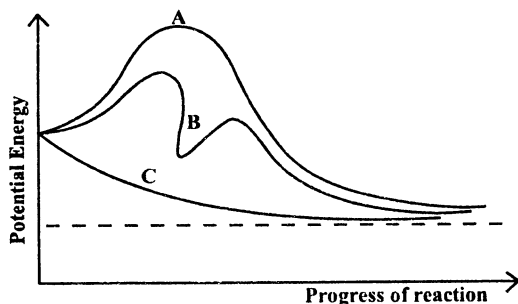
In thermodynamics, the triple point of a substance is the temperature and pressure at which the three phases (solid, liquid and gas) of that substance coexist in thermodynamic equilibrium. For water, the triple point occurs at 0.01°C and a pressure of 611.73 pascals.

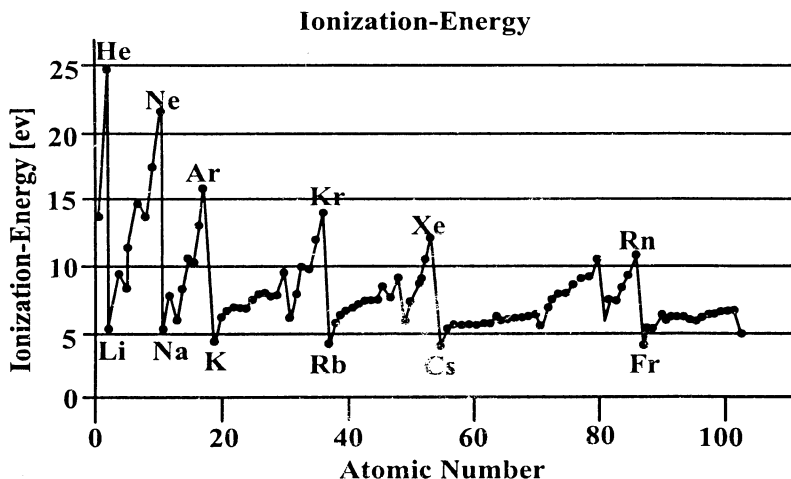
On the other hand, the critical point is the temperature and pressure at which a phase boundary ceases to exist and it is not possible to differentiate between the two respective phases.

Which of the following statements is incorrect?

- (a) When the temperature of water, which was at the triple point, is decreased without altering the pressure, it passes into the solid state.
- (b) When the pressure of water, which was at the triple point, is decreased without altering the temperature, it passes into vapour state.
- (c) If the temperature of compressible water is increased and its pressure is decreased, it will attain gaseous state.
- (d) If water at the critical point is depressurised while keeping the temperature constant, it will solidify.

12. The above graph shows the potential energy versus reaction progress for three chemical reactions. The troughs indicate stable substances.



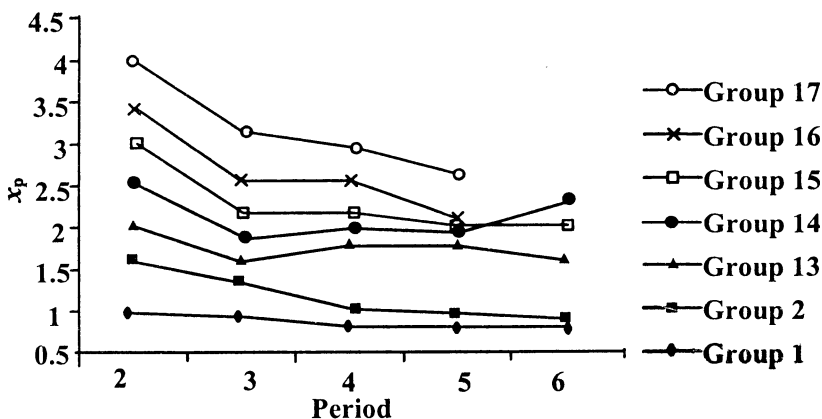


Which of the following statements is incorrect?

- (a) Among all the elements, Helium (He) has the highest ionisation energy.
- (b) The ionisation energy for the inert gases decreases as we move down the group.
- (c) The ionisation energy of Mercury (Hg) is more than that of Potassium (K).
- (d) Across any period, the ionisation energy keeps increasing as we move to the right.

14. The electronegativity of an element describes the tendency of an atom of that element to attract electrons towards itself.

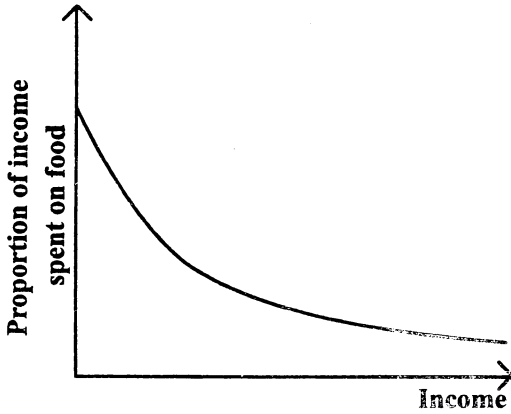
The following graph shows the variation of the electronegativity of elements as we move down a group, for different groups.



Using the Periodic table and the above graph, determine which of the following statements is incorrect.

- (a) As we move across to the right in Period 2, the electronegativity of the elements increases.
- (b) Elements of Group 17 have the highest electronegativity in a group.
- (c) Chlorine (Cl) is the element with the highest electronegativity.
- (d) The electronegativity of Lithium (Li) is less than that of Iodine (I).

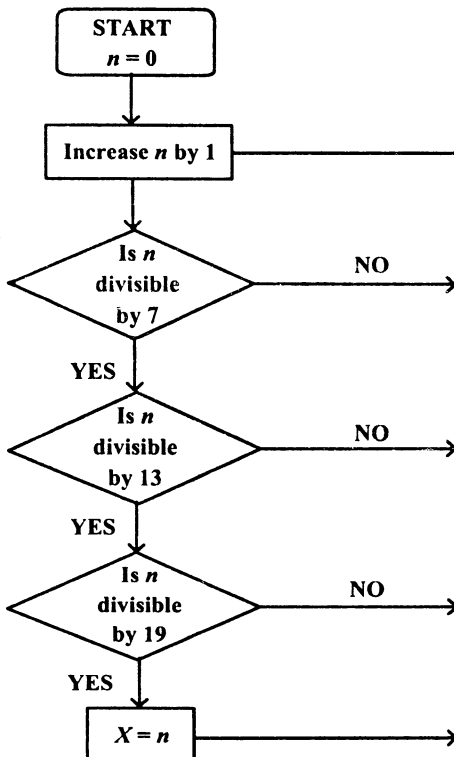
15.



The above curve is called the Engel's law. Based on the curve which of the following statements can be inferred?

- (a) People in lower income groups spend more on food than people in higher income groups.
- (b) People in higher income groups need less food.
- (c) People in higher income groups have a larger proportion of their income available for spending on non-food items.
- (d) People in lower income groups save less than people in higher income groups.

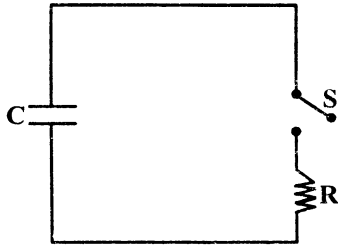
16. By using the flow chart given below, find the remainder when the fourth possible value of X is divided by 24.



- (a) 2 (b) 4 (c) 6 (d) 8

Answer questions 17 to 18 based on the following information:

Consider the electrical circuit shown below. S denotes a switch, R denotes a resistor and C denotes a capacitor.



The switch S is closed at time $t = 0$.

The capacitor C was charged initially with charge Q_0 .

The rate of discharge of the capacitor $\left(\frac{dQ}{dt}\right)$

is directly proportional to the charge (Q) on the capacitor, at any instant. It can be shown that

$$\frac{dQ}{dt} = \frac{-Q}{RC},$$

where C is the capacitance and R is the resistance.

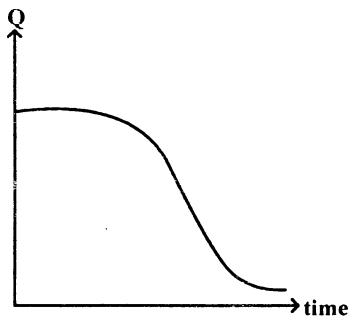
Using this and applying integration techniques,

$$\text{We find that } Q = Q_0 e^{\frac{-t}{RC}}.$$

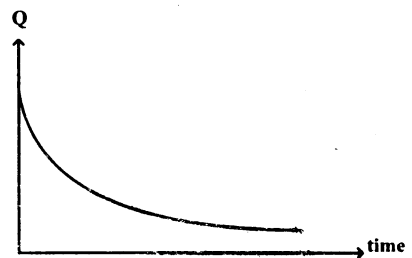
This equation gives the variation of the charge on the capacitor with time.

17. Which of the following graphs correctly represents the variation of the charge on the capacitor with time?

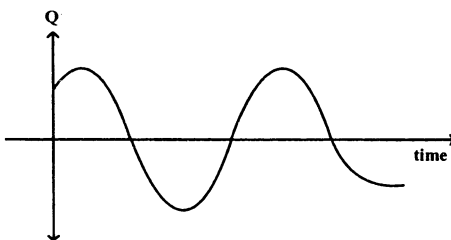
(a)



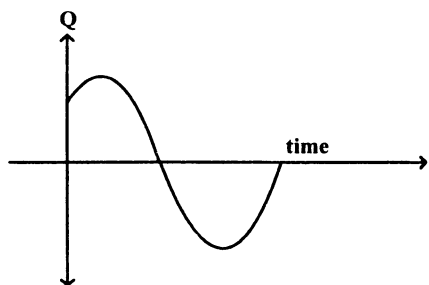
(b)



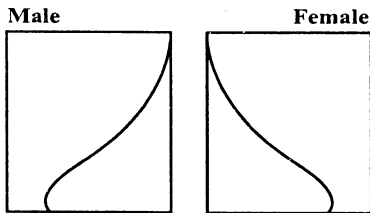
(c)



(d)



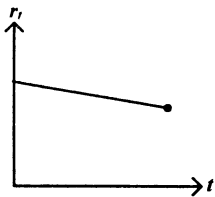
18. Which of the following statements regarding the above circuit is incorrect?
- (a) The time required for the charge on the capacitor to halve to $Q_0/2$ is unchanged if R is doubled and C is halved.
 - (b) The time required for the charge on the capacitor to decrease from any value k to $k/2$ is independent of k .
 - (c) Theoretically, it takes infinite time for the capacitor to discharge completely.
 - (d) The time required for the charge on the capacitor to halve to $Q_0/2$ is doubled if R and C are both doubled.
19. Which of the following is true for the given population pyramid? The horizontal axis shows the population of the region and the vertical axis shows the age.



- (a) The average life expectancy is low.
 - (b) The death rate is high.
 - (c) Both 1 and 2 are correct.
 - (d) Neither 1 nor 2 is correct.
20. Future value (or price) of certain commodity is given by,

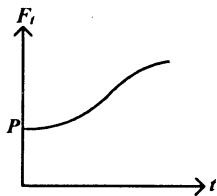
$$F_t = P \times e^{r_t t}$$

Here, F_t is the future value of the commodity at time t , r_t is risk free rate at time t , and t is the time.

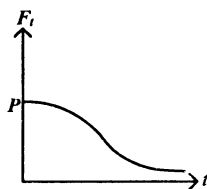


If the graph above depicts the risk free rate over time period t , then which of the following diagrams depicts the future value of the commodity over time period t ?

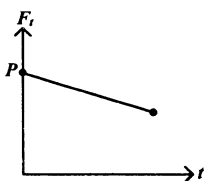
(a)



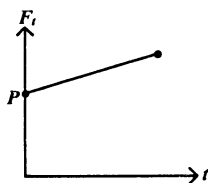
(b)



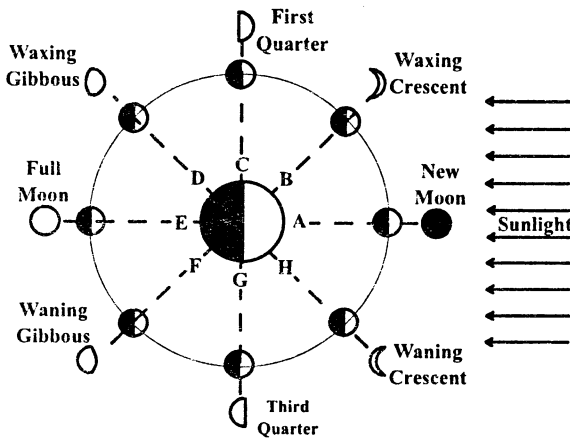
(c)



(d)



Answer questions 21 to 24 based on the following information:



The diagram above shows different phases of the moon. The moon is orbiting the earth in the anticlockwise direction and the earth rotates around its axis in the anticlockwise direction. The dotted line from the earth to moon shows the line of sight of the observer.

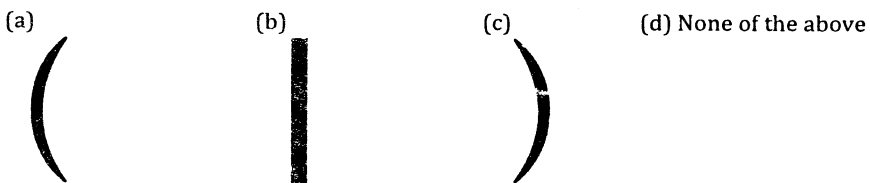
The orbits of the earth and the moon are in different planes though the diagram above is drawn in a single plane.

A Crescent refers to that phase of the moon which appears less than half illuminated. A Gibbous moon is the one which appears more than half illuminated.

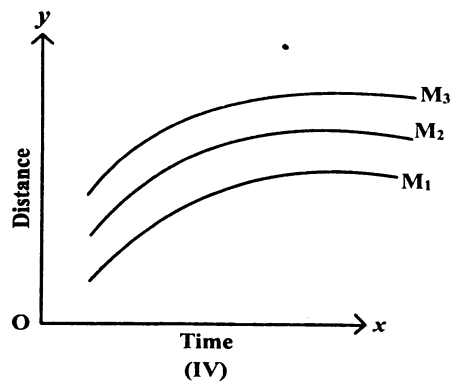
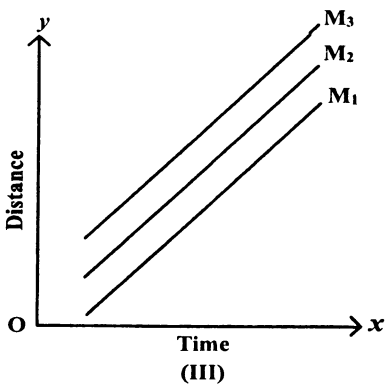
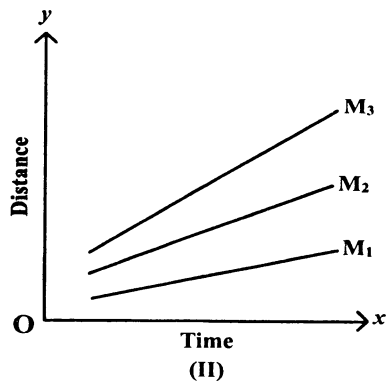
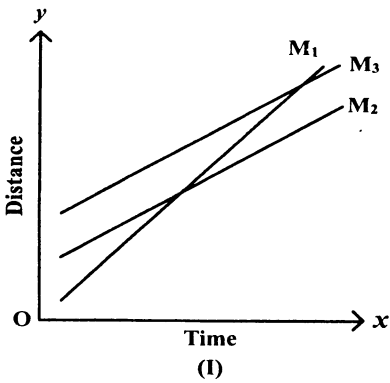
A waxing moon is a moon that is growing in size and a waning moon is a moon that is shrinking in size. Thus, the phases of the moon from the new moon to the full moon are waxing phases and the phases of the moon from the full moon to the new moon are waning phases.

The Moon is 3500 km in diameter and the Earth, 12,800 km wide. The distance between the centres of the earth and the moon is approximately 3,84,000 km.

21. A person standing at position B in the figure sees the waxing crescent of the moon. Which phase of the moon does a person standing at position H see at the same time?
 (a) Waxing Gibbous (b) Waning Gibbous (c) Waxing Crescent (d) Waning Crescent
22. The moon is at such a position that A experiences a new moon night. What phase of the moon will E see?
 (a) Full Moon (b) New Moon (c) First Quarter (d) Third Quarter
23. The waxing crescent can
 (a) never be seen during day time. (b) never be seen during night time.
 (c) never be seen during afternoon. (d) be seen only during night time.
24. In a solar panel used for converting solar energy into light energy, a certain kind of mirror is used wherein the sun rays after reflection meet at a particular point. Assuming that the sun is at an infinite distance from the mirror, which of the following is the correct diagram for the mirror used in Solar panels?



25. Three friends, M1, M2 and M3, have their speeds in the ratio 1 : 2 : 3. Which of these figures shows the correct relationship between the distance covered by each of them over a certain period of time?



(a) I

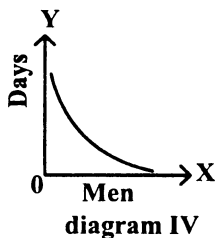
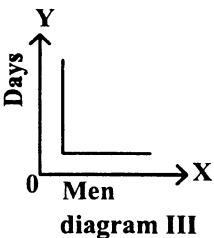
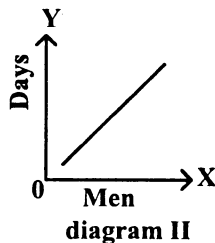
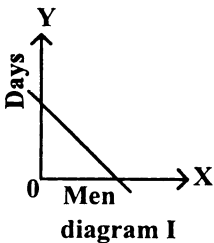
(b) II

(c) III

(d) IV

26. Consider the following diagrams:

x men, working at constant speed, do a certain job in y days. Which one of these diagrams shows the relation between x and y ? [UPSC 2013]



(a) diagram I

(b) diagram II

(c) diagram III

(d) diagram IV

DATA INTERPRETATION AND ANALYSIS

TEST 1

1. The total number of buildings constructed across all 6 cities in 2006 = 9207
The total number of buildings constructed across all 6 cities in 2009 = 10384

$$\therefore \text{The required percentage} = \left(\frac{9207}{10384}\right) \times 100 = 88.66\%$$

Hence, **option d**.

2. The total number of buildings constructed in 2005 = 8636
The total number of buildings constructed in 2008 = 9990

$$\therefore \text{The required difference} = 9990 - 8636 = 1354$$

Hence, **option a**.

3. The total number of buildings constructed across all 6 cities over the given period = 8636 + 9207 + 9570 + 9990 + 10384 + 11104 = 58891

Hence, **option e**.

4. Required percentage increase

$$= \frac{[(2045 - 1950) \times 100]}{1950} = 4.87\%$$

Hence, **option e**.

5. The total number of buildings constructed in Allahabad over the given period = 8572. The total number of buildings constructed in Pune over the given period = 11394

$$\therefore \text{The required ratio} = 8572 / 11394$$

Hence, **option e**.

6. Total number of students who appeared for the examination in 2008 = 325 + 312 + 768 + 690 + 614 = 2709
Total number of students who appeared for the examination in 2009 = 430 + 325 + 815 + 716 + 576 = 2862

$$\text{Required ratio} = 2709 : 2862 = 301 : 318$$

Hence, **option b**.

7. The pass percentage of each school in 2010 is given below:

$$\text{School A} = \left(\frac{390}{412}\right) \times 100 = 94.66\%$$

$$\text{School B} = \left(\frac{290}{345}\right) \times 100 = 84.05\%$$

$$\text{School C} = \left(\frac{714}{845}\right) \times 100 = 84.49\%$$

$$\text{School D} = \left(\frac{596}{728}\right) \times 100 = 81.86\%$$

$$\text{School E} = \left(\frac{354}{542}\right) \times 100 = 65.31\%$$

Thus, School A had the highest pass percentage.

Hence, **option a**.

8. For school C, the passing percentage for each year is

$$2006 : \left(\frac{448}{745}\right) \times 100 = 60.13\%$$

$$2007 : \left(\frac{426}{712}\right) \times 100 = 59.69\%$$

$$2008 : \left(\frac{321}{768}\right) \times 100 = 41.79\%$$

$$2009 : \left(\frac{756}{815}\right) \times 100 = 92.76\%$$

$$2010 : \left(\frac{714}{845}\right) \times 100 = 84.49\%$$

$$2011 : \left(\frac{628}{932}\right) \times 100 = 67.38\%$$

Thus, the pass percentage was lowest for school C in 2008.

Hence, **option c**.

9. Total number of students who appeared from all the schools together in 2011

$$= 365 + 334 + 932 + 744 + 522 = 2897$$

Total number of students who passed from all the schools together in 2011

$$= 340 + 305 + 628 + 686 + 104 = 2063$$

\therefore The required percentage

$$= (2063/2897) \times 100 = 71.21\%$$

Hence, **option d**.

10. Average number of students that passed from school B over the given period

$$= (264 + 270 + 272 + 276 + 290 + 305) / 6 = 279.5$$

Average number of students that passed from school D over the given period

$$= (320 + 380 + 455 + 648 + 596 + 686) / 6 = 514.2$$

\therefore The required ratio = 279.5 : 514.2

Hence, **option b**.

TEST 2

11. Number of boys studying Commerce is 416 which is 40% of the total number of boys.

∴ Total number of boys in the institute

$$= \frac{(416 \times 100)}{40} = 1040$$

Since girls are 35% of the total number of students in the institute, boys form the remaining 65%.

∴ Total number of girls in the institute

$$= \frac{(1040 \times 35)}{65} = 560$$

45% of the girls study Arts.

∴ The total number of girls studying Arts

$$= \frac{(45 \times 560)}{100} = 252$$

Number of boys and girls studying Commerce is in the ratio of 4 : 1.

∴ The total number of girls studying Commerce = $416/4 = 104$

∴ The total number of girls studying Science

$$= 560 - 252 - 104 = 204$$

25% of boys study Science

∴ Number of boys studying Science

$$= \frac{(25 \times 1040)}{100} = 260$$

∴ The total number of boys studying Arts

$$= 1040 - 416 - 260 = 364$$

Thus we get the following table:

Subject	Boys	Girls
Arts	364	252
Commerce	416	104
Science	260	204

Thus, 204 girls study Science in the institute.

Hence, **option e**.

12. Consider the table obtained in the solution to the first question.

Number of girls studying Commerce = 104

Number of boys studying Science = 260

∴ Required percentage

$$= (104 / 260) \times 100 = 40\%$$

Hence, **option d**.

13. Consider the table obtained in the solution to the first question.

Number of boys studying Arts = 364

Number of girls studying Arts = 252

∴ Required ratio = $364 : 252 = 13 : 9$

Hence, **option c**.

14. Consider the table obtained in the solution to the first question.

Total number of boys and girls in the institute is 1040 and 560 respectively.

∴ The total number of students in the institute = $1040 + 560 = 1600$

Hence, **option a**.

15. Total number of people in the city who read newspapers

$$= 5478 + 1420 + 2684 + 2060 + 4686 + 4062 + 2466 + 1540 + 3542 = 27938$$

5478 people read only Times of India, 2466 people read Times of India as well as Employment News and 3542 people read Times of India as well as The Hindu.

∴ The required percentage

$$= (5478 + 2466 + 3542) \times 100 / 27938$$

$$= (11486/27938) \times 100$$

$$= 41.11\% \approx 41\%$$

Hence, **option b**.

16. Total number of people in the city reading some newspaper = 27938

4062 people read only Employment News, 2466 people read Times of India as well as Employment News and 1540 people read Economic Times as well as Employment News.

∴ Required percentage

$$= (4062 + 2466 + 1540) \times 100 / 27938$$

$$= (8068/27938) \times 100 = 28.88\% \approx 29\%$$

Hence, **option c**.

17. Total number of people in the city reading some newspaper = 27938

2684 people read only Hindustan Times and 2060 people read only DNA.

∴ The required Percentage

$$= (2684 + 2060) \times 100 / 27938$$

$$= (4744/27938) \times 100 = 16.98 \approx 17\%$$

Hence, **option d**.

18. Total number of people in the city who read some newspaper = 27938

4686 people read only The Hindu.

∴ The required percentage

$$= 4686 \times 100/27938$$

$$= 16.77 \approx 17\%$$

Hence, **option e**.

19. Total number of students = 2400

The ratio of girls to boys is 7 : 5

∴ Total numbers of girls

$$= \frac{(2400 \times 7)}{12} = 1400$$

and, total number of boys

$$= \frac{(2400 \times 5)}{12} = 1000$$

The total number of boys taking table-tennis classes = $(30 \times 1000)/100 = 300$

The total number of girls taking carom classes = $1400/4 = 350$

The total number of students taking carom classes is 650.

\therefore The total number of boys taking carom classes = $650 - 350 = 300$

The total number of boys taking chess classes = $1000/5 = 200$

\therefore The total number of boys taking badminton classes are = $1000 - 300 - 300 - 200 = 200$

\therefore The total number of girls taking chess classes = $3 \times 200 = 600$

\therefore The total number of girls taking badminton classes = $(3/5) \times 200 = 120$

\therefore The total number of girls taking table-tennis classes = $1400 - 350 - 600 - 120 = 330$

Thus, the final table becomes,

Classes	Girls	Boys
Chess	600	200
Badminton	120	200
Table-Tennis	330	300
Carom	350	300

Hence, the required ratio = $330 : 200 = 33 : 20$
Hence, **option c**.

20. Consider the table obtained in the solution to the first question.

Number of girls taking carom classes = 350

Total number of children in the school = 2400

$$\therefore \text{Required percentage} = \left(\frac{350}{2400}\right) \times 100$$

$$= 14.58\%$$

Hence, **option a**.

21. Consider the table obtained in the solution to the first question.

The total number of children taking badminton classes = $120 + 200 = 320$

Hence, **option c**.

22. Consider the table obtained in the solution to the first question.

The total number of girls taking table-tennis classes = 330

Hence, **option c**.

TEST 3

23. Profit = Sales - Expenses

Profit for each year (in thousands) is :

$$2006 : 100 - 80 = 20$$

$$2007 : 115 - 85 = 30$$

$$2008 : 125 - 90 = 35$$

$$2009 : 130 - 110 = 20$$

$$2010 : 145 - 105 = 40$$

$$2011 : 120 - 115 = 5$$

Thus, the profit is maximum in 2010.

Hence, **option b**.

24. Expenses in 2008 = 90 thousand

Expenses in 2010 = 105 thousand

$$\therefore \text{Required ratio} = 90 : 105 = 6 : 7$$

Hence, **option a**.

25. Consider the solution to the first question.

The profit in 2006 as well as in 2009 is 20000.

Therefore, the required difference = 0.

Hence, **option e**.

26. Consider the solution to the first question.

Profit (in thousands) over the given period

$$= 20 + 30 + 35 + 20 + 40 + 5 = 150$$

Hence, **option c**.

27. Sales in 2006 = 100 thousand

Sales in 2008 = 125 thousand

Therefore,

$$\text{Percentage} = \left(\frac{125}{100}\right) \times 100 = 125\%$$

Hence, **option e**.

28. Section D has $\left(\frac{30}{100}\right) \times 2500 = 750$ students

10% of this group are girls i.e. there are 75 girls in section D.

$$\therefore \text{Number of boys in section D} = 750 - 75 = 675$$

Hence, **option d**.

29. Section B has 12% of the students

\therefore Angle subtended by the number of students in section B = $(12 \times 360)/100 = 43.2^\circ$

Hence, **option a**.

30. Number of girls in section C = 24% of 25% of 2500

Number of girls in section E = 16% of 15% of 2500

$$\text{Therefore, Ratio} = \frac{24 \times 25 \times 2500}{16 \times 15 \times 2500} = 5 : 2$$

Hence, **option b**.

31. Number of girls in section F

$$= 0.24 \times 0.07 \times 2500 = 42$$

\therefore Number of boys in section F

$$= 175 - 42 = 133$$

Number of girls in section A

$$= 0.16 \times 0.11 \times 2500 = 44$$

$$\text{So percentage} = \frac{133}{44} \times 100 = 302\%$$

Option (c) is the closest answer.

Hence, **option c**.

32. Sections D and E have 30% and 15% of the total students respectively.

$$\text{Difference} = \frac{30 - 15}{100} \times 2500 = 375$$

Hence, **option e**.

TEST 4

33. The maximum difference between exports and imports within the same year is 20 (in the year 1999).

Hence, **option e**.

34. Exports in 1999 = 110

$$\text{Exports in 2003} = 125$$

Therefore,

$$\text{Ratio} = \frac{110}{125} = 22 : 25$$

Hence, **option d**.

35. Imports in 2000 = 175

$$\text{Exports in 2003} = 125$$

Therefore,

$$\text{Percent} = \left(\frac{175}{125} \right) \times 100 = 140\%$$

Hence, **option c**.

36. Sum of exports in 1999 and 2000

$$= 110 + 185 = 295$$

$$\text{Sum of imports in 2002 and 2003}$$

$$= 110 + 135 = 245$$

$$\therefore \text{The required difference} = 295 - 245 = 50$$

Hence, **option b**.

37. The total difference between exports and imports over the given period

$$= (110 + 185 + 115 + 110 + 125) - (90 + 175 + 120 + 110 + 135) = 15$$

Hence, **option a**.

38. Total runs scored

$$= \text{Average runs} \times \text{Total number of matches}$$

$$\text{Total runs scored by India} = 225 \times 50 = 11250$$

$$\text{Total runs scored by Pakistan}$$

$$= 220 \times 35 = 7700$$

$$\therefore \text{Difference} = 11250 - 7700 = 3550$$

Hence, **option b**.

39. Total runs = Average runs \times Total matches

$$\text{Total runs by Australia} = 250 \times 45$$

$$\text{Total runs by W.Indies} = 210 \times 40$$

$$\text{Ratio} = \frac{250 \times 45}{210 \times 40} = 75 : 56$$

Hence, **option a**.

40. This need not be calculated. It can be answered by observation.

Total runs

$$= \text{Average runs} \times \text{Total number of matches}$$

Since Pakistan has the lowest average as well as the lowest total number of matches, Pakistan has the minimum total runs.

Hence, **option e**.

41. W.Indies has the lowest difference between average runs and best runs.

Hence, **option d**.

42. Total runs

$$= \text{Average runs} \times \text{Total number of matches}$$

$$\text{Total runs} = 200 \times 55 = 11000$$

Hence, **option c**.

TEST 5

43. Observe that in each graph, the total percentage of players playing the format is greater than 100 which implies that some players are contracted to more than one team. We see from the pie charts that highest degree of coincidence is in team E which is 20%.

Hence, **option d**.

44. As the number of players is the same in all 3 formats, we can directly compare percentages.

For T20,

$$A + B + C + D + E = 34 + 10 + 35 + 12 + 8 = 111$$

For ODI

$$A + B + C + D + E = 10 + 40 + 15 + 21 + 35$$

$$= 121$$

For Tests

$$A + B + C + D + E = 27 + 22 + 24 + 26 + 23$$

$$= 122$$

Since the number is greatest for tests, this format has maximum overlaps.

Hence, **option c**.

45. As compared to the average performance, Raul is highest in terms of percentage of positive deviation from average.

$$\text{For Ronaldo it is } \frac{1.5 - 3}{3} \times 100 = -50\%$$

$$\text{For Maradona it is } = \frac{2 - 2}{2} - 100 = 0\%$$

$$\text{For Raul it is } = \frac{2.5 - 1.5}{1.5} = +66.66\%$$

$$\text{For Roberto it is } = \frac{5 - 4}{5} = +20\%$$

Thus, Raul has done the best for his team.

Hence, **option c**.

46. Let the number of matches = x .

$$\text{For Team A number of goals} = 3x \times r \quad T \quad 12x$$

$$\text{For Team B number of goals} = 2x \times r \quad \text{Teamx}$$

For Team C number of goals = $1.5x \times r$ Teax
 For Team D number of goals = $4x \times r$ Teamx
 Goals scored by Ronaldo = $1.5x$
 Goals scored by Maradona = $2x$
 Goals scored by Raul = $2.5x$
 Goals scored by Roberto = $5x$
 \therefore Average of other players

$$= \frac{12x + 10x + (9 - 2.5)x + 16x}{[4 + 5 + (6 - 1) + 4]x}$$

$$= \frac{44.5x}{18x} = \frac{89}{36} = 2.47$$

which is less than 2.5.

Hence, **option a**.

47. 50% of A moves to B means that 22.5% of total moves to B.

Also $\frac{15}{4}\%$ also moves to B (from E).

$$\therefore B' \text{ s share} = 20\% + 22.5\% + \frac{15}{4}\%$$

$$= 46.25\%$$

Hence, **option c**.

48. Percentage change in the number of students of branch A

$$= \frac{(180 - 140)}{140} \times 100 = 28.57\%$$

Percentage change in the number of students of branch C

$$= \frac{(220 - 150)}{150} \times 100 = 46.67\%$$

Difference in percentage change =
 |% change of branch A - % change of branch C|
 = $|28.57 - 46.67| = 18.1$

Hence, **option d**.

49. Number of students in branch B = 280

Number of students in branch C = 150

$$\therefore \text{Percentage difference} = \frac{280 - 150}{150} \times 100$$

$$= 86.67\%$$

Hence, **option a**.

50. Since no base year is given, the percentage change for 2006 is taken with respect to the corresponding figure for 2005.

In 2006, number of students in branch B

$$= 125$$

In 2005, number of students in branch B

$$= 150$$

$$\therefore \text{Percentage change} = \frac{125 - 150}{150} \times 100$$

$$= 100.67$$

Hence, **option c**.

TEST 6

51. The average annual sales of company Q during the given period i.e. 1991 to 1996

$$= \frac{\text{Total sales for all six years}}{\text{Total number of years}}$$

$$= \frac{785 + 1040 + 980 + 1060 + 980 + 1200}{6}$$

$$= \text{Rs. } 1007.5 \text{ crores}$$

Hence, **option c**.

52. Sales of company T in 1995 = Rs. 460 crores

Sales of company P in 1992 = Rs. 900 crores

Since, 50% of 900 is 450, the answer has to be greater than 50%.

Hence, options 2 and 3 can be eliminated.

Hence, the sales of company T in 1995 as a percentage of the sales of company P in 1992 = $(460/900) \times 100 = 51\%$

Hence, **option a**.

53. The poorest tribe has 12% of wealth.

The richest tribe has 30% of wealth.

$$\therefore \text{Richest} - \text{Poorest} = 30\% - 12\% = 18\%$$

$$\therefore \text{The required solution is } \frac{18}{30} \times 100 = 60\%$$

Hence, **option b**.

54. % change in 3 star from 2009 to 2010

$$= \frac{34 - 32}{32} \times 100 = 6.67\%$$

This is also equivalent to $1/16$.

\therefore Increase in sales for Diary Milk

$$= 6.67\% \text{ of } 80$$

$$= \frac{1}{16} \times 80 = 5 \text{ units}$$

$$\therefore \text{Total expected sales} = 80 + 5 = 85 \text{ units.}$$

Hence, **option c**.

55. The ratios are:

Dapple	Damdung	Denq	Dony
$\frac{730}{795}$	$\frac{165}{230}$	$\frac{139}{204}$	$\frac{337}{902}$

Since the common difference between the numerator and denominator is 65 & all of them are proper fractions, "Dony" has the highest value.

Since, in proper fractions, when the difference between the numerator and denominator is the same, higher the value of numerator (and denominator), higher is the value of the fraction

Hence, **option d**.

Note: There is no need to calculate the value for Dhilips as it is not mentioned in the options.

56. Average profit of all companies

$$= \frac{A + B + C + D}{4}$$

$$= \frac{40 + 60 + 30 + 50}{4} = 45 \text{ crores}$$

Profit of company C = 30 crores

$$\therefore \text{Profit of company C} = 30 \frac{30}{45} \times 100$$

$$= \frac{2}{3} \times 100 = 66.67\%$$

Hence, **option a.**

57. The Average price/ Km

$$= \frac{3 \times 85 + 2 \times 60 + 4 \times 30 + 1 \times 40}{3 + 2 + 4 + 1}$$

$$= 53.5 \text{ per person/Km}$$

Hence, **option d.**

58. Total number of cars (in lakhs) sold in January = 47 + 59 + 74 + 85 = 265

Total number of cars (in lakhs) sold in March = 54 + 56 + 45 + 65 = 220

$$\therefore \text{Fall in sales} = 265 - 220 = 45$$

Hence, **option b.**

59. Number of engineering students from Maharashtra = 80 lakhs

Number of MBBS students from Kerala = 80 lakhs

Number of Law students from Gujarat = 80 lakhs

$$\therefore \text{Required ratio} = 80 : 80 : 80 = 1 : 1 : 1$$

Hence, **option c.**

60. Number of visitors (in lakhs) in 2010 = 199

Number of visitors (in lakhs) in 2011 = 169

Number of visitors (in lakhs) in 2012 = 139

Number of visitors (in lakhs) in 2013 = 149

Thus, 2010 saw the highest number of visitors.

Hence, **option a.**

61. Total number of Physics professors

$$= 32 + 8 = 40$$

Physics professors in the age-group 35 to 44

$$= 40\% \text{ of } 40 = 16$$

Hence, **option b.**

62. Note that apart from Chemistry, the number of males is more than the number of females for all the other subjects given in the options.

So, the ratio for Chemistry is less than 1 while that for the other 3 subjects (Physics, Mathematics and Economics) is more than 1.

Now, among these 3 subjects, the number of females is the same i.e. 8.

So, the subject with the highest number of males has the highest ratio.

Since Physics has the maximum number of males (32), it has the highest ratio of males to females.

Hence, **option a.**

63. Total number of Psychology professors

$$= 4 + 6 = 10$$

Number of female Psychology professors = 6

\therefore Percentage of female Psychology professors

$$= \frac{6}{10} \times 100 = 60\%$$

Hence, **option c.**

64. Total number of Physics professors

$$= 32 + 8 = 40$$

Number of Physics professors in the age group 25 - 34 = 30% of 40 = 12

Since 25% of professors in this group are female, the remaining 75% are male.

So, number of male Physics professors in the age group 25 - 34 = 75% of 12 = 9

Hence, **option a.**

65. Total number of Psychology professors

$$= 4 + 6 = 10$$

This is 2% of the total number of professors.

$$\therefore \text{Total no. of professors} = \frac{100}{2} \times 10 = 500$$

Hence, **option b.**

GRAPHS AND DIAGRAMS

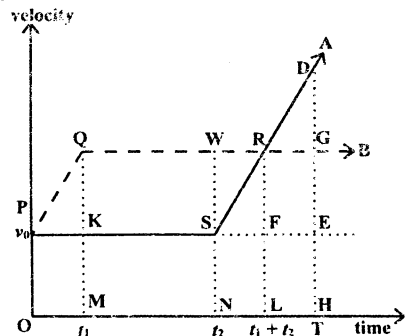
TEST 1

1. Curve 1 is a semi-circular curve whereas a rapid outbreak will be described by an exponential curve. Curve 3 never reaches zero and hence it doesn't show the eradication of the disease.

The curves 2 and 4 show an exponential increase in the number of patients suffering from the disease but curve 4 shows a linearly decreasing graph which is not ideal in this question.

Hence, **option b.**

2.



We can see that PQRS is a parallelogram.

By Newton's first law of motion, velocity of B at point Q and thereafter = $v_0 + a_0 t_1$

Let the two particles meet at time T .

Velocity of A at time $T = v_0 + a_0(T - t_2)$

At time T , distance travelled by A

$$= A(\text{OPSN}) + A(\text{RSNL}) + A(\text{RLHD})$$

At time T , distance travelled by B

$$= A(\text{PQRS}) + A(\text{OPSN}) + A(\text{RSNL}) + A(\text{RLHG})$$

Distance travelled by A = Distance travelled by B

$$\therefore A(\text{RLHD}) = A(\text{PQRS}) + A(\text{RLHG})$$

$$\therefore A(\text{RLHG}) + A(\Delta\text{RGD}) = A(\text{PQRS}) + A(\text{RLHG})$$

$$\therefore A(\Delta\text{RGD}) = A(\text{PQRS})$$

$$\therefore \frac{1}{2} \times \text{GD} \times \text{GD} = \text{PS} \times \text{QK}$$

$$\therefore \frac{1}{2} (T - t_1 - t_2) \times (v_0 + a_0(T - t_2) - v_0 - a_0 t_1)$$

$$= t_2 \times (v_0 + a_0 t_1 - v_0)$$

$$\therefore [T - (t_1 + t_2)]^2 = 2t_1 t_2$$

$$\therefore T = t_1 + t_2 + \sqrt{2t_1 t_2}$$

Hence, **option c**.

3. The distance between A and B at $t = t_1$ is equal to the difference between the areas under the two curves.

$$\text{This is equal to } \frac{v_1(t_B - t_A)}{2}.$$

So, option (a) is correct.

Relative velocity of a particle P with respect to another particle Q is equal to $(v_P - v_Q)$. So, relative velocity of A with respect to B at $t = t_B$ is equal to the velocity of A because the velocity of B at that instant is zero. Using similarity in triangles, we can determine that the velocity of A at this instant is

$$\frac{v_1(t_B - t_A)}{t_1 - t_A}$$

Hence, option (b) is incorrect.

A is ahead of B for t for t_1 as the area under the velocity-time graph of A is greater than that of B. So, option (c) is correct.

At $t = t_1$, the velocities of both A and B are equal. Hence, their relative velocities with respect to each other are zero. Option (d) is also correct.

Hence, **option b**.

4. The average velocity is nothing but total displacement divided by total time taken. Note that though the electron has travelled a distance that is greater than 10 nanometers its effective displacement is still 10

nanometers. The time taken is 2 seconds and hence its speed is 5 nanometers per second.

Hence, **option a**.

5. The distance travelled by the grain of sand is equal to the circumference of the circular rim. This is equal to $2\pi(2) = 4\pi$.

Hence, **option d**.

6. The time that the tip of the pendulum takes to travel from its mean position to one extreme is equal to the time that it takes to travel back to the mean position from the extreme. Also, the speed of the bob does not increase or decrease linearly in any of these directions. So option (a) and (d) are eliminated. Option (b) shows finite intervals where the speed remains constant and other finite intervals where the speed is zero. The bob of the pendulum has a zero speed only momentarily at the extreme ends. So option (c) is eliminated.

Hence, **option c**.

7. The concentration of the reactants decrease and that of the products increase in any chemical reaction. Only curve B satisfies this condition.

Hence, **option b**.

8. Pressure and Volume are inversely proportional to each other at a fixed temperature.

$$\therefore PV = \text{constant } (k)$$

Only option (d) shows an inverse relationship.

Hence, **option d**.

9. $PV = nRT$

Isothermal process: Temperature constant

$$\therefore PV = \text{constant}$$

\therefore Pressure and volume are inversely proportional

Isobaric process: Pressure constant

$$\therefore V \propto T$$

Isochoric process: Volume constant $\therefore P \propto T$

The first process is an isothermal expansion. Thus the volume increases in this process and the pressure decreases as they are inversely proportional to each other ($PV = \text{constant}$ at fixed temperature). If we follow the arrow, curve 4 shows that the pressure remains constant and curve 3 shows increase in volume as well as pressure and hence these two curves cannot be the correct answer. The second process is an isochoric one and the third one is an isobaric compression. In an isobaric compression, the

pressure remains constant and the volume decreases. As temperature is directly proportional to volume, the temperature also decreases. Hence to reach the original temperature T , there should be an increase in temperature in the second process (isochoric). Increase in temperature will take place with increase in pressure when the volume is constant as they are directly proportional to each other. Only curve 2 shows such a characteristic i.e. decrease in pressure followed by increase in pressure (and temperature) and then decrease in volume (and temperature) to reach the original temperature T .

Hence, **option b**.

10. At the time of refraction, the ratio of sine of angle of incidence (in the denser medium) to that of the angle of refraction (in the rarer medium), which is nothing but the slope of the line, will be equal to the ratio of the refractive index of the rarer medium to that of the denser medium = $1/1.5$

When the angle of incidence becomes fairly large and becomes larger than the critical angle, the light will be reflected and will never leave the denser medium. For reflection, the angle of incidence and angle of reflection are equal and the ratio of the sines of the angles (slope) is 1.

Only the graph in option (a) satisfies these conditions.

Hence, **option a**.

TEST 2

11. All the above given statements except for statement 4 is correct. As shown in the diagram, if water at critical point is depressurised while keeping the temperature constant, it will attain gaseous state and it won't solidify.
Hence, **option d**.
12. It can be seen that the potential energy of reaction A increases as the reaction progresses which means that it absorbs energy. Hence option (a) is true.
There is a trough in the middle of reaction B and hence a stable intermediate is formed there. The reaction C doesn't need any energy to commence and hence it is a spontaneous reaction.
Hence, **option d**.
13. We can see that in the 6th period the ionisation energy drops after Hg and then rises again upto Rn.
Thus, it is not true that across any period the ionisation energy keeps increasing as we move to the right.
The other three statements can be easily seen to be true from the graph.
Hence, **option d**.
14. The element in Group 17 and Period 2 has the highest electronegativity. This element is Fluorine (F). Thus option (c) is incorrect.
The other options can be verified to be true from the graph and the periodic table.
Hence, **option c**.
15. The curve shows us that the proportion of income spent on food decreases as income increases.
It does not mean that people earning higher incomes spend less on food. Thus options (a) and (b) cannot be inferred.
The curve does not tell us anything about saving patterns. Hence, option (d) cannot be inferred.
Option (c) talks about a higher proportion of income available for non-food spending among people with higher incomes, which can be directly inferred from the diagram.
Hence, **option c**.
16. The value of n will keep on increasing by 1 till n becomes divisible by 7.
At this point, it will be checked whether n is divisible by 13 or not.
If n is not divisible by both 7 and 13, n 's value will be increased by 1.
If n is divisible by 7 and 13, it will be checked whether n is divisible by 19 or not.
Thus the process will look for a number that is divisible by 7, 13 and 19.
The first such number is 1729.
The fourth such number will be 4×1729 .
The remainder obtained when 4×1729 is divided by 24 is 4.
Hence, **option b**.
17. $Q = Q_0 e^{-\frac{t}{RC}}$
We see that the charge on the capacitor is an exponentially decreasing function.
Hence, **option b**.
18. $Q = Q_0 e^{-\frac{t}{RC}}$
Consider option (a).
Let $Q = Q_0/2$

$$\therefore \frac{Q_0}{2} = Q_0 e^{-\frac{t}{RC}}$$

$$\therefore e^{\frac{t}{RC}} = 2$$

$$\therefore \frac{t}{RC} = \ln 2$$

$$\therefore t = RC \times \ln 2$$

Now, let R change to $2R$ and C change to $C/2$.

$$\frac{Q_0}{2} = Q_0 e^{-\frac{t}{2RC}} = Q_0 e^{-\frac{t}{RC}}$$

This leads us to $t = RC \times \ln 2$

\therefore The time required for the charge on the capacitor to halve to $Q_0/2$ is unchanged if R is doubled and C is halved. So option (a) is correct.

Consider option (b).

Suppose the charge on the capacitor is k at $t = t_1$.

$$\therefore Q_1 = Q_0 e^{-\frac{t_1}{RC}}$$

Let t_2 be the time required for this charge to halve to $(k/2)$.

$$\therefore \frac{k}{2} = Q_0 e^{-\frac{t_2}{RC}}$$

Solving this gives $(t_2 - t_1) = RC \ln 2$

Thus, the time required for the charge to halve is independent of the initial charge.

So option (b) is correct.

Consider option (c).

If the capacitor discharges completely, $Q = 0$

$$\therefore = Q_0 e^{-\frac{t_1}{RC}}$$

$$\therefore Q_0 \neq 0, e^{-\frac{t_1}{RC}} = 0$$

$$\therefore -\frac{t_1}{RC} = \ln 0 = n$$

\therefore Theoretically, it takes infinite time for the capacitor to discharge completely.

So option (c) is correct.

Option (d) can be easily proven to be incorrect.

Hence, **option d.**

19. As age increases, the population steadily decreases. This means that average life expectancy is low and death rate is high. Thus option (a) and option (b) are correct. Hence, **option c.**
20. As, F_t is an exponential function of t , we can easily rule out the last two options. Now, we can easily observe that, $a \times e^b > a$ if $b > 0$. Now, for the given period, r_t is always positive.

Hence, F_t will always be greater than P .

Hence, we can eliminate 2nd option also.

Hence, **option a.**

21. It is important to note here that the earth and the moon are not in the same plane. Also as the distance between the two bodies is very large, the persons at B and H will both see the waxing crescent of the moon. Hence, **option c.**

22. A sees a new moon. So the moon is between the earth and the sun. When the moon is in such a position, E will not see the moon at all and will hence experience a new moon too. Hence, **option b.**

23. The waxing crescent is visible to people only on the part of earth experiencing day time and to people who are just experiencing the time after sunset. So it can never be seen during night time. Hence, **option b.**

24. According to law of reflection, the angle made by the incident ray and the reflected ray with the normal (the line that is perpendicular to the surface of the mirror) is equal.

It is given that the source is at an infinite distance from the mirror and the rays after reflection converge to meet at a point.

This phenomenon takes place in a concave mirror (a curved portion of a spherical glass in which the inside portion acts as the mirror).

Hence, **option c.**

25. If time is constant, distance is directly proportional to speed.

So, in a fixed time, distance covered by M1, M2 and M3 is also in the ratio 1 : 2 : 3.

The distance covered w.r.t time is given by a straight line. Also, the gap between the distance covered by M2 and M1 keeps increasing with time. The same logic applies to M3 and M2.

This relationship is correctly shown in figure (II).

Hence, **option b.**

26. The number of men working is inversely proportional to the number of days required to complete the job. So, if x men working at constant speed finish some work in y days, the equation of work is of the form $xy = \text{constant}$.

This is represented by a rectangular hyperbola as shown in diagram IV.

Hence, **option d.**